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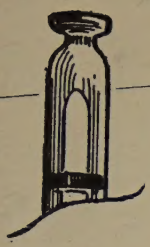

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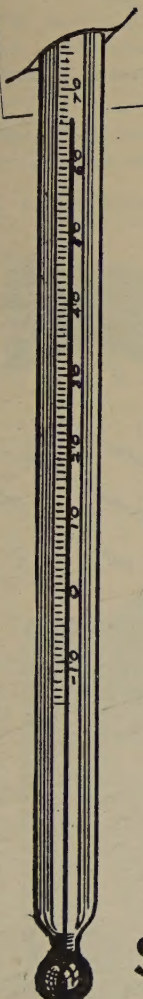

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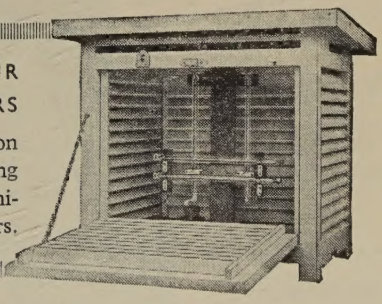

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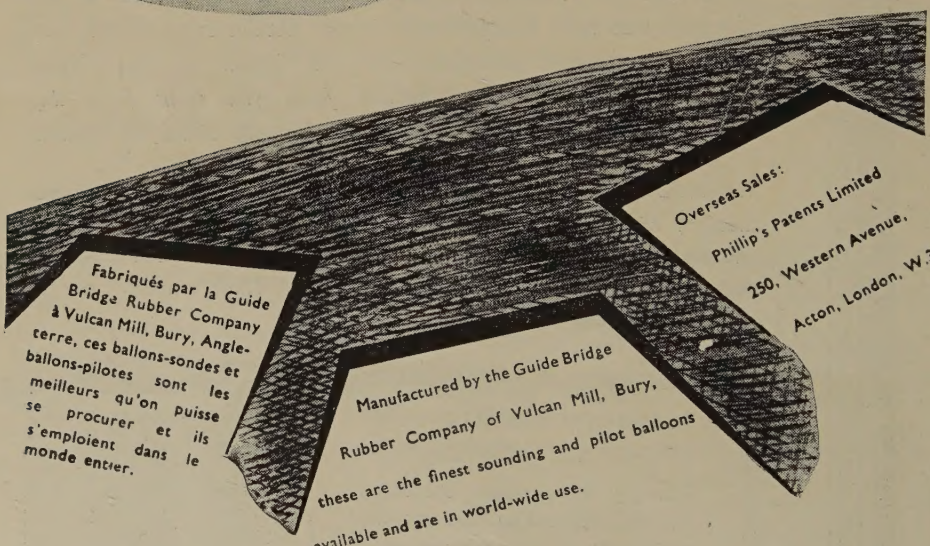

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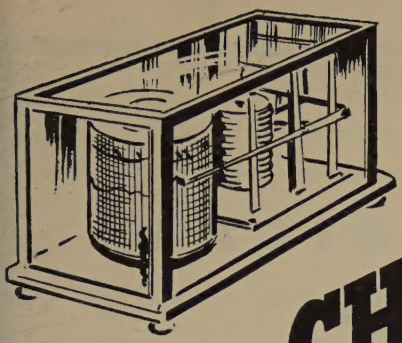

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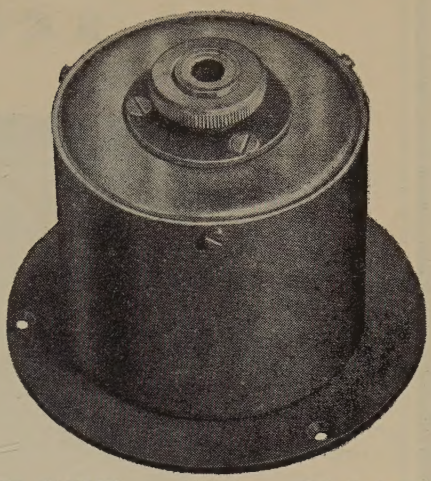

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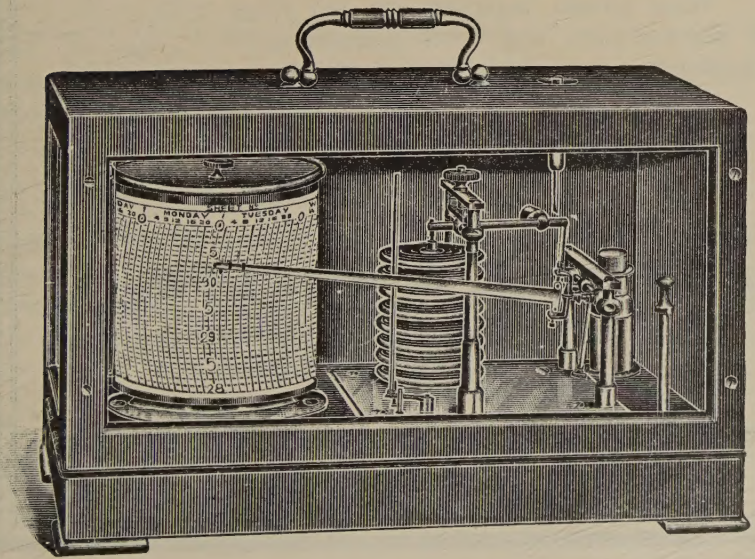

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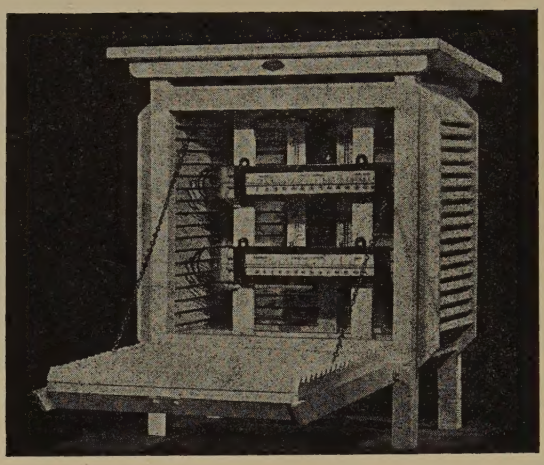

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Some observations of air flow over the sea

By J. S. HAY

Chemical Defence Experimental Establishment, Ministry of Supply, Porton, nr. Salisbury.

(Communicated by C. J. M. Aanensen. Manuscript received 21 October 1954, in revised form 4 March 1955)

SUMMARY

Wind speeds were measured over the sea at five heights up to 8 m. Though the fetch was limited, the wind profiles obtained are considered representative of flow over the sea.

The wind speed varies logarithmically with height when the temperature gradient is small. Assuming that laboratory laws of flow are applicable, values of the roughness parameter z_0 are deduced. It is found that z_0 increases from about 0·02 cm in a wind of 5 m/sec to 0·25 cm at 9 m/sec. The relationship between wind speed and z_0 however is not unique for the state of the surface is determined by other factors beside wind speed. In inversion conditions the wind-speed distribution departs from the logarithmic form a few metres above the surface. The results obtained are compared with those of some other workers.

Bi-directional vanes were used to obtain gustiness measurements at the site. While the vertical gustiness is found to be of similar magnitude to that over land surfaces of comparable roughness, the cross-wind gustiness appears to be greater over the sea.

1. INTRODUCTION

From laboratory studies of turbulent boundary layers it has been established that, in the case of aerodynamically smooth flow, the mean wind speed u varies with height z above the surface according to the law

$$\frac{u}{u_*} = 5{\cdot}5 + 5{\cdot}75 \log_{10} \frac{u_* z}{\nu}, \quad z > \frac{30\,\nu}{u_*} \qquad (1)$$

where $u_* = \sqrt{\frac{\tau_0}{\rho}}$ is the 'shearing stress' velocity, τ_0 is the shear stress per unit area of surface and ν and ρ are the kinematic viscosity and density of the fluid. In the case of aerodynamically rough flow, we have

$$\frac{u}{u_*} = 5{\cdot}75 \log_{10} \frac{z}{z_0}, \quad z > z_0 \qquad (2)$$

where z_0 is the roughness parameter of the surface. In the regime between these two types, the flow is said to be transitional. The type of flow prevailing in any given set of circumstances was found by Nikuradse to depend on the following criteria :

$$\left.\begin{array}{ll} \text{aerodynamically rough flow} & \dfrac{u_* z_0}{\nu} > 2{\cdot}5 \\ \text{transitional flow} & 2{\cdot}5 > \dfrac{u_* z_0}{\nu} > 0{\cdot}13 \\ \text{aerodynamically smooth flow} & 0{\cdot}13 > \dfrac{u_* z_0}{\nu} \end{array}\right\} \qquad (3)$$

Wind profiles over land surfaces have been studied extensively, notably by Deacon (1953). In particular, it may be noted that, in neutral stability conditions, the wind-speed distribution can generally be expressed by Eq. (2). Over water, similar investigations have been less numerous, with inconsistent results, partly due to increased experimental difficulties. The intimate relationship between wind profile, and surface stress, and the resulting surface currents makes such investigations of interest to oceanography. Results are also of value in meteorological studies of diffusion over water.

2. The site

The observations were made on a large floating platform which was moored in 25 m of water about 800 m east of a coastline in the United Kingdom which ran roughly north-south. Tidal currents were about 2 knots at the most. The shore was mainly cliff, rising to 60-90 m, though south-west from the platform was a strip of sandy beach. Inland from the cliffs, the ground was fairly level for several miles.

The platform was rectangular in form, with its longer axis parallel to the shore. It was listed slightly so that the edge nearer to the shore was at mean sea level. The surface of the platform sloped upward at an angle of about 6°. Observations were only made in off-shore winds, between SW. and NW. The platform could be swung within these limits to bring the low edge perpendicular to the mean wind direction.

The anemometer mast was erected close to the low edge of the platform, so that the anemometers were effectively over the sea. They were set up at heights of 0·5, 1, 2, 4 and 8 m above the edge of the platform. Because of its great size, any rolling or pitching of the platform was slight and rather slow, so the anemometer mast remained nearly vertical and the speeds indicated by the anemometers did not include any significant component due to lateral movement of the anemometers themselves.

Owing to wave movement and swell, the instantaneous heights of the anemometers above the sea surface differed from those quoted above by as much as $\pm 0{\cdot}3$ m and, over a period of a few minutes, this resulted in a possible error in average height of the anemometers of 5 to 10 cm. The effect of such errors will be discussed later.

One unfortunate result of using a floating platform was that additional eddies must have been created when the water level retreated down the side of the platform, exposing momentarily a bluff obstacle to the wind. The eddies, however, did not appear to affect the profiles.

3. Instrumental

The anemometers employed were sensitive electric-contact anemometers similar to the type described by Crawford (1951). They were calibrated in a wind tunnel in the April prior to these observations and again in the following October. The later calibrations were found to be within 5 cm/sec of the original, so errors due to a change in calibration are negligible. The anemometers were connected to a bank of electric counters (telephone-message registers), the readings of which were recorded photographically every 30 sec. The exposure occupied 1 sec during which the counters were stopped. In the 29-sec interval between exposures the counters operated normally. On 16 Sept. this apparatus was replaced by a bank of counters which were switched manually and read by eye, readings being noted every 4 min.

This paper is mainly concerned with wind speeds meaned over a period of 4 min, either of continuous sampling or from eight 29-sec intervals. In work of this nature, such a period is usually accepted as being sufficiently long to smooth out the effects of short-period eddies and yet not long enough to mask the effects of large-scale turbulence, which may be regarded as changes in mean wind speed and direction. The errors in the derived speeds, due to the fact that the counters did not run continuously for the 4 min in the photographic records, were found to be negligible, amounting to 0·5 per cent at at the most in a few cases. To make fullest use of the available data, it was necessary on a few occasions to take means over other periods, usually 3·5 min.

4. Discussion of results

Air temperature, at about 1 m, and sea temperature were measured at intervals during each series of observations to give air-sea temperature differences which will be regarded as measures of stability in the lowest layers.

The conditions experienced during the observations are given in Table 1.

TABLE 1. Summary of conditions during wind-profile observations

Date 1952	Series of obsns	Total period of obsns (min)	Range of air-sea temperature differences (°F)	Stability category	Wind direction	Fetch (m)	Swell	Wave height (m)	Weather
30 July	A	40·5	+ 2·3 to + 3·5	Inversion	SW. to WSW.	1200	NNE. about 1 m	0·3-0·5	Cloudy
31 July	B	28·5	+ 1·1 to − 0·8	Isothermal	SW. to WSW.	1200	NNE. 0·3 m or less	0·3-0·4	Partly cloudy, becoming cloudy
31 July	C	30·0	− 0·3 to + 1·2	Isothermal	SW.	1200	NNE. 0·3 m or less	0·4	Partly cloudy
25 Aug.	E	55·5	− 0·6 to − 1·9	Lapse	NW.	800	N. 1 m	0·2	Cloudy
16 Sept.	D	44·0	0 to + 1·0	Isothermal	SW. to W.	1200	Nil	0·4-0·5	Cloudy

In general, temperature differences over a moderate range of height in the air are known to be smaller than the air-sea temperature difference. In view of the lack of observations of the former and the restricted range of the latter, there would seem to be little justification in discriminating thermal stability beyond a grouping of the observations into the broad categories inversion, zero temperature gradient (isothermal) and lapse. The appropriate classifications are given in Table 1.

On 31 July the observations were interrupted by a shower and have been split into two groups, series B before and series C after the shower. On 16 Sept. four observations were made at intervals in the morning while the remaining seven observations were made over successive periods in the afternoon. Conditions were almost the same during the morning and afternoon of that day.

The observations will now be considered under each stability heading, beginning with ' isothermal,' for wind profiles then assume their simplest form.

(*a*) *Zero temperature gradient*

The ratio of the speed at each height to that at 1 m was found. $R_{0\cdot5}$, R_4, etc. will be used to denote the ratios $u_{0\cdot5}/u_1$, u_4/u_1, etc. where $u_{0\cdot5}$ = wind speed at 0·5 m, etc. In series C and D, no systematic variation in any of the ratios with time or wind speed was evident. In series B however there was seen to be a definite drop in $R_{0\cdot5}$ with time, with a slight tendency for R_8 to rise. These changes were associated with a slow but sustained increase in wind speed. The observations of this series were therefore split into two groups, B_1 consisting of the observations at the lower wind speeds and B_2 those at the higher wind speeds.

The mean ratios are given in Table 2 and are shown plotted in Fig. 1 against $\log_{10} z$. It will be seen that the plot is in each case approximately linear, so the lines

$$\frac{u}{u_1} = a \log_{10} z + b \quad . \quad . \quad . \quad . \quad (4)$$

giving the best fit with the observed data were derived by the method of least squares. They have been drawn in Fig. 1. Wind-speed ratios calculated from these equations with appropriate values of z are included in Table 2, together with the discrepancies between observed and calculated ratios. From the excellent agreement, there seems to be little doubt that on these occasions the profiles were logarithmic from 0·5 to 8 m.

While each ratio varied little in each series, it is clear that there is a variation from one series to another. From Table 2 and Fig. 1 it will be seen that the gradient becomes steeper as the wind speed increases. This must be due to increased drag at the surface, i.e. an increase in surface roughness.

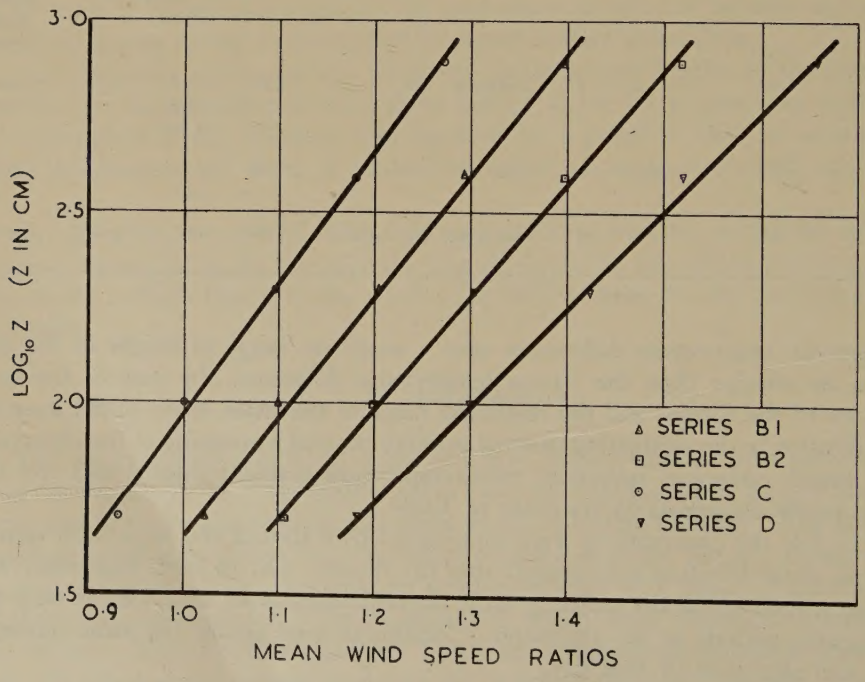

Figure 1. Profiles of mean wind-speed ratios in zero temperature-gradient conditions. The wind-speed ratio scale is shown for the left-hand profile. The other profiles are displaced to the right in steps of 0·1.

TABLE 2. MEAN WIND-SPEED RATIOS WITH ZERO TEMPERATURE GRADIENT

Series of obsns	No. of obsns	Mean and, in brackets, range of 2-m wind speeds (cm/sec)		Mean wind-speed ratios $R_{0.5}$	R_1	R_2	R_4	R_8	z_0 cm	u_* cm/sec
B1	4	557	Observed	0·923	1·000	1·105	1·193	1·297		
			Calculated	0·916	1·010	1·104	1·198	1·292	0·059	27·4
		(520-596)	Obs-Cal %	+ 0·8	− 1·0	+ 0·1	− 0·4	+ 0·4		
B2	4	656	Observed	0·907	1·000	1·103	1·197	1·317		
			Calculated	0·901	1·003	1·104	1·206	1·308	0·107	35·0
		(628-689)	Obs-Cal %	+0·7	− 0·3	− 0·1	− 0·8	+ 0·7		
C	7	531	Observed	0·931	1·000	1·097	1·181	1·270		
			Calculated	0·923	1·009	1·095	1·180	1·266	0·029	24·0
		(498-567)	Obs-Cal %	+ 0·8	− 0·9	+ 0·2	+ 0·1	+ 0·3		
D	11	920	Observed	0·882	1·000	1·123	1·218	1·354		
			Calculated	0·883	0·999	1·115	1·231	1·348	0·257	55·0
		(799-1038)	Cal-Obs %	− 0·1	+ 0·1	+ 0·7	− 1·1	+ 0·7		

Assuming Eqs. (1) and (2) to be applicable to flow over the sea, we may now consider the type of flow on these occasions. It is most likely to have been aerodynamically smooth during series C, when the wind speed was lowest. Substituting the mean value of u_2, 2 m being the logarithmic mean height, and the value of ν in Eq. (1), u_* was determined and the appropriate law found to be

$$\frac{u}{17{\cdot}3} = 5{\cdot}5 + 5{\cdot}75 \log_{10} \frac{17{\cdot}3\, z}{0{\cdot}145}$$

Wind-speed ratios may be calculated from this equation as follows :

	$R_{0.5}$	R_1	R_2	R_4	R_8
Observed	0·931	1·000	1·097	1·181	1·270
Calculated	0·940	1·000	1·060	1·120	1·178

From the poor agreement between observed and calculated ratios, it appears safe to conclude that the flow was not smooth over the periods covered by this or the other series.

Taking the flow to be aerodynamically rough during each series, it will be seen from Eqs. (2) and (4) that appropriate values of u_* and z_0 may be readily derived from the constants a and b and the mean value of u_1. The minimum value of the parameter $\frac{u_* z_0}{\nu}$

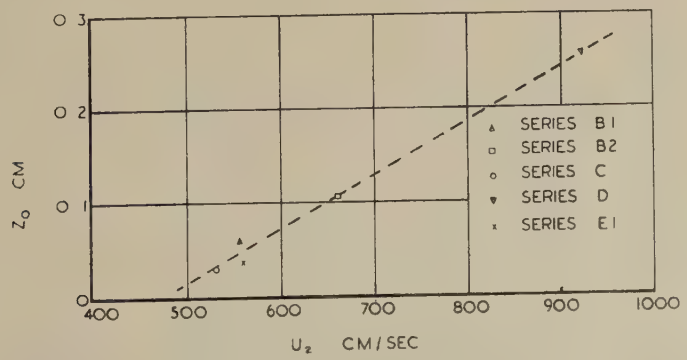

Figure 2. Variation of the roughness parameter with wind speed.

was found to be 4·8, so the criteria (3) indicate that the flow was in fact rough on all occasions. Values of u_* and z_0 are given in Table 1 and z_0 plotted against mean u_2 in Fig. 2. In the latter, the dashed line has been drawn roughly to indicate the trend of increasing surface roughness with increasing wind speed. The figures for B1 and B2 are of special interest for they show that the surface was becoming rougher during the course of the observations.

(*b*) *Lapse*

The observations in series E, when there was a lapse of temperature, will now be considered. Unfortunately the operation of the recording apparatus was very poor on this occasion and the available data are thus rather limited. From the readings obtained a few mean speeds over variable periods were derived, the periods being as follows : 4·5, 21, 4, 4, 3·5 and 18·5 min.

On deriving wind-speed ratios for each period, a tendency for $R_{0·5}$ to increase and for R_8 to decrease with time was apparent. As with series B, it seemed inadvisable to take mean ratios for the whole series. A division of the observations with respect to wind speed however did not seem appropriate, for the wind-speed variations were very irregular. Instead, the observations were grouped according to time, the first three forming series E1 and the last three series E2. For each of these sub-groups, the mean wind-speed ratios were derived for each height, the individual ratios being weighted according to the length of time over which they were measured. The mean wind-speed ratios are given in Table 3 and are shown plotted against $\log_{10} z$ in Fig. 3. It will be seen that both plots are almost linear; the lines of best fit are drawn in Fig. 3. Calculated ratios and discrepancies from observed ratios are included in Table 3. From the close agreement, it may be concluded that under the lapse conditions experienced on this occasion, the profiles were logarithmic from 0·5 to 8 m.

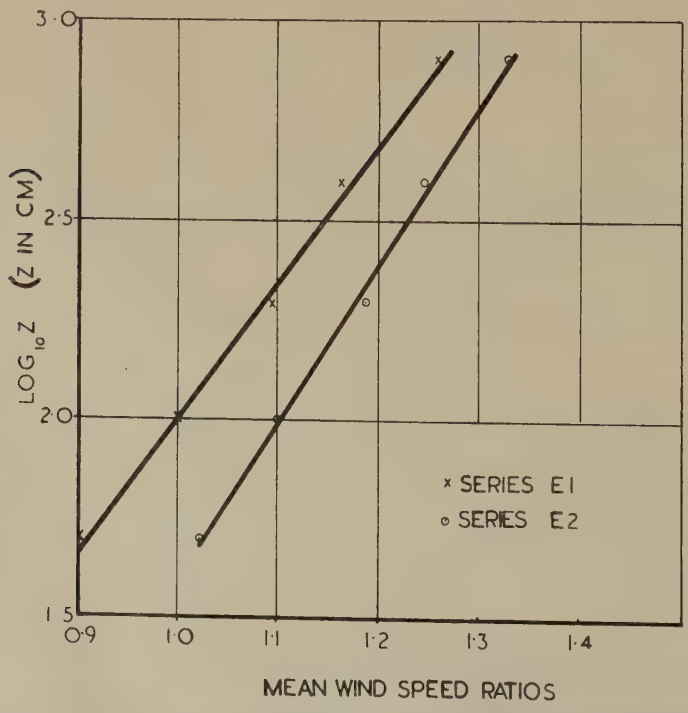

Figure 3. Profiles of mean wind-speed ratios in lapse conditions. The wind-speed ratio scale for the right-hand profile is displaced 0·1 to right.

TABLE 3. MEAN WIND-SPEED RATIOS FOR SERIES E

Series of obsns	No. of obsns	Mean and, in brackets, range of 2-m wind speeds cm/sec		$R_{0.8}$	Mean wind-speed ratios R_1	R_2	R_4	R_8	z_0 cm	u_* cm/sec
E1	3	559	Observed	0·902	1·000	1·094	1·162	1·257		
			Calculated	0·910	0·997	1·084	1·172	1·259	0·036	25·8
		(521-677)	Obs-Cal %	− 0·9	+ 0·3	+ 0·9	− 0·9	− 0·2		
E2	3	581	Observed	0·923	1·000	1·086	1·146	1·228		
			Calculated	0·925	1·001	1·077	1·152	1·228	—	—
		(519-605)	Obs-Cal %	− 0·2	− 0·1	+ 0·8	− 0·5	0		

From Table 3 it will be seen that for series E2 the ratio R_8 is smaller than that for series C, so there is a stronger possibility of the flow having been smooth. When tested in the same manner as before, however, the possibility had to be dismissed. Assuming now that the flow was rough, it was found that $z_0 = 0{\cdot}010$ cm and $\frac{u_* z_0}{\nu} = 1{\cdot}6$. According to the criteria (3) then, the flow was not rough but rather transitional. The z_0 value quoted is thus not applicable. In the case of series E1, $\frac{u_* z_0}{\nu} = 6{\cdot}4$ so that the flow was rough. The surface was thus tending to become smoother as time passed, a change which could not be attributed to wind-speed variations. Moreover the waves and swell did not appear to change appreciably during the course of the observations. As no tidal observations were made, it is not possible to say if this had any effect.

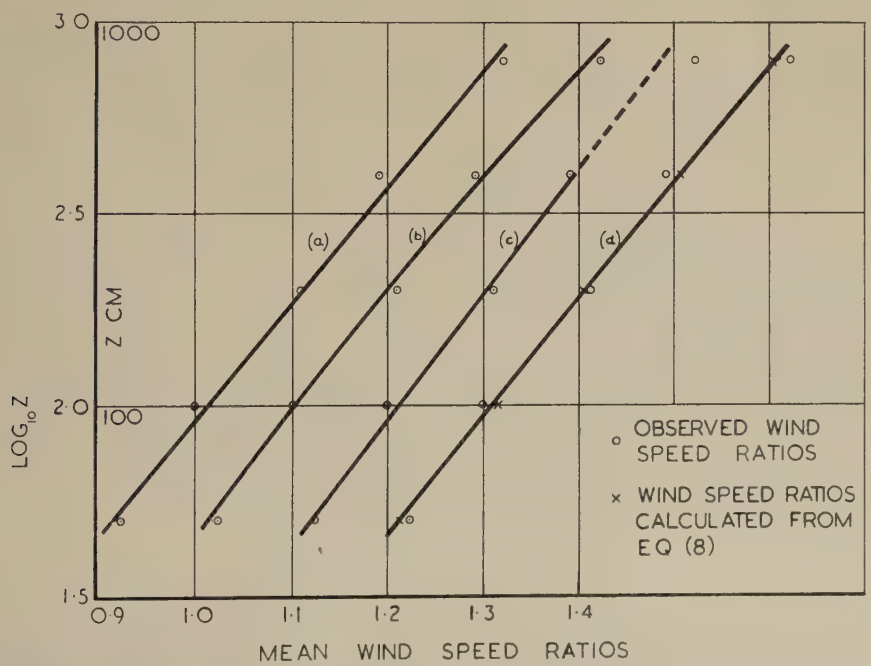

Figure 4. Profiles of mean wind-speed ratios in inversion conditions. The wind-speed ratio scale is shown for the left-hand profile. The other profiles are displaced to the right in steps of 0·1.

During these observations, the sea track was the shortest possible, about 800 m. It has been seen, however, that the mean profiles were logarithmic from 0·5 to 8 m without significant deviations. This suggests that the flow up to 8 m at the platform was fully determined by passage over the sea and not influenced by previous travel over land and abrupt descent to sea level at the cliff face.

In view of the logarithmic wind-speed distribution we may assume that the stability conditions for this series were effectively the same as for the series already considered, i.e. close to isothermal conditions. The z_0 value for E1 has thus been added to Fig. 2.

(c) *Inversion*

Ten observations in series A now remain to be discussed. There was no systematic variation in any of the ratios so the mean ratio was derived for each height. These are given in Table 4 and are shown plotted against $\log_{10} z$ in Fig. 4.

TABLE 4. MEAN WIND-SPEED RATIOS FOR SERIES A.

	Mean wind-speed ratios					
	$R_{0.5}$	R_1	R_2	R_4	R_8	
Observed	0·923	1·000	1·110	1·191	1·322	
Calculated by Eq. (5)	0·912	1·011	1·110	1·209	1·308	Mean u_4
Obs-Cal %	+ 1·2	− 1·1	0·0	− 1·5	+ 1·1	579 cm/sec
Calculated by Eq. (7)	0·919	1·011	1·103	1·194	1·286	Range of u_8
Obs-Cal %	+ 0·4	− 1·1	+ 0·6	− 0·3	+ 2·7	530-637 cm/sec
Calculated by Eq. (8)	0·912	1·017	1·104	1·208	1·304	
Obs-Cal %	+ 1·2	− 1·7	+ 0·5	− 1·4	+ 1·4	

Following the same technique as before, a logarithmic law was fitted to these data and was found to be

$$\frac{u}{u_1} = 0{\cdot}329 \log_{10} z + 0{\cdot}353 \qquad (5)$$

giving a value $z_0 = 0{\cdot}085$ cm

This is drawn in Fig. 4 (a) and ratios calculated from it are included in Table 4 together with discrepancies from observed ratios. It will be seen that the overall agreement is rather poorer than for any other series, though the maximum discrepancy is still only 1·5 per cent.

In Fig. 4 (b), a curve which has been fitted to the points by eye appears to give a closer fit. This is concave to the wind-speed ratio axis, similar to findings over land in inversion conditions. We may enquire then if a distribution law of other than logarithmic form would fit the observed data better, such as that proposed by Deacon (1953), namely

$$\frac{u}{u_*} = \frac{2{\cdot}5}{(1-\beta)} \left[\left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right], \qquad z > z_0 \qquad (6)$$

Here β takes values < 1 and > 1 in stable and unstable conditions respectively and $\beta \to 1$ as conditions approach neutral, Eq. (6) then reducing to Eq. (2). A mean value of β, which varies with height, is determined appropriate to the layer being considered. The value of z_0 in Eq. (6) is that obtained over the same surface under neutral conditions. In this

instance, however, we are faced with the difficulty that a particular value of z_0 cannot be assumed for it is such a variable quantity. One possibility is to assume the value quoted above but as Eq. (6) tends to Eq. (2) with decreasing z, it would appear desirable to give more weight to the lower heights in its evaluation. It was found that a log law

$$\frac{u}{u_1} = 0{\cdot}304 \log_{10} z + 0{\cdot}403 \quad . \quad . \quad . \quad . \quad (7)$$

would give a good fit up to 4 m as may be seen from Table 4 and Fig. 4 (c) but on extrapolating to 8 m, there is a discrepancy of 2·7 per cent from the observed ratio. A value of 0·057 cm. for z_0 was derived, in reasonable accord with figures already obtained. Using this and the values of $R_{0{\cdot}5}$ and R_8, a Deacon type curve was obtained in a slightly modified form of Eq. (6) as

$$\frac{u}{u_1} = \frac{2{\cdot}5\, u_*}{u_1(1-\beta)}\left[\left(\frac{z}{z_0}\right)^{1-\beta} - 1\right] = \frac{2{\cdot}5 \times 26{\cdot}0}{522 \times (1 - 0{\cdot}985)}\left[\left(\frac{z}{0{\cdot}057}\right)^{1-0{\cdot}985} - 1\right] \quad . \quad (8)$$

This is drawn in Fig. 4 (d) and ratios calculated from it are included in Table 4. It will be seen that discrepancies from the observed ratios are in fact greater than those obtained using the log law. There is thus little to commend the application of the Deacon law to this set of data, using a value of z_0 as derived above. If the ratios R_1 and R_4 are used to determine the constants in Eq. (6) instead of $R_{0{\cdot}5}$ and R_8, the agreement achieved is of the same order.

General conclusions regarding inversion conditions can scarcely be drawn from one set of observations, apart possibly from saying that the wind speed varies with the logarithm of the height close to the surface, but further away appears to increase more rapidly than this.

5. Effect of errors in height measurements

It was stated earlier that the anemometers were at specific heights above the low edge of the platform, which was at mean sea level, this being midway between the wave crests and troughs. As the platform was being listed each day, the correct attitude was estimated by eye. With waves breaking over the edge of the platform, it is obvious that this could not be judged precisely. The specified heights might thus have been in error on some days, though it is unlikely that the errors exceeded a matter of 5 to 10 cm.

To investigate the effects of such errors, the data in series C and D were used for they cover the extreme wind speeds experienced. Assuming various height errors d, the lines of best fit relating $\frac{u}{u_1}$ and $\log(z - d)$ were determined. Ratios calculated from them were mainly within 1 per cent of the observed ratios, 1·4 per cent in an extreme case, so that any tendency to curvature by adjusting height was not marked. Derived z_0 values are as follows (all values in cm) :

d		-10	-5	0	$+5$	$+10$
z_0	Series C	0·053	0·040	0·029	0·020	0·014
	Series D	0·410	0·329	0·257	0·193	0·147

From these figures it would appear that the z_0 values already quoted are at least correct to within a factor of about 2. It is also seen that the height errors are insufficient to explain the increase in z_0 from series C to series D, which must be due largely to the increased wind speed.

It might be considered that a significant curvature of the profiles in lapse conditions, series E, was obscured by underestimation of height. Allowing for a height error $d = -10$ cm, a logarithmic profile is still obtained in the case of series E2, but with series E1 there is some evidence of a curvature concave to the height axis. There might be some doubt then about the z_0 value quoted for this series. It has, however, been considered worthy of retention in Fig. 2 in view of the consistency with the other points.

In the same way, an underestimation of height might account for the curvature in inversion conditions, series A. To test this, lines of best fit relating $\frac{u}{u_1}$ and $\log(z - d)$ for values of d from -5 to -50 cm were determined. Comparing ratios calculated from them with observed ratios, it was found that the closest agreement was obtained with $d = -35$ cm. A height error of this order was scarcely possible so the curvature does appear genuine.

6. Comparison with the results of other workers

A comparison of the findings presented in this paper with the results of other workers is not without interest. Rossby and Montgomery (1936) concluded that the air flow is aerodynamically rough, with $z_0 = 0{\cdot}6$ cm, in winds exceeding about 7 m/sec, and transitional or aerodynamically smooth at lesser speeds. In the present work it has been found that the flow may be rough with wind speeds as low as 5 to 6 m/sec at 2 m. Fig. 2, despite the limited data, shows no abrupt change in surface roughness around 6 to 7 m/sec, but rather a roughly linear variation of z_0 with wind speed, at least up to about 9 m/sec. The maximum value of z_0 observed was 0·26 cm. To have approached the 0·6 cm of these authors would have required an error of more than 10 cm in estimating the mean level of the sea and this is considered unlikely. There is also no support for Model (1942) who, from an analysis of data obtained by Bruch, suggested that the flow is always rough, with z_0 independent of wind speed and having a value of about 0·07 cm. Deacon (1953), using data obtained by Johnson, quotes a value of 0·02 to 0·04 cm for z_0 in winds averaging 10 m/sec at 21·3 m, roughly 8 m/sec at 2 m. This is rather lower than the value suggested by Fig. 2.

Instead of z_0, a related quantity, the shear-stress coefficient, is frequently discussed in the literature. It is defined by

$$C = \frac{\tau_0}{\rho u^2}$$

where u is usually taken as the mean wind speed at 10 m. Values of C derived from the various series are plotted against u_{10} in Fig. 5. The meagre data suggest a linear variation,

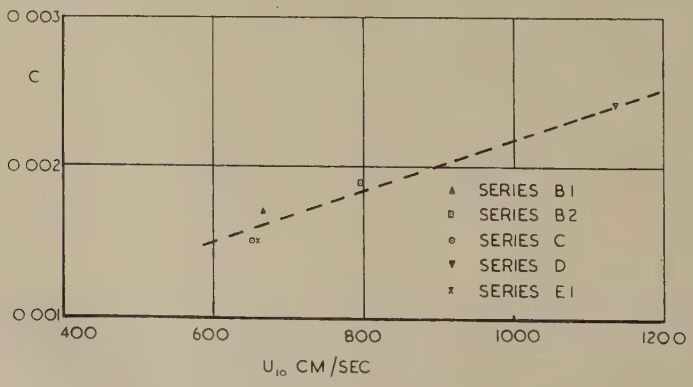

Figure 5. Variation of the shear-stress coefficient C with wind speed.

at least in the range 6 to 12 m/sec, as indicated roughly by the dashed line. The maximum value of C is 0·0024 (limiting values 0·0021 and 0·0026 if a height error of $\pm$ 10 cm is assumed), comparable with the figure 0·0026 quoted by Munk (1947) for $u_{10} > 8$ m/sec. At lesser speeds, however, Munk gives $C = 0{\cdot}0008$ whereas Fig. 5 shows $C = 0{\cdot}0015$ at about 6 m/sec (limiting values 0·0017 and 0·0013), provided the flow is aerodynamically rough. Fig. 5 is also not in agreement with results obtained over short sea tracks by Roll (1952).

Objection to the above comparison may be raised on the grounds that the fetch in these experiments was rather limited. While it was always long enough for the profiles up to 8 m to be fully representative of the flow over the sea, it is quite possible that the waves formed were not in equilibrium with the wind. These results would thus only be applicable to this particular site. On the other hand, it is now accepted by many workers that the roughness characteristics of the sea surface are determined by the ripples superimposed on the fundamental waves rather than by the waves themselves. In this case the results presented here would be of more general application.

7. Gustiness over the sea

Some gustiness measurements were also made, independently of the profile measurements, using two Sheppard pattern bi-directional vanes. They were set up close to the low edge of the platform at heights of 1 and 2 m. Traces over periods of 5 min were obtained while wind speeds at 2 m and air-sea temperature differences were noted simultaneously. Only 19 traces at each height were obtained so any conclusions drawn will necessarily be rather tentative.

The method of analysis of the traces was that described by Best (1935). The smallest oval was drawn which included most of the trace, not more than 15 loops being allowed outside the oval. From the diameters of the oval and the dimensions of the instrument, the gustiness values g_y and g_z were derived. According to Best, the lateral gustiness, g_y, obtained in this way is the ratio with respect to the mean wind speed of the ' mean extreme ' eddy velocity in the horizontal at right angles to the mean wind direction. g_z is defined similarly for the vertical direction.

(*a*) *Variation of gustiness with wind speed and stability*

The respective gustiness values have been plotted against wind speed at 2 m in Fig. 6, the points being distinguished according to air-sea temperature difference. The scatter in the points is quite appreciable, particularly in the case of the g_y values which range through some 30 per cent on either side of the mean. This feature is also observed over land and is due to the variable amount of large-scale eddying. The values of g_z range through about 15 per cent on either side of the mean. No systematic variation of g_y or g_z with wind speed or stability is evident. This is consistent with the fact that the wind-speed profiles are logarithmic at these heights in the range of wind speed and stability conditions covered.

Over a surface of constant roughness, Deacon (1953) found that g_y and g_z were approximately independent of the wind speed under neutral conditions. They did, however, vary with surface roughness. In the present investigation, the mean wind speed at 2 m ranged from 2·6 to 6·7 m/sec, so that z_0 probably varied from 0·01 or less to 0·1 cm, according to Fig. 2. Within this very limited range of surface roughness, then, the data presented here show that g_y and g_z are independent of z_0 to within the accuracy of the experiments. It is interesting to compare the results obtained here with those obtained

over land surfaces. Mean values of g_z and g_y/g_z at 2 m and their standard errors are given as follows, together with values over snow and desert surfaces, as quoted by Deacon in his Table 7.

	z_0 (cm)	g_z (rad)	g_y/g_z	u_z m/sec
Sea	0·01 — 0·1	0·199 ± 0·004	2·22 ± 0·11	2·6 — 6·7
Natural snow surface	0·1	0·205	1·37	—
Desert near Karachi	0·03	0·190	1·33	3 — 7

The agreement of the g_z values is quite striking. It shows that the flow up to 2 m at least was fully characteristic of the sea surface and not influenced by previous passage over land. The lateral gustiness, however, appears to be greater over the sea for the particular site, as is seen from the greater ratio g_y/g_z. Two possible explanations come to mind. Firstly, the extreme lateral components of eddy velocity at the platform may be remnants of large-scale eddies created at the cliffs. If this was the case, a decrease of g_y might be expected with increasing fetch. In this investigation the fetch only varied from 800 to 1,200 m, and it is doubtful if this range is sufficient to show an effect. No dependence of

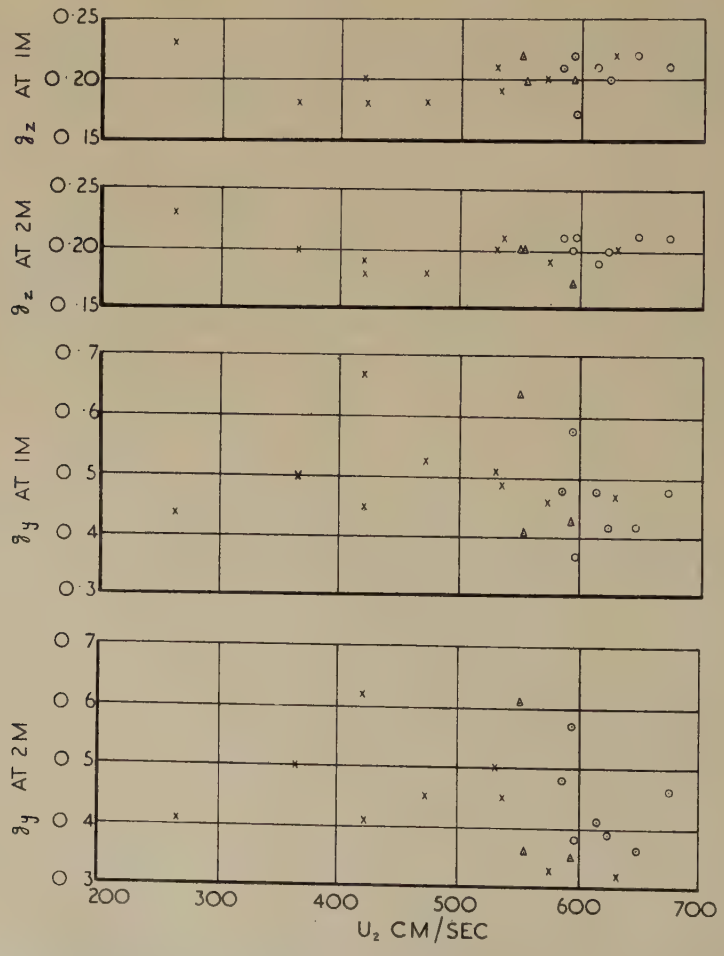

Figure 6. Variation of gustiness with wind speed and air-sea temperature difference. Mean air-sea temperature differences (°F): × = + 0·4, ⊙ = + 1·7, △ = + 2·6.

the ratio g_y/g_z on fetch is in fact evident in this series. Secondly, it may be attributed to the geometrical form of the sea surface, which was much more irregular than land surfaces of similar roughness length z_0. The water level varied from point to point by as much as 1 m at times and, particularly with the wind blowing across the swell, the crests and troughs were distributed more or less at random.

(*b*) *Variation of gustiness with height*

As only two heights were employed, little can be said about this aspect. The mean value of g_y and g_z at 1 and 2 m and their standard errors were found to be as follows :

Height	1 m	2 m
g_z	$0{\cdot}203 \pm 0{\cdot}004$	$0{\cdot}199 \pm 0{\cdot}003$
g_y	$0{\cdot}486 \pm 0{\cdot}017$	$0{\cdot}440 \pm 0{\cdot}020$

Over this interval, then, there is practically no change in the vertical gustiness. There does appear to be definite drop in the lateral gustiness however.

8. Conclusions

From the observations of winds over the sea reported in this paper, the main findings may be summarized as follows :

(a) The wind-speed profile is logarithmic in zero temperature-gradient conditions. When the wind speed at 2 m exceeds 5 m/sec, the flow is generally aerodynamically rough, though it may be of the transitional type up to 6 m/sec on occasions. As the wind speed increases from 5 to 9 m/sec, the roughness parameter z_0 of the sea surface increases from about 0·02 to 0·25 cm. Other undetermined factors influence the state of the surface so that these figures are approximate.

(b) In inversion conditions, the wind speed varies logarithmically with height close to the surface but at greater heights it increases more rapidly.

(c) The vertical gustiness over the sea is similar to that observed over land surfaces of comparable roughness. The lateral gustiness however appears to be greater over the sea, but whether or not this may be attributed to the siting of the instruments is not clear.

Acknowledgments

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Wind stress on water : an hypothesis

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Summary

The form drag of wind on an irregular surface is found to equal the product of U^2 (U is anemometer wind-speed) and a function of the two-dimensional spectrum of this surface. For a solid surface this function is, of course, independent of U and the form drag is proportional to U^2, as observed over land. Over water, Neumann's frequency spectrum and the glitter measurements by Cox and Munk make this function proportional to U, and hence the form drag proportional to U^3. The computed form drag is not far out of line with measurements by Van Dorn and others.

The dependence of form drag on the ' beam-width ' and frequency spectrum of the surface roughness is discussed. The essential roughness statistic comes close to being the mean-square *slope* of the surface, rather than being related to wave height or some other roughness *length*, as usually assumed. The slope statistics are governed by the high-frequency part of the wave spectrum, and this feature accounts for the pronounced reduction of form drag by surface ' slicks,' and the relatively small effect of a limited fetch. The total drag, $c_1 U^2 + c_2 U^3$ (skin friction + form drag), must be larger at very high winds than if it followed a U^2 law, as usually assumed.

1. Introduction

The problem of wind stress on water plays an essential part in studies of ocean circulation and storm tides, and of the momentum balance of atmospheric circulation. The present note is an attempt to connect results from recent experimental determinations of wind stress with results from measurements of wave statistics. The idea was already inherent in an early paper by Jeffreys (1925), but he was unable to achieve quantitative agreement, largely on account of dealing only with a single-frequency wave system. A more recent treatment by Kapitza (1949) does not go beyond the work of Jeffreys, as far as I can see. Francis's experiments (1951, 1954a, 1954b) have been most stimulating, and much of what will be said here is already implied in Section 5 of his last paper.

2. Wind stress

Our empirical knowledge of wind stress is based largely on measurements of (1) the tilt, or ' set-up,' of a water surface under wind action, and (2) the deviation from geostrophic flow of the wind near the surface. In the past these two methods have given quite different results. The situation has now changed. In an important paper presented at the UGGI meeting in Rome, Charnock was able to point out that a shortcoming in the past use of method (2) can account for the discrepancy. Nearly all the ' better ' data are now reasonably consistent. They indicate that the stress increases with the ' anemometer ' wind-speed U more rapidly than U^2 but less rapidly than U^3. Over land, the U^2 law appears to be satisfactory. This is essentially also the conclusion reached by Francis (1954b) in a recent summary of all measurements by method (1), and by Groen (personal communication) from measurements over the Wadden Sea.

Van Dorn's (1953) measurements by method (1) give a reasonable picture of the present state of affairs. Curve B, Fig. 1, represents the stress on the natural surface; line A refers to conditions after the application of a detergent has greatly reduced the ' roughness ' of the water. The simplest interpretation, but not the only one, is to associate

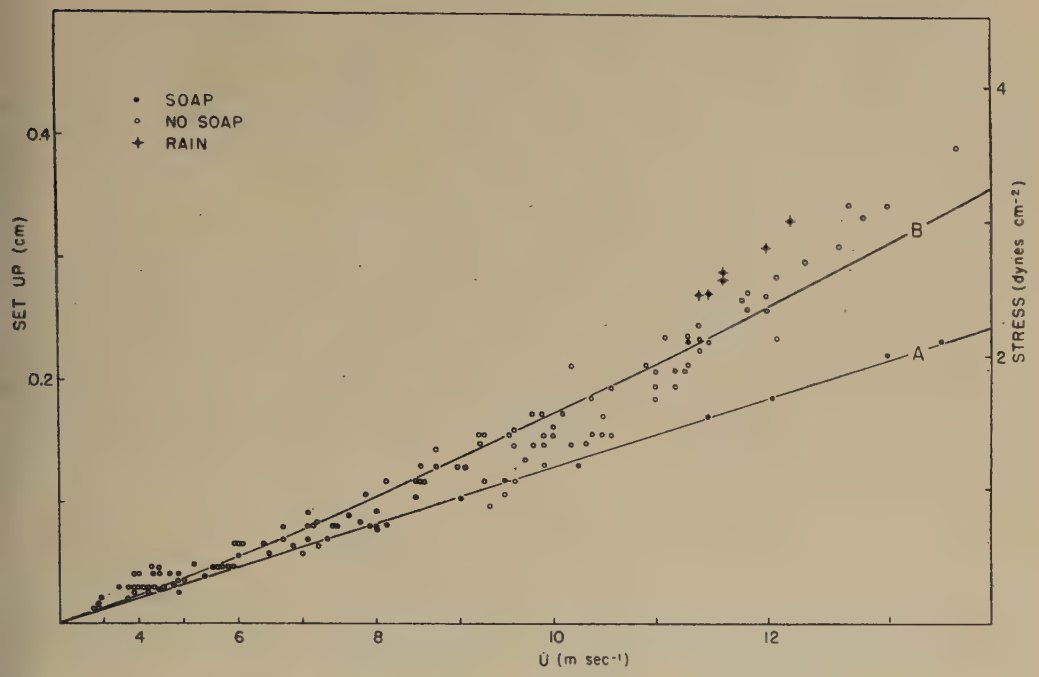

Figure 1. Van Dorn's measurements in Mission Bay model-yacht pond. The abscissa is linear in U^3, where U is the wind speed at the 10-m level. The conversion of the measured values of ' set-up ' (left scale) to stress (right scale) presumes a negligible stress on the bottom, as found by Van Dorn.

the quadratic stress relation (curve A) to the skin friction, τ_s, and to assign the incremental stress between curves A and B to the form drag, τ_f, against the waves. An increment in stress associated with the development of waves has also been observed in the laboratory (Keulegan 1951; Francis 1951, 1954a). According to the usual law for turbulent flow over polished plates

$$\tau_s = c\,\rho\,U^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ρ is air density. Van Dorn obtains $c = 0{\cdot}0025$ and $0{\cdot}0010$ from curve A, depending on whether U is measured at 1 m or 10 m. Curve B has been drawn by assuming the incremental set-up to be given by

$$\tau_f = c'\,\rho\,(g\nu)^{-1/3}\,U^3, \qquad c' = 0{\cdot}68 \times 10^{-6} \quad . \quad . \quad . \quad (2)$$

where ν is the kinematic viscosity of the fluid. The factor $(g\nu)^{-1/3}$ has been introduced from dimensional considerations.* The above value of c' was computed by the method of least squares, and refers to the anemometer at 10 m. The standard deviation of the observed τ_f from that given by (2) is 0·23 dynes cm^{-2}.

Following Keulegan (1951), Van Dorn fitted his data by assuming the incremental stress to vary as $(U - U_0)^2$, where U_0 is an empirical constant, the ' formula ' velocity. This gives a somewhat better fit than the cubic law, which is seen to be low at high winds, and high at moderate winds. The cubic law was selected because it follows from the hypothesis to be discussed in this paper.

* The dimensionless number $U_{min}\,(g\nu)^{-1/3}$, for the least wind to maintain waves against viscosity, appears in Jeffreys's (1925) sheltering hypothesis. Experiments with water ($\nu = 0{\cdot}01$ cm^2 sec^{-1}) and a concentrated sugar solution (0·018 to 0·06 cm^2 sec^{-1}) by Keulegan (1951) and with oil (2·51 cm^2 sec^{-1}) by Francis (1954a) are consistent with U_{min} proportional to $\nu^{1/3}$.

3. Sheltering hypothesis

Let $h(x, y, t)$ designate the elevation of the sea surface, and $p(x, y, t)$ the pressure at the surface. In both cases we refer to departures from mean values. The components of the normal stress for small surface slopes are $p \cdot \partial h/\partial x$, $p \cdot \partial h/\partial y$, and $-p$. Their mean values,

$$\tau_x = < p \cdot \partial h/\partial x >, \quad \tau_y = < p \cdot \partial h/\partial y >, \quad \tau_z = 0 \qquad . \qquad . \qquad (3)$$

define the average stress due to the normal pressure, i.e., the components of the form drag τ_f. For a statistically uniform surface $< >$ designates either space or time averages.

To proceed further we must know something about the distribution of pressure over a rough surface. The principal evidence comes from wind-tunnel experiments with a *solid* sinusoidal bottom, $h = a \cos kx$, and the wind blowing along the x-axis. Suppose we measure p along the wave profile by a series of pitot tubes, and represent the resulting observations by a Fourier series

$$p(x) = A_1 \cos kx + A_2 \cos 2kx + \ldots$$
$$+ A_1' \sin kx + A_2' \sin 2kx + \ldots$$

Then since $\partial h/\partial x$ varies as $\sin kx$, the only term that will contribute to τ_x is the one in $\sin kx$. This term is out of phase with the wave profile. The irrotational theory (Lamb 1932, p. 373) gives only an in-phase pressure variation

$$-\rho U^2 k a \cos kx \qquad . \qquad . \qquad . \qquad . \qquad (4)$$

with a minimum in pressure over the crests, and a maximum over the troughs. The required term is therefore outside the scope of the irrotational theory. But in a turbulent regime such a term can be found, and is in fact prominent. It is associated with the phenomenon of the lee vortex. The following passage is taken from Jeffreys's (1925) paper :

> Water flowing past a sphere does not in general flow all round it; the particles that strike the front of the sphere leave it soon after they have passed the centre, and the region behind the sphere is occupied by eddying liquid with little or no systematic motion relative to the sphere. By analogy one may suggest that if waves are once formed on water, the main air current, instead of flowing steadily down into the troughs and over the crests, merely slides over each crest and impinges on the next wave at some point intermediate between the trough and the crest. The region sheltered from the main air current contains an eddy with a horizontal axis, while smaller eddies exist along the boundary between this eddy and the main current. If such a theory is correct, the pressure of the air will be greater on the slopes facing the wind than on those away from it; for the deflexion of the air upwards when it strikes the exposed slopes implies a reaction between the air and the water.

By analogy with the problem of the thrust of a current against a plane lamina inclined to the direction of flow, Jeffreys infers that the reaction (to be designated by p) is proportional to the air density, the square of the wind speed, and the surface slope :

$$p = s \rho U^2 \cdot \partial h/\partial x = -s \rho U^2 k a \sin kx \qquad . \qquad . \qquad . \qquad (5)$$

Jeffreys calls s the ' sheltering coefficient.' By the preceding argument it must be less than unity : we shall find it to be of the order of a few per cent. A comparison of expressions (4) and (5) shows that the sheltering is out of phase with the irrotational term and only s times as large. But this relatively small component in the pressure variations over the waves is the one important to our problem.

In the final analysis the assumed form of expression (5) must rely on experimental evidence. The appeal to the extensive experimental work with inclined *isolated* laminae

is not altogether convincing, because there one includes also the effect of circulation *around* the laminae. The critical experiment of measuring pressure variations along the profiles of water waves has not yet been accomplished. Wind-tunnel experiments with *solid* corrugated surfaces have been performed by Stanton *et al.* (1932) and Motzfeld (1937) (see Table 1). In three out of five of Stanton's experiments the sheltering term actually exceeds the irrotational term, and yet s is small. This is contrary to expectation. Individual values of s vary widely, and they do not lend any support to the law assumed by Jeffreys. Motzfeld's results, on the other hand, are definitely encouraging.* To a first approximation Motzfeld finds low pressure at the crests and high pressure over the troughs, as predicted by (4). In addition, there are small but consistent deviations which are roughly in accord with (5), as indicated by the relative constancy of the s-values computed from his data for runs (a)-(c) (Table 1). Yet between runs (a) and (b) the height was doubled, and between (a) and (c) the length was cut in half. The coefficient was markedly higher for experiment (d), for which the wave crests were sharply edged. The latter experiment was carried out for four different values of wind speed, yet s remained the same, in accord with the assumed quadratic dependence of the pressure term on wind speed.

The critical dependence of s on the shape of the crest indicates that Jeffreys's sheltering expression (5) cannot be complete, and that it should depend on other factors (curvature at crest, perhaps) as well. Experiments by Fromm (1923) with flow through roughened pipes indicate also a possible role of skewness. Using a 30°-60° saw-tooth roughness, he found a drag twice as great, when the flow was directed against the steep slopes, as when it was directed oppositely. The following development is certainly weakened by having to ignore these circumstances. Motzfeld's result is hardly unexpected, inasmuch as one is accustomed to find a more-pronounced flow separation in the lee of sharp edges than in the lee of smoothly-curved surfaces. Perhaps the tiny corrugations just leeward of the crests of wind waves (Munk 1954) have a similar effect to the sharp edges in Motzfeld's sheet-metal waves; this could explain why the waves fail to be *regenerated* when the corrugations are smoothed by the addition of a detergent (Van Dorn 1953). The hypothesis is that the smoothing leads to a smaller value of s, and a corresponding reduction of energy input into waves by normal pressures. The suggestion is due to Charles Cox (personal communication).

TABLE 1. Results from wind-tunnel measurements

		a (cm)	λ (cm)	U (cm sec^{-1})	s
Stanton	(a)	1·1	10·8	325	0·036
	(b)	1·1	10·8	470	0·047
	(c)	2·2	21·6	330	0·068
	(d)	2·2	21·6	580	0·090
	(e)	1·5†	7·6	1400	0·006
				Average	0·049
Motzfeld	(a)	0·75	30	110	0·034
	(b)	1·5	30	110	0·024
	(c)	0·72	15	110	0·030
	(d)	1·0‡	15	63, 116, 139, 178	0·098, 0·098, 0·098, 0·098
				Average	0·047

* This was pointed out to me by Capt. I. T. A. Hunt (USA). Francis has written to me that inasmuch as Motzfeld's measurements were rather near to the upwind end of the tunnel it is possible that the flow pattern was affected by entry conditions.

† Performed in a different wind tunnel from other measurements.

‡ Sharp 120° crests; other measurements over sinusoidal waves. The value of s was computed allowing for the sharp crests according to equation 6 (3) of Jeffreys (1925); for sinusoidal waves of equal length and height one obtains $s = 0{\cdot}107$ instead of 0·098.

4. Sheltering hypothesis for water surface

The next problem is to extend the 'sheltering' law (5) to a water surface. The simplest case is that of a sine wave progressing in the x-direction. Accordingly we write

$$h = a \cos (kx - \omega t) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the wave number, k, and the frequency, ω, are related according to the usual deep-water theory,

$$\omega^2 = g\,k \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The phase velocity is then given by

$$C = \omega/k = g/\omega \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The obvious procedure is to write

$$p = \pm s\,\rho\,(U - C)^2 \cdot \partial h/\partial x \quad . \quad . \quad . \quad (9)$$

in the place of (5), with the sign depending on whether U is greater or less than C. This is what Jeffreys has done. The sheltering pressure is now proportional to the square of the wind speed, relative to the moving crests. The form drag, according to Eq. (3), equals

$$\tau_f = \tau_x = s\,\rho < \pm (U - C)^2 (\partial h/\partial x)^2 > \quad . \quad . \quad (10)$$

But there are difficulties if this procedure is to be extended to the case of several sine waves. In the resulting interference pattern, the quantity C has now no immediately observable interpretation, such as the one given by Eq. (8). We shall reinterpret it as the 'facet velocity.' Consider a facet of water surface at a fixed distance h above the undisturbed water level. The velocity C of this facet is then defined by

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + C\frac{\partial h}{\partial x} = 0 \quad . \quad . \quad . \quad . \quad (11)$$

The trouble is that C may become infinite at crest and trough, where, according to Eq. (9), this leads to an infinite pressure jump. The mean-square pressure is also infinite. The form drag $< p \cdot \partial h/\partial x >$ remains, however, finite. Evidently the contributions by the infinite pressure jumps to the form drag are small. Only when the wave amplitudes are perfectly steady, that is, when $\partial h/\partial t = 0$, is the infinite pressure jump avoided. Jeffreys was concerned with this singular case.

As an example, suppose the interference pattern gives rise to a profile of slowly varying amplitude,

$$h = a\,(1 + \epsilon\,\omega t) \cos (kx - \omega t),$$

with $\epsilon \ll 1$. Then the pressure pattern is given by (Fig. 2),

$$p = \pm s\,\rho\,(U - g/\omega)^2 \cdot \partial h/\partial x$$

except within a small distance $\xi = \epsilon^2/k$ to either side of the crest (or trough), where it is given by

$$p = \pm s\,\rho\,(g/\omega)^2\,\epsilon^2\,k\,a\,(k\xi)^{-1}.$$

The average contribution to τ_x by the pressure spikes is only ϵ^4 that of the remaining pressure pattern.

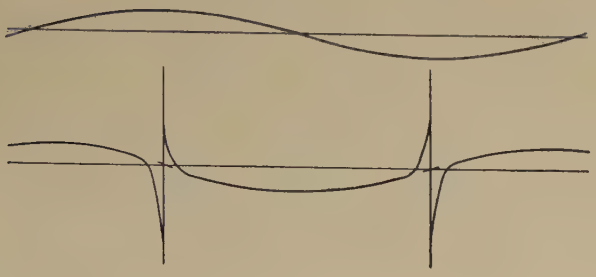

Figure 2. The upper curve is the wave profile, the lower curve the pressure according to Eqs. (9) and (11).

This discussion is not intended to give the reader the impression that the pressure spikes are to be taken seriously. They arise only from the adopted definitions of p and C. What has been shown is that the spikes can be accepted without appreciably affecting the present discussion. An alternative procedure is to introduce a simple modification of Jeffreys's pressure pattern (9), which avoids the infinite spikes, and yet is in accord with experimental evidence :

$$p = s\rho U(U - C) \cdot \partial h/\partial x = s\rho U\left(U\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}\right) \quad . \quad . \quad (12)$$

The expression in the brackets represents the vertical movement of the water surface as seen by an observer drifting with the anemometer wind.

For the two-dimensional case, we introduce the vector slope ∇h of magnitude $|\nabla h|$ in the direction θ relative to the x-axis. Then

$$C = -\frac{\partial h/\partial t}{|\nabla h|}, \qquad \cos\theta = \frac{\partial h/\partial x}{|\nabla h|}$$

and

$$\tau = < p\,\nabla h >.$$

Taking first the spike pressure, it would seem reasonable to write it in the form

$$p = \pm s\rho < (U \cdot \nabla h + \partial h/\partial t)^2 \frac{1}{|\nabla h|} >$$

so that

$$\tau_x = s\rho < \left(U\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}\right)^2 \cos\theta > \quad . \quad . \quad . \quad (13a)$$

$$\tau_y = s\rho < \left(U\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}\right)^2 \sin\theta > \quad . \quad . \quad . \quad (13b)$$

For the revised pressure pattern (12) we write

$$p = s\rho U(U \cdot \nabla h + \partial h/\partial t)$$

and

$$\tau_x = s\rho U < U\frac{\partial h}{\partial x}\frac{\partial h}{\partial x} + \frac{\partial h}{\partial x}\frac{\partial h}{\partial t} > \quad . \quad . \quad . \quad (14a)$$

$$\tau_y = s\rho U < U\frac{\partial h}{\partial x}\frac{\partial h}{\partial y} + \frac{\partial h}{\partial y}\frac{\partial h}{\partial t} > \quad . \quad . \quad . \quad (14b)$$

There is, of course, a good deal of arbitrariness in any of these derivations. For example, one could write $U \cos\theta$ instead of U outside the parenthesis in Eq. (14), or define C in terms of the velocity of a facet of constant *slope*. The laboratory experiments do not distinguish between these alternatives, nor between Eqs. (13) and (14), because all of these expressions are equivalent when $\cos\theta$ and $\partial h/\partial t$ are zero. I shall use Eq. (14) because the calculations are then by far the simplest; this avoids also the infinite pressure peaks. Any refinement should await further experimental evidence. But one further remark should be made here. It is nearly certain that the essential results reached in this investigation do not depend critically on whether the pressure varies as $(U - C)^2$ or $U(U - C)$, or in some other similar way.

5. Transformation into wave spectra

The quantities in Eq. (14) can be transformed into a more familiar form. A useful tool is the correlation (or mean displaced product)

$$\psi(a, b, \tau) = \langle h(x, y, t)\, h(x + a, y + b, t + \tau) \rangle$$

between the elevations at a position x, y and time t, and at $x + a$, $y + b$, and $t + \tau$. The following identities have been established (for example, Eckart 1953) :

$$-\frac{\partial^2 \psi}{\partial a^2}\bigg|_{a=b=\tau=0} = \left\langle \frac{\partial h}{\partial x}\frac{\partial h}{\partial x} \right\rangle = \int_0^\infty \int_{-\pi}^{\pi} k^2 \cos^2\theta\, S(k, \theta)\, k\, dk\, d\theta$$

$$-\frac{\partial^2 \psi}{\partial a\, \partial \tau}\bigg|_{a=b=\tau=0} = \left\langle \frac{\partial h}{\partial x}\frac{\partial h}{\partial t} \right\rangle = -\int_0^\infty \int_{-\pi}^{\pi} \omega k \cos\theta\, S(k, \theta)\, k\, dk\, d\theta,$$

and similarly for b. Here $S(k, \theta)$ is the wave spectrum (units cm^4); $S(k, \theta)\, k\, dk\, d\theta$ designates the contribution, to the total mean-square elevation, by waves with wave numbers in the interval $k - \frac{1}{2}\delta k$ to $k + \frac{1}{2}\delta k$, and which travel in the direction from $\theta - \frac{1}{2}\delta\theta$ to $\theta + \frac{1}{2}\delta\theta$ relative to the x-axis. Eq. (14) can now be written

$$\tau_x = s\rho U \int_0^\infty \int_{-\pi}^{\pi} [U \cos\theta - C(k)]\, k^2 \cos\theta\, S(k, \theta)\, k\, dk\, d\theta, \quad . \quad . \quad \text{(15a)}$$

$$\tau_y = s\rho U \int_0^\infty \int_{-\pi}^{\pi} [U \cos\theta - C(k)]\, k^2 \sin\theta\, S(k, \theta)\, k\, dk\, d\theta. \quad . \quad . \quad \text{(15b)}$$

These are the required equations for relating the form drag to the wave spectrum.

6. Comparison of special solutions with observations

To proceed further we have to specify $S(k, \theta)$.

(a) *Cross-wind symmetry*

One does not ordinarily expect systematic differences between waves moving at some given angle θ to the right and to the left of the generating wind, and this is in fact confirmed by the glitter observations (Cox and Munk 1954). Hence $S(k, +\theta) = S(k, -\theta)$, and the contributions from $+\theta$ cancel those from $-\theta$ in Eq. (15b), so that

$$\tau_y = 0 \quad . \quad . \quad . \quad . \quad . \quad \text{(16)}$$

One can imagine some intriguing exceptions. Wherever there are marked changes in wind direction over short distances, such as behind islands, in intense concentrated

cyclones, or near storm fronts, the sea may be sufficiently out of alignment with the local wind to produce a stress at an angle with the mean local wind. A similar effect could result from wave refraction in shallow water. Mr. Henry Stommel suggested to me in a letter that the absorption of waves by surface-slick bands and the resulting skewness in wave direction would lead to a drag component into the slick. This in turn would cause a convergence at the slick, as observed, and thereby a maintenance of the slick itself.

(*b*) *Narrow beam, high frequency*

The assumption is that the spectrum is so concentrated in high-frequency waves travelling nearly with the wind, that $\cos\theta \simeq 1$ and $C \ll U$ for all waves that contribute appreciably to the integral in Eq. (15a). Therefore

$$\tau_x = s\,\rho\,U^2\,\sigma^2 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (17)$$

where

$$\sigma^2 = \int_0^\infty \int_{-\pi}^{\pi} k^2\,S\,k\,dk\,d\theta$$

is the mean square slope.* From measurements of sun glitter, Cox and Munk (1954) obtain a linear relation between σ^2 and the 'anemometer' wind U. This relation can be written in the dimensionless form

$$\sigma^2 = b\,(g\nu)^{-1/3}\,U \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (18)$$

where $b = 1{\cdot}1 \times 10^{-4} \pm 0{\cdot}1 \times 10^{-4}$. Substituting into Eq. (17) gives

$$\tau_x = s\,b\,\rho\,(g\nu)^{-1/3}\,U^3 \qquad . \qquad . \qquad . \qquad . \qquad (19)$$

The important feature is that the form drag varies like U^3, whereas the skin friction, Eq. (1), varies like U^2. Hence the total stress increases with wind speed more rapidly than U^2, but less rapidly than U^3, in accordance with Francis's and Charnock's conclusion (Section 2). Over land, the equivalent roughness statistic does not vary with wind speed, and hence the quadratic law should apply.

The next question is whether the computed form drag is of the right magnitude. Comparison of Eqs. (2) and (19) identifies $s\,b$ as the theoretical value of c' under the present assumptions. Jeffreys (1925) suggested a value of $s = 0{\cdot}27$ on the basis of the initial formation of waves. Stanton's and Motzfeld's experiments both yielded average values of about 0·05 (Table 1), but no significance must be attached to the close agreement between the two investigations. Sverdrup and Munk (1947) estimated $s = 0{\cdot}01$ from the observed rate of wave growth. With $b = 1{\cdot}1 \times 10^{-4}$ from the glitter measurements, these three estimates of s yield

$$s\,b = 30 \times 10^{-6}, \qquad 5 \times 10^{-6}, \qquad 1 \times 10^{-6},$$

compared with $c' = 0{\cdot}7 \times 10^{-6}$ from Van Dorn's measurements. The computed values are high, but not too far out of line, and one is tempted to consider more realistic models.

(*c*) *Wide beam, high frequency*

The previous assumption of a narrow beam is not in accord with observations. Allowance for variability in wave direction can be expected to reduce the computed values of τ_x. From the ratio of up/down- to cross-wind components of mean-square slope in a trade-wind sea, Cox and Munk inferred a 'beam width' in excess of 130° for the

* Eq. (17) follows also directly from Eq. (14a) for $\partial h/\partial t \ll U \cdot \partial h/\partial x$.

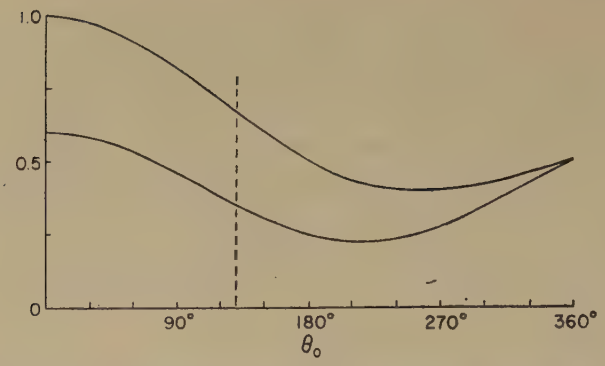

Figure 3. The beam factor $f(\theta_0)$ relating beam width θ_0 to form drag. Upper curve : high-frequency spectrum (Section 6 (c)); lower curve : Neumann spectrum (Section 6 (d)). The beam width of a trade-wind sea is 130°.

relatively short waves that contribute to the slope spectrum. They adopted a model for which

$$S(k, \theta) = F(k) \quad \text{for} \quad -\tfrac{1}{2}\theta_0 < \theta < +\tfrac{1}{2}\theta_0$$

and zero otherwise. The mean-square slope is then given by

$$\sigma^2 = \theta_0 \int_0^\infty k^2 F(k)\, k\, dk,$$

and Eq. (15a) becomes

$$\tau_x = s\rho U^2 \theta_0^{-1} \int_{-\frac{1}{2}\theta_0}^{\frac{1}{2}\theta_0} \cos^2\theta\, d\theta \cdot \theta_0 \int_0^\infty k^2 F(k)\, k\, dk$$

$$= s\, b\, f(\theta_0)\, \rho (g\nu)^{-1/3} U^3, \qquad (20)$$

where

$$f(\theta_0) = \tfrac{1}{2}\left(1 + \frac{\sin\theta_0}{\theta_0}\right)$$

is a ' beam factor ' (Fig. 3). For a very narrow beam, $f(\theta_0)$ approaches 1, and (20) reduces to the narrow-beam solution (19). For a directionally isotropic sea, $f(\theta_0) = \frac{1}{2}$. For the observed 130° beam, $f(\theta_0) = 0{\cdot}67$ and the computed values of τ_x are two-thirds of those for the pencil-beam pattern.

(d) *Wide beam, Neumann spectrum*

A further reduction results if we discard the assumption of a high-frequency spectrum. The lower frequencies are less effective because their phase velocity approaches the wind speed. The best present estimate for the spectrum of a fully-developed sea is that given by Neumann (1953). For the case of the sharply-bounded directional beam discussed in the preceding section, Neumann's spectrum can be written

$$S_\beta(\beta, \theta) = F(\beta) = \pi^3 \theta_0^{-1} r\, g^{-3} U^5 \beta^4 \exp(-2\beta^2) \qquad (21)$$

in the range $-\frac{1}{2}\theta_0 < \theta < \frac{1}{2}\theta_0$, and zero otherwise. Here β is the dimensionless parameter

$$\beta = \frac{C}{U} = \frac{1}{U}\left(\frac{g}{k}\right)^{1/2} \qquad (22)$$

which has been called the ' wave age ' by Sverdrup and Munk (1947). The ' wave-age spectrum ' $S_\beta(\beta, \theta)$ has the dimensions cm^2; it is related to the preceding ' wave-number '

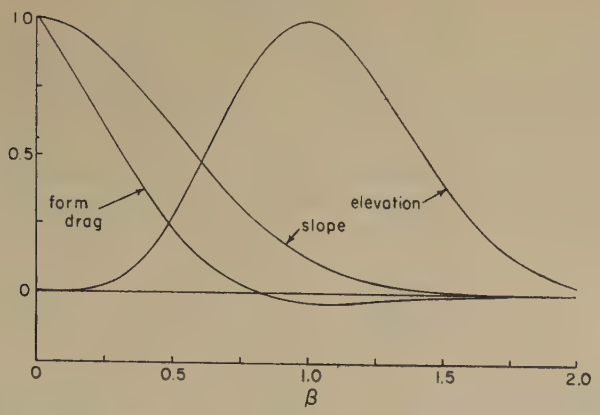

Figure 4. Contributions to the mean-square elevation, mean-square slope and form drag by waves of different wave age β.

spectrum $S(k, \theta)$ according to $S_\beta(\beta, \theta)\, d\beta\, d\theta = S(k, \theta)\, k\, dk\, d\theta$, with β expressed as a function of k according to (22). For short fetches and short duration the energy is concentrated in waves of small β. The spectrum of a fully-developed sea is peaked near $\beta = 1$. The parameter r is a constant with dimension (time)$^{-1}$. By setting

$$r = \frac{4b}{\pi^3 (2\pi)^{1/2}} \left(\frac{g^2}{\nu}\right)^{1/3} \quad . \quad . \quad . \quad . \quad (23)$$

and integrating $k^2 S$ over all values of wave age and direction we recover the empirical relationship, equation (18). All this has been discussed by Cox and Munk (1954).

Eq. (15a) now becomes

$$\tau_x = s \rho g^2 U^{-2} \int_0^\infty \int_{-\pi}^{\pi} \cos\theta\, (\cos\theta - \beta)\, \beta^{-4}\, S(\beta, \theta)\, d\beta\, d\theta$$
$$= s\, b\, f(\theta_0)\, \rho\, (g\nu)^{-1/3}\, U^3 \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where

$$f(\theta_0) = \tfrac{1}{2}\left(1 + \frac{\sin\theta_0}{\theta_0}\right) - \left(\frac{2}{\pi}\right)^{1/2} \frac{\sin \tfrac{1}{2}\theta_0}{\theta_0}$$

is a beam factor (Fig. 3). Again we have recovered a U^3 law. The beam factor now varies from 0·60 for a pencil beam to 0·25 for a 180° beam pattern. For $\theta_0 = 130°$ and $s = 0{\cdot}27,\ 0{\cdot}05,\ 0{\cdot}01$, one obtains

$$s\, b\, f(\theta_0) = 10 \times 10^{-6}, \quad 2 \times 10^{-6}, \quad 0{\cdot}4 \times 10^{-6}.$$

Van Dorn's observed value, $0{\cdot}7 \times 10^{-6}$, lies between the last two computed values. This is as far as the comparison will be carried. Had we used a different sheltering model, the computed values would be different, but probably not by more than a factor of two.

7. The contribution to form drag by various frequencies

For the case of a fully-developed sea, the energy spectrum S (Eq. (21)), the corresponding slope spectrum $k^2 S$ (or $\beta^4 S$) and the integrand of Eq. (24) are plotted in Fig. 4. The latter is averaged with respect to θ for a trade-wind sea ($\theta_0 = 130°$). The integrand is actually the co-spectrum of pressure and up/downwind slope. The three curves represent, on an arbitrary scale, the contributions to mean-square elevation, mean-square slope, and form drag, by component waves whose age is in the neighbourhood of β. The noteworthy feature is the large contribution to form drag by the young (or high-frequency) waves. Two consequences of this are examined in Sections 8 and 9.

The extent to which the capillaries and the high-frequency gravity waves contribute cannot yet be determined. If we accept the Neumann spectrum as it stands, then the capillaries can be neglected. In effect this is what has been done here. However, there is some indication that the Neumann spectrum is defective in the capillaries, and it is by no means impossible that both sources are important. In that event Eqs. (21) and (22) must be modified, and the lower limit to the integration in Eq. (24) replaced by $\beta_{\min} = C_{\min}/U$ ($C_{\min} = 23$ cm sec^{-1}). This will result in a substantial reduction of τ_x for low winds.

8. Limited fetch and duration

In Section 6 we have dealt with the case of a 'fully-arisen sea' (Neumann 1953) where it is assumed that all spectral components have attained their equilibrium values. If the wind fetch or duration is small, the effect will be that only the high frequencies (small β) have attained their equilibrium values, whereas the low frequencies have less than their equilibrium values. For prediction purposes Neumann introduces what is essentially an abrupt high-pass filter. This is equivalent to specifying some critical wave age, say β_m, which depends on wind speed, fetch and duration. The spectrum is presumed to be fully developed for $\beta < \beta_m$, and zero for $\beta > \beta_m$.

Suppose now that $\beta_m = 0{\cdot}5$. It will be seen from Fig. 4 that the mean-square elevation is a very small fraction of the equilibrium value. The mean-square slope has more than half its equilibrium value. The form drag, however, has nearly its equilibrium value. The form drag is therefore much less affected by limitations in fetch and duration than the wave amplitude. This accounts, in a qualitative way, for the fact that values of stress by the tilt method (Section 2) for different water bodies have not differed by orders of magnitude, even though the dimensions of the water bodies have.

One could attempt a quantitative treatment by setting an upper limit, β_m, in the integration for τ_x, and then specifying from empirical considerations the dependence of β_m on wind speed, fetch and duration; but this would be carrying the present state of the art too far. We must however point out that such a modification would impair rather than improve, the fit of the computed form drag to the observations, for it would diminish the computed drag at high winds, where it already is too low.

9. Slicks

The opposite example, that of a low-pass filter, is provided by the action of oil or detergent spread on the water surface. Suppose for the moment that the spectrum, for β_m *less* than 0·5, is dissipated by the surface-active agent. This would leave the mean-square elevation virtually unchanged, reduce the mean-square slope by a large factor and essentially eliminate the form drag. In a general way this corresponds to what is observed. But, for moderate and strong winds, β_m is likely to be far smaller, perhaps 0·2, and, according to Fig. 4, a substantial part of the stress should remain, in contradiction to Van Dorn's results. There are however certain complications when the surface contamination entirely covers a limited fetch, and these tend in the direction of a further reduction of τ_x. The contamination wipes out the tiny corrugations on the crests of the waves. Although their direct contribution to the form drag is negligible, they may affect the flow pattern and hence the value of s. A reduction in s leads to a proportional reduction in τ_x, but, in addition, as noted in Section 4, it also apparently leads to a failure for the waves to be regenerated, and this leads to a reduction of the spectrum S, and a further reduction in τ_x.

10. Discussion

In this paper two experimental measurements have been related; those dealing with wind stress (in particular, form drag); and those dealing with the wave spectra. The procedure is somewhat analogous to the derivation of drag coefficients in engineering problems. This is not going back to fundamentals ! But neither has the drag coefficient of a simple cylinder been accounted for from fundamental principles.

The starting point is a sheltering-type hypothesis for the pressure distribution over a wavy surface. But there is much uncertainty in extending available laboratory evidence for one-dimensional sheet-metal waves to an irregular sea surface. The principal weakness of the present treatment is, I think, the assumption that the pressure drop at a point in the lee of a wave crest depends only on the slope *at that point*. It may turn out that the sheltering from upwind waves is an important factor. One then has to go to a more complicated statistics, involving correlation between slope at one point and elevation at another. Progress will not be easy, and further experimental work is essential, and could be rewarding.

There may also be an opportunity of extending the present method to other surfaces with random roughness, provided these surfaces are ' analytical.' Much of the present evidence of drag on rough surfaces is based on experiments with surfaces on which sand grains have been glued; these are poor examples of analytical surfaces.

The present results indicate that the form drag should be proportional to the product of U^2 with a certain statistical description of surface roughness which comes close to the mean-square slope. Over land the surface roughness is independent of wind speed, and the U^2 law applies. Over water the mean square slope increases linearly with U, and hence a U^3 law follows. This would account for the unique behaviour of fluid surfaces. However, the agreement is only fair, and there is definite indication that the computed form drag is low for high winds. The discrepancy is of the same order as the observed increase in form drag caused by rain. It does not seem unreasonable to presume that, on windy days, the flux of spray, up or down, could be as large as the downward flux of rain water. If this is so, then spray picked up by the wind, and accelerated before returning to the surface, represents yet another appreciable mechanism for transfer of momentum from wind to water. With or without spray, the stress at very high winds should be greater than has previously been estimated on the basis of a U^2 law.

Many difficulties remain. At what elevation is the wind to be measured ? What is the effect of lapse rate ? Can skin drag be added to form drag ? Is C the classical phase velocity, or does it exceed this as a consequence of being impelled by the wind ? Does it involve also the orbital velocity of the larger waves present ? But regardless of these difficulties, I am convinced that the form drag is much more closely related to the slope statistics than the elevation statistics. I believe this conclusion is a departure from the usually accepted view.* There one deals with Reynolds numbers, or with roughness coefficients, and in either case a length-parameter is involved which is supposed to be related to wave height. If this were so, then the effect on form drag of a six-foot sea would be equivalent to that of a six-foot tide.* If this were so, one would expect a critical dependence of wind stress on fetch, and no measurable effect due to a detergent, spread on the water surface. If, on the other hand, the high-frequency slope spectrum is the essential element of surface roughness, then the dependence of wind stress on fetch should be slight, as observed; and the reduction by a slick should be considerable, again as observed.

* But Neumann (1949) and Francis (1954b) have stated that the drag is caused by the small ripples. Francis (1951) also found that high but relatively long waves, generated by a plunger, did not appreciably increase the set-up from wind blowing over water.

Acknowledgments

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Determination of wind stress on the surface of Lough Neagh by measurement of tilt

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Summary

Continuous records of water level were taken at four points on the perimeter of Lough Neagh in N. Ireland during April 1949. The values of level at each point were measured over three-hourly intervals to eliminate the effect of seiches and the differences in the values of level so obtained were found for all pairs of stations and are correlated with the wind speed and direction. The temperature difference between water and air is taken as a measure of the atmospheric stability.

It is shown that atmospheric stability has a very marked effect on the tilt of the water surface due to the wind, the slope of the surface for a given wind speed (measured at a height of 12 m over land near the Lough) being about twice as great in unstable conditions as in stable conditions. It is also shown that the stress coefficient C increases with the length of water under the action of the wind under all conditions of atmospheric stability.

Surface waves were also measured on the lake over this period but no conclusive evidence could be obtained to show that atmospheric stability has any effect on the generation of surface waves by the wind.

1. Introduction

In studying the energy interchange between the atmosphere and water masses it is useful to determine the stress of the wind on the water surface. The method used in this investigation is an old one, that of measuring the slope of the surface in an area of water under the action of the wind, but the measurements in this investigation are much more detailed than in most previous attempts, because there was available a continuous record of the water level for a period of three weeks. Furthermore, as the water mass in question was an inland lake there were no complications due to the effect of tides and the transport of water from other areas. The area of water investigated was Lough Neagh in Northern Ireland. This lake was considered suitable as it is approximately a rectangle of sides 27 km and 14 km, with depth varying from 7 to 12 m, and it is situated in flat country and is well exposed to winds from all directions. There is also the advantage that it is close to the meteorological observatory at the R.A.F. station Aldergrove.

The observations were made during April 1949 by the National Institute of Oceanography as part of a programme of research in collaboration with the Department of Meteorology, Imperial College, London. The instruments used to record the water level were developed by Mr. L. A. Baxter.

2. Description of instruments

The level recorders are self-contained, semi-portable, units following basically the principle of the standard tide recorder. A still well is used of such dimensions that oscillations of a period less than 15 min are greatly attenuated in it. The still well contains a float suspended by a fine flexible wire passing over two pulleys to a counter weight. Each pulley carries $1\frac{1}{4}$ turns of the wire in order to prevent slip and is mounted on highly-polished bearings to minimize friction. Each pulley has attached to it a smaller pulley of one-third its diameter to which is fixed another fine wire under tension. This wire carries a block holding a pen of the siphon type recording on 6-in.-wide paper driven

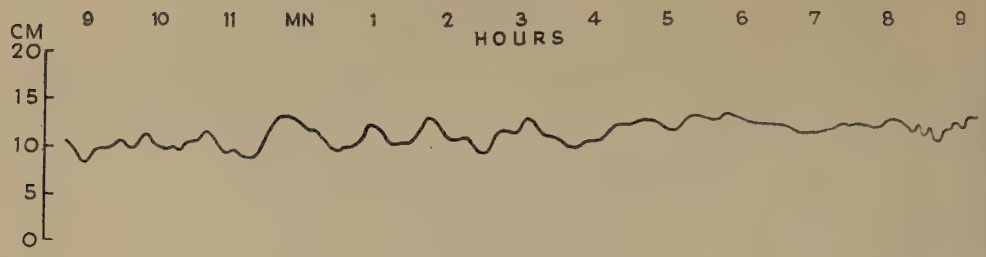

Figure 1. Example of a water-level record.

Figure 2. Map of Lough Neagh, showing positions of recording stations.

at 1 in./hr by an 8-day clockwork motor. An increase in lake level causes the level of the water in the still well to rise and lift the float, rotating the primary wheels which govern the movement of the pen. The movement of the pen is one-third that of the float and the record is a faithful reproduction of the movements of the float. An example of a water-level record is shown in Fig. 1. The surface waves on the lake were measured by a pneumatic-type wave recorder.

3. Site of measuring points

The water-level recorders were installed at Toome Bridge, Newport Trench, Ellis Cut and Langford Lodge as shown on the map in Fig. 2. Each site was either in a small harbour as at Newport Trench and Langford Lodge or in a small protected creek as at Toome Bridge and Ellis Cut, so that the recorders were not unduly exposed to the waves. The surface-wave recorder was laid at a depth of 5ft. near the Antrim end of the lake.

4. Method of analysis of records

Records of water level were taken at Ellis Cut, Newport Trench and Langford Lodge from 5 to 27 April and at Toome Bridge from 10 to 27 April. The effects of seiches and of the finite time taken to reach equilibrium were reduced by taking measurements every $\frac{1}{4}$ hr and averaging every 3 hr. Before the actual differences in level at each station could be determined it was necessary to find the levels when the lake was uninfluenced by the wind. To do this, periods when the wind was less than 3 m/sec for 3 hr or more were noted and the three-hourly mean levels were found during such periods for the four stations. These mean values were found to be unchanged during the period covered by the investigation, there being no appreciable changes in level due to the effect of evaporation and the flow of rivers. The zero value was subtracted from the three-hourly mean values for each station to give the changes in level due to the action of the wind. The observed differences in level had a range of 22 cm.

The wind speeds and directions (at a height of 12 m) were obtained from the Dines anemometer at the nearby R.A.F. station at Aldergrove. The wind speed observed varied from 0 to 13 m/sec. The speed and direction were judged by eye from the record at half-hourly intervals. It was assumed initially that, if V denotes the wind speed, the wind stress is proportional to V^2 in magnitude and that the north and east components of the stress are proportional to $V^2 \cos\theta$ and $V^2 \sin\theta$ where θ is the angle which the wind direction makes with the north. $V^2 \cos\theta$ and $V^2 \sin\theta$ were found for every half-hour and then meaned for the same three-hourly intervals as the records of water level. Six combinations of differences in level can be found from the four stations, and for convenience the stations will be denoted by numbers : 1 for Toome Bridge, 2 for Newport Trench, 3 for Ellis Cut and 4 for Langford Lodge. The difference in level between Toome Bridge and Newport Trench will be denoted by $\Delta Z\,(2-1)$ and so on. Regression equations between ΔZ, $V^2 \cos\theta$, and $V^2 \sin\theta$ were found for each pair as follows, ΔZ being in mm and V in m/sec. The number of three-hourly mean values used to derive the equations varied from 122 to 161.

TABLE 1. Variation with wind speed and direction of the difference in water level between all pairs of measuring stations.

	Correlation coefficient
$\Delta Z\,(3-1) = 0{\cdot}62\,V^2\cos\theta - 0{\cdot}40\,V^2\sin\theta$	0·84
$\Delta Z\,(3-2) = 0{\cdot}34\,V^2\cos\theta - 0{\cdot}32\,V^2\sin\theta$	0·79
$\Delta Z\,(4-1) = 0{\cdot}32\,V^2\cos\theta - 0{\cdot}21\,V^2\sin\theta$	0·88
$\Delta Z\,(2-1) = 0{\cdot}25\,V^2\cos\theta$	0·84
$\Delta Z\,(4-2) = 0{\cdot}08\,V^2\cos\theta - 0{\cdot}19\,V^2\sin\theta$	0·84
$\Delta Z\,(3-4) = 0{\cdot}31\,V^2\cos\theta - 0{\cdot}14\,V^2\sin\theta$	0·74

5. Discussion of results

The correlations obtained as shown in the last column are remarkably high, especially since the wind values were not measured over the lake but at Aldergrove, a few miles away. Since the equations are in the form $\Delta Z = aV^2\cos\theta + bV^2\sin\theta$, the implied direction of the stress for a given slope is $\tan^{-1} b/a$. These angles have been obtained for each pair of stations and are compared with the direction of the line joining the pairs of recorders. These angles are in good agreement except in the case of the last pair where the direct line crosses a promontory of land.

TABLE 2. COMPARISON OF THE IMPLIED DIRECTION OF STRESS WITH THE BEARING OF THE LINE BETWEEN THE TWO MEASURING STATIONS FOR ALL PAIRS OF STATIONS.

Stations	Bearing of line	Implied direction of stress
1 — 3	163°	147°
2 — 3	142°	137°
1 — 4	148°	147°
1 — 2	12°	0°
2 — 4	102°	113°
3 — 4	7°	336°

Francis (1951) from measurements in a wind tunnel and Durst (1949) from measurements at sea have concluded that the stress is proportional to the cube of the wind speed. It was decided to investigate this possibility and, as a trial, ΔZ was correlated with $V^3 \cos\theta$ and $V^3 \sin\theta$ for the case of 1 and 3. The result found was :

$$\Delta Z\,(3-1) = 0{\cdot}065\ V^3 \cos\theta - 0{\cdot}048\ V^3 \sin\theta.$$

This had a correlation coefficient of 0·85 and a most effective wind direction of 144° as against 0·84 and 147° found with the squares. It is therefore not possible to determine conclusively the exact form of the power law from the data, but it was decided to use the square law since it affords a better basis for comparison with previous work. It is interesting to note that in a recent paper (1954), Francis concludes that there is not yet enough evidence to decide definitely the value of the index in the power law.

6. DETERMINATION OF WIND STRESS

The tangential stress of the wind and the stress coefficient have been calculated from the slope of the water surface by means of a formula developed by Ekman (1905) :

$$\sin\psi = \lambda\tau/g\rho h$$

where ψ is the angle the water surface makes with the horizontal, τ is the tangential stress due to the wind, h is the depth of water, ρ is the density of water, and $\lambda = (\tau + \tau_D)/\tau$ where τ_D is the value of the horizontal stress due to the return flow of the water at the bottom. If the water is deep $\tau_D = 0$, but if the depth is small τ_D varies between 0 and $\frac{1}{2}\tau$ so that λ varies between 1 and 3/2. Van Dorn (1953) found the bottom stress to be negligible at a depth of about 2 m and so it is justifiable to neglect it in Lough Neagh where the depth is about 12 m. λ is therefore taken to be 1. The slopes induced in the lake are small enough to approximate $\sin\psi$ to $\tan\psi$, which is given by the ratio of the difference in level between the two points to the direct distance between them.

The mean depths across the six sections considered were found from the Admiralty chart of the lake. This procedure was not quite straightforward as the direct line between the two positions often passed over areas of land. It was decided to take the mean depth in each case over the nearest section parallel to the original section which went entirely across water.

The equations $\Delta Z = aV^2 \cos\theta + bV^2 \sin\theta$ were expressed in the form $\Delta Z = AV^2 \cos\phi$ where $\phi = \theta - \tan^{-1} b/a$ and is approximately the angle between the direction of the wind and the line joining the two stations, so that $\Delta Z = AV_1^2$ where V_1^2 is approximately the component of V^2 along the line joining the two points. The stress coefficient C is given by the equation $\tau = C\rho_A V_1^2$ where ρ_A is the density of the air.

7. Effect of atmospheric stability on slope of surface

Before evaluating C it was decided to ascertain the effect of atmospheric stability on the tilt, the difference between the temperature of the lake and the air being taken as a measure of the atmospheric stability. During the period investigated some measurements of the lake temperature were taken, and as these only varied slightly from day to day it was possible to interpolate values for days when no readings were taken. It was not possible to do this with the observed air temperatures as the day-to-day variation was too large and values of temperature taken at Aldergrove had to be used. The observations were divided into three groups : (1) stable, consisting of 32 observations, where the air temperature was more than 2°F higher than the water temperature, (2) neutral, consisting of 32 observations, where the difference between the water and air temperature was less than 2°F, (3) unstable, consisting of 58 observations, where the temperature of the lake was more than 2°F higher than that of the air. These three groups were investigated statistically in the same manner as the combined data, and values of $\Delta Z/V_1^2$ and C found for each pair of stations for all three groups, as well as those found from the combined data, are given in Table 3. The standard errors are indicated in the case of $\Delta Z/V_1^2$. The difference between the values for stable and unstable conditions is greater than or equal to twice the sum of the two standard errors for all cases except 1–2 and is significant.

TABLE 3. Variation of stress coefficient C with atmospheric stability for all pairs of stations.

Stations	Separation (km)	Mean depth (m)	$\Delta Z/V_1^2$ From all data	$\Delta Z/V_1^2$ Stable	$\Delta Z/V_1^2$ Neutral	$\Delta Z/V_1^2$ Unstable	$C \times 10^3$ From all data	$C \times 10^3$ Stable	$C \times 10^3$ Neutral	$C \times 10^3$ Unstable
1 – 3	29·0	12·3	0·74	0·48 ± ·06	0·67 ± ·10	0·96 ± ·17	2·47	1·60	2·23	3·20
2 – 3	18·5	9·3	0·47	0·29 ± ·05	0·44 ± ·11	0·72 ± ·11	1·85	1·14	1·74	2·84
1 – 4	18·5	12·5	0·38	0·27 ± ·04	0·37 ± ·05	0·50 ± ·08	2·02	1·43	1·96	2·65
1 – 2	13·3	9·3	0·25	0·26 ± ·04	0·23 ± ·03	0·31 ± ·06	1·37	1·43	1·26	1·70
2 – 4	12·9	11·3	0·21	0·12 ± ·02	0·21 ± ·04	0·26 ± ·05	1·45	0·82	1·44	1·79
3 – 4	12·0	7·2	0·34	0·21 ± ·04	0·29 ± ·08	0·52 ± ·10	1·60	0·99	1·37	2·45

8. Effect of length of wind's path over the water on slope of surface

Values of C are plotted against l, the separation between the measuring stations, for the combined data in Fig. 3 and for the three stability groups separately in Fig. 4. The straight lines correspond to :

$$C = (0{\cdot}8 + 0{\cdot}06\, l) \times 10^{-3} \text{ for all conditions}$$

$$C = (0{\cdot}65 + 0{\cdot}03\, l) \times 10^{-3} \text{ for stable conditions}$$

$$C = (0{\cdot}8 + 0{\cdot}05\, l) \times 10^{-3} \text{ for neutral conditions}$$

$$C = (0{\cdot}95 + 0{\cdot}085\, l) \times 10^{-3} \text{ for unstable conditions.}$$

The winds were measured at a height of 12 m.

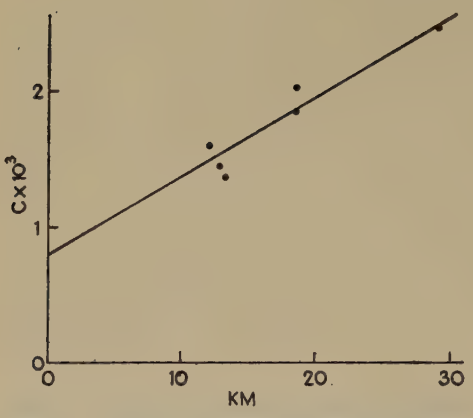

Figure 3. Variation of stress constant C with length of wind's path over the water, using combined data.

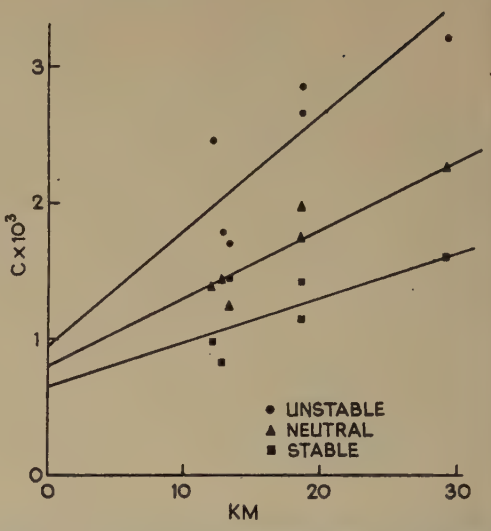

Figure 4. Variation of stress constant C with length of wind's path over the water, for three stability groups.

The increase of C with l indicates that $\Delta Z/l$ increases with l so that ΔZ is given by a quadratic expression in l. This would imply that the slope increases with the length of water from the upwind shore and the difference in level between two points well down wind would be greater than between two similar points nearer the upwind end. The measurements in Lough Neagh are not sufficient to show such an increase in slope with distance, and the wind direction varied so much that it does not give rise to any systematic differences between the meaned slopes measured at the pairs of stations used in Table 3.

The values of 10^3 C at small values of l are not very different from those obtained by Van Dorn (1953) when he worked with an artificial lake of length 240 m and depth 2 m, and measured the winds at a height of 10 m.

The values of C obtained at 18·5 km and 29·0 km are comparable with those collected by Neumann (1948) and Hela (1948) for the Baltic Sea under comparable wind conditions but with longer stretches of water. It is therefore unlikely that the equations for C given above can be extrapolated very far outside the values of length actually worked with.

9. Effect of stability on wave generation

Since the effect of atmospheric stability on the slope of the water surface is significant, it seemed desirable to investigate whether it has a similar effect on the generation of waves. The wave heights were measured every hour from 10 to 26 April, the maximum wave height in a series of 50 waves being used for the comparison. The hourly values were then meaned for every 3 hr. The wind speed and direction were determined from the Aldergrove wind records. The effect of variation of fetch was eliminated by selecting from the data only the three-hourly means (a total of 37 cases) corresponding to times when the wind was approximately SW. so that the fetch was always about 25 km, this being the distance from the extreme south-west corner of the lake to the north-east corner where the recorder was situated. These cases are plotted against the corresponding three-hourly wind values in Fig. 5. The points are grouped into three classes corresponding to the three stability groups. The variation with stability is not nearly so marked as in the case of tilt, although there may be a slight tendency for the wave heights to be greater under unstable conditions, but this is far from conclusive.

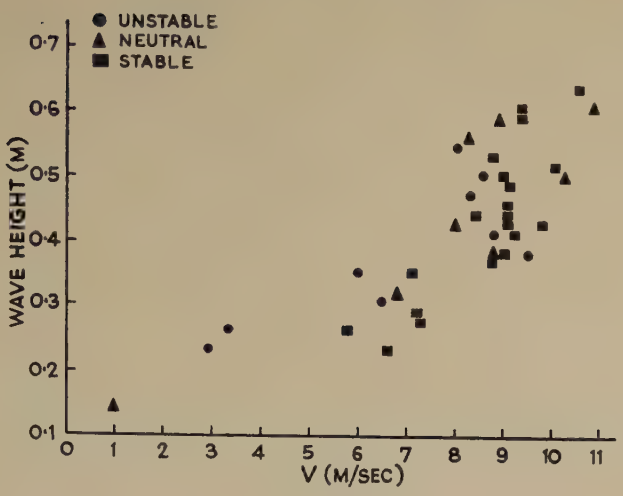

Figure 5. Variation of wave height with wind speed.

10. Conclusions

Measurements on Lough Neagh show that atmospheric stability has a very marked effect on the tilt of the water surface due to the wind, the slope of the surface for a given wind strength (measured at 12 m height) being about twice as great in unstable conditions as in stable conditions.

They show that the stress coefficient C increases with the length of water under the action of the wind under all conditions of atmospheric stability.

The observations give some indication that wave heights are somewhat greater for a given wind speed (measured at 12 m height) when the air is colder than the water, but the evidence is not conclusive.

Acknowledgments

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Theory of airflow over mountains: IV–Separation of flow from the surface

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SUMMARY

Separation of flow may be two-dimensional when the wake or eddy is closed, or three-dimensional when the air is continuously replaced. It often occurs at a salient edge. When it occurs the effect of the mountain on the high-level flow is reduced. If the airflow is temporarily induced to follow the ground in an airstream favourable to separation, very large vertical velocities may result. Eddies may be shed periodically from the lee slope of a ridge. Lee waves inhibit separation and so do katabatic winds and downdraughts in heavy rain. Convection makes it more likely over lee slopes but inhibits it at the top of windward slopes.

1. The friction layer and the boundary layer

Throughout this paper it is supposed that the flow is laminar at high levels and more or less in the same direction as near the surface, where, however, there may be regions of reversed flow or stagnant air produced by hills and mountains.

Separation of a boundary layer occurs when the air is moving up the pressure gradient; up to the point of separation the boundary layer is thickening. The *friction layer* of an airstream on the other hand is theoretically, at any rate when the ground is uniform, of uniform thickness; but the air is moving nearly along the isobars, and a steady-state friction layer is attained when the work done by the pressure gradient through the component of flow across the isobars balances that expended against friction. An airstream over a flat uniform surface thus achieves a steady velocity profile. This velocity profile is disturbed when the air passes over a hill and pressure gradients in the direction of the flow are created. These pressure gradients are also experienced by the air close to the surface which is more retarded by friction than the main body of the airstream and most of what we usually call the friction layer. If friction together with the adverse pressure gradient produces stagnation at any point, then reversed flow at the surface takes place on the downstream side of that point, in the manner familiar in the study of aerodynamics.

If in approaching a hill crest the air near the surface is speeded up, the steady-state friction layer is disturbed; the flow across the synoptic-scale isobars is not sufficient to overcome the drag (nor can flow across these isobars make any significant difference to the velocity in the short distances considered in flow over hills), and so a new velocity profile is set up. This may be thought of as consisting of a *boundary layer* within the friction layer, which grows as the air passes through the disturbed region.

When the air is decelerated on passing the hill crest the air in this boundary layer is moving more slowly than the air above it, whose flow pattern determines the pressure gradients at the surface, so it is liable to separation. It is to be noted that the air outside this boundary layer, while not appreciably affected by friction at the hill, may nevertheless be well within the friction layer. The steady-state velocity profile of the friction layer may be regarded as the velocity profile of the oncoming airstream which flows over the hill, an airstream which is not subject to surface drag until its profile is disturbed from this state at the hill.

This justifies the use of the equations of motion of an inviscid fluid for the flow over a mountain so long as separation does not take place, even though the velocity profile of the air current in question may have been determined by some eddy viscosity, and the friction layer over the level ground extends to greater heights than the mountain; for the boundary-layer thickness is still only a fraction of the mountain height.

After the air has passed over a mountain the steady-state friction-layer profile is gradually resumed. Starting with a gradient wind at all levels, the friction-layer profile would be approached asymptotically but would be achieved to within 10 per cent in a distance characteristic of the situation and determined by the pressure gradient, the earth's rotation, and the kind and magnitude of viscosity. The boundary layer can only grow for distances which are a small fraction of this characteristic distance, otherwise it becomes modified by flow across the isobars. In practice this distance is large compared with the horizontal dimensions of mountains for the flow over which the process of separation is important. For example, gradients of the pressure perturbation in flow over a hill can easily be of the order of 1 mb/km in the direction of flow, but the air must travel a distance of the order of 100 km to pass 1 mb down the pressure gradient over a level surface. The departures of the wind from the gradient wind are of the same order in the two cases.

2. Types of separation

(*a*) *The air-mass type*

In the Alps the air in the valleys may often be moving along the mountain sides while the wind blows across the top in a different direction. There is generally an inversion of temperature separating the two regimes and where this meets the hillsides the upper flow separates from the surface to pass over the lower mass of air. Often wave clouds

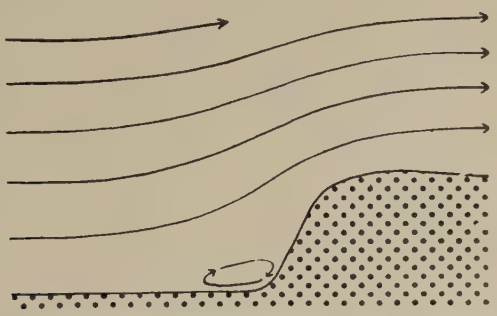

Figure 1. 'Bolster' formed at foot of steep windward-facing slope. Separation occurs in the boundary layer where the streamlines bend upwards in front of the hill but where the ground is level.

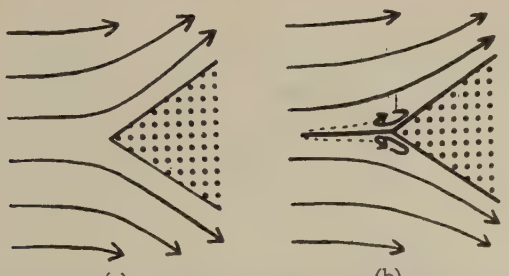

Figure 2. Model demonstration that the bolster is a boundary-layer effect. It does not form at the apex of a wedge pointed into the airstream unless a solid surface is placed upwind from the apex. The pecked line shows the thickening boundary layer.

are formed in the upper mass and the wind is then sometimes called a ' high föhn.' If the synoptic situation is such that the lower air mass is drawn away, the föhn wind then descends the lee hillside. Although, occasionally, valleys in Britain are filled with stagnant cold air, the phenomenon presents a special aspect of separation of the flow from the surface which is mainly of interest in regions of high mountains and is best studied on a synoptic scale. It is not pursued further here.

(*b*) *The two-dimensional aerodynamic type*

When the flow is two-dimensional the air composing an eddy in which there is reversed flow at the ground must originally have been moving along the ground with the stream. The eddy is formed where the pressure gradient is in the opposite direction to the flow because the flow is widening out : the surface layers, being already retarded by friction, are slowed to a standstill; and if the adverse pressure gradient extends beyond this point the flow at the surface will then be in the opposite direction and separation of the flow will occur at the point of zero velocity. We now give some examples.

(i) *Bolsters.* If there is a steep windward-facing slope the streamlines higher up follow a smoother path so that the streamlines fan apart at the approach to the foot of the hill. There must therefore exist an adverse pressure gradient up to the foot of the slope, and reverse flow is therefore possible, as indicated in Fig. 1. It is easily demonstrated in a wind tunnel that this is an effect due to the drag of the surface because no bolster occurs at the windward tip of a two-dimensional wedge unless there is a solid plane surface along the plane of symmetry on the upwind side.

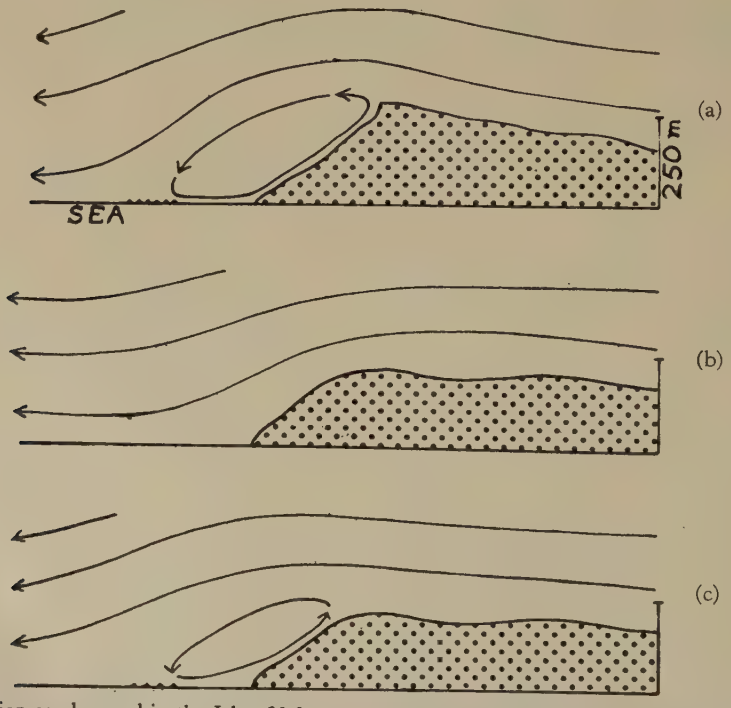

Figure 3. Separation as observed in the Isle of Man on 25 April 1954. (a) Separation was permanent where there was a salient edge at the top of the hill. The sea was ruffled where the downdraught impinged upon it. (b) Sometimes where the hill was rounded smoke was seen to pass straight down the lee slope. (c) At other times an eddy was formed, the smoke passing over the top of it. From time to time in cases (a) and (c) thermals were seen to originate on the lee slope and penetrate upwards across the streamlines. The motion in all cases was unsteady, and the ruffling of the sea moved about. The vertical scale is not exaggerated.

(ii) *Lee eddies.* The region over a steep lee slope is often filled with an eddy, the flow separating from near the top of the hill. A good example of this was seen on the west coast of the Isle of Man in an east wind (Fig. 3). The pattern of the airflow was revealed by the smoke from numerous heather fires. The impact of the air on the calm sea at the downwind end of the eddy caused a disturbance of the surface rather like gusts of wind on a field of tall corn. In spite of the fairly strong winds the noise of the tiny waves at the foot of the lee slope was clearly audible from close to the top of the slope : this indicated that the upslope wind was stronger a few metres from the surface than close to it so that the sound was refracted towards the surface.

It is very common for the wind to blow up the lee slope of a hill, a fact which I have often thankfully noticed on long cycling journeys against the prevailing wind, and which Mr. A. J. Whiten has also remarked upon to me in the same connexion. It most often occurs when the air is well mixed to a height well above the hill tops. If it is very stably stratified then either a strong downslope wind blows, of which some föhn winds are good examples, or the flow separates from the top leaving below a separate air mass, whose motion is not necessarily related to the motion above and need not be up the slope.

(iii) *Rotors and lee waves.* When a train of lee waves occurs in an airstream over the level ground to the lee of a mountain ridge the surface pressure is a maximum under the wave crests and a minimum under the wave troughs. Since there is an adverse gradient from a trough to the succeeding crest separation is likely to occur there, and has often been observed. Manley (1945) reports that under the helm-bar clouds the surface wind may be opposed to the main stream. Förchtgott (1950) tells how Colorado beetles carried by the wind were deposited in the stagnant air under a series of lee waves in the lee of the Ore mountains. Roper (1952) gives a diagram showing haze accumulated in a band under a wave parallel to a range of hills in Derbyshire, with calm air even at a hill crest. In the Owens Valley in California dust from the ground is seen to rise, sometimes to over 20,000 ft, under the first wave in the lee of the Sierra Nevada, and many photographs of this have been published (e.g., Saudek and Eldredge 1953). The phenomenon is also common in Silesia and other places where lee waves are observed.

Lyra (1943) explained the apparent rotation of some wave clouds by assuming that they were inside the eddies formed in this way. This would require the effect of surface drag to be appreciable to well above cloud base. It seems more likely that the eddy is in fact very shallow, perhaps one or two hundred feet deep and that the cloud appears to rotate because the wind at the top is greater than at the bottom. The rotation would be more obvious if the cloud were cumuliform and the motion of the parts were clearly visible, than if it were lenticular and smooth. Ordinary cumulus is often seen to be in apparent rotation about a horizontal axis when there is moderate wind shear, and the impression that the lower part of wave clouds is moving against the main stream is induced by the reversed surface wind at the level of the observer and by fixed average position of the cloud as a whole. Time-lapse films have revealed a few short-lived cases of reversed flow in the bottoms of the clouds in the Owens Valley rotor, studied by the Sierra Wave Project, but these may be evidence that the clouds are partly caused by convection from the ground where the flow is reversed and that the bubbles still have at cloud base a horizontal velocity in the same direction as in the layers in which they originate.

(iv) *Salient edges.* Separation occurs very readily at a sharp edge in the profile of a hill. The air may be almost stagnant in the sheltered region, as it is in the classical theory of free stream lines from sharp edges. In an east wind at Camphill, where there is a westward-facing cliff, the flow pattern has been explored by means of smoke, and it

was found that the discontinuity of velocity extended as far from the edge as could be traced with the smoke – about 100 m. Smoke emerging from a generator suspended from a 10-m pole held out from the cliff, rose slowly on account of the heat from the generator, and when it reached the discontinuity it was carried swiftly away. A smoke puff exploded at the discontinuity was shorn in two (Fig. 4). The position at which separation occurs is often determined by a salient edge, as for instance in the case illustrated in Fig. 3. At other places, on that occasion, where the hills were more rounded the point of separation moved about over a distance of about half a mile downwind from the hill tops.

(*c*) *The three-dimensional aerodynamic type*

In this case the air flowing up the lee slope of a hill has come round the sides. Smoke released in this air passes away to the lee and does not accumulate in the eddy as in the two-dimensional case. Förchtgott (1951) has described his observations of the flow round a conical hill, and I have observed similar motions round the Rock of Gibraltar. The motion is indicated in Fig. 5, and it can be seen that the crest of the wave produced by the hill is found downwind of the hill top. Banner clouds are often seen to the lee of the hill crest, and this is particularly noticeable at Gibraltar where the motion at the surface on the lee side is shown by the pattern of disturbances on the surface of the bay. The rock is not conical but is about two miles long lying north to south. At the top there is a salient edge which causes separation always to take place in an east or west wind. Another fine example has been given by Loewe and Radok (1955); they describe the wave cloud over the conical Heard Island, and it is displaced to the lee. A similar motion is often seen over conical slag heaps in the Midlands of England.

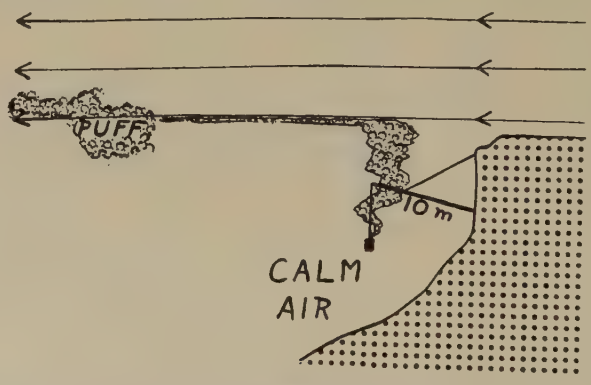

Figure 4. A ' free streamline ' leaving the top of a small cliff, at the top of the 500-ft slope at Camphill, in an east wind. Smoke puffs, and a smoke generator suspended from a 10-m pole showed calm air below the streamline to at least 100 m from the cliff. A smoke puff exploded on the discontinuity was shorn in two. The smoke from the generator rose slowly because of the heat from the generator.

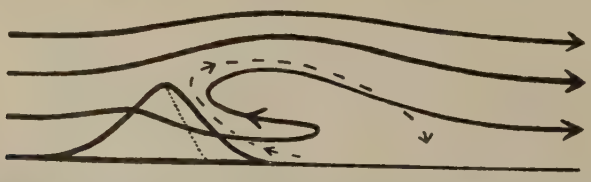

Figure 5. The streamlines originating in a vertical plane slightly to one side of the middle of a conical hill. The lowest streamline shown is displaced sideways also, out of the plane of the diagram, as it moves round the side of the hill and into the lee. The pecked line shows the path of smoke in the central plane. The dotted line shows the line along which separation takes place. The air blows through the eddy and does not remain circulating in it, as in a two-dimensional one.

3. The relation of separation to the behaviour of the air stream

(*a*) *The effect of standing eddies*

Sketches and photographs of clouds in air currents blowing over the Alps suggest hat however disturbed the flow may be close to the ground it may at the same time be xtremely smooth higher up. Ragged low clouds, often shifting violently, may have nooth lenticulars above them. Above some level, therefore, the flow follows steady reamlines, and the lowest streamline of this pattern may follow the ground contour n places. Time-lapse pictures made during the Sierra Wave Project (shown at the .S.T.I.V. meetings in 1952 and on other occasions) showed very smooth flow over ery rugged mountains, the smaller corrugations being filled by eddies, steady and nsteady and only the broad outline of the mountain range being followed steadily by he smooth flow.

In the classical theory of free streamlines from a sharp edge the pressure is constant long the streamline. The pressure gradients are less when the shape assumed by the reamlines has some freedom than when it is constrained to coincide with the ground hape. When the wake of a body is elongated the pressure gradients and curvature f the streamlines outside the wake are less than when the wake is small or non-existent. f an eddy forms over part of a mountain the gradients of pressure and velocity outside he eddy, that is to say in the smooth flow above, will therefore be less than if no eddy orms. Another way of saying this is that *the effect of the mountain on the airflow is lessened hen separation occurs*. In particular the effect of a three-dimensional hill is much less han that of a two-dimensional ridge, because, in addition to other effects, the kind of eparation described in Section 2 (*c*) is possible.

In the atmosphere pressure gradients must either be produced by standing-wave notions in stably-stratified air currents, as in the case of lee waves, or by large gradients f velocity as in the aerodynamic flow over a small hill, or they may accompany the ccelerations of unsteady motion. This last possibility is discussed in Section 3 (*e*) below; or the rest we simply note that anything which inhibits separation will increase the ffect of the mountain on the airflow.

(*b*) *Airstreams suitable for waves*

The kinds of airstream in which lee waves are likely to be set up have been described t length (see Scorer I, II and III and Corby 1954). If they are set up their phase is uch that they induce strong flow at the surface down the lee slope of the ridge producing hem. This theoretical result has been confirmed by the observations of Larsson (1954), who notes that the first wave is less than one wavelength from the mountain. Thus the

igure 6. Comparison of the complete disturbance with the *hill disturbance* without the lee waves, showing hat the presence of lee waves prevents separation on the lee slope. It may occur under the first lee-wave crest.

first trough occurs on the lee slope. Because a minimum of the pressure perturbation is produced at some distance from the crest, separation does not occur until the air has passed the first trough and is approaching the crest of the first lee wave (Fig. 6). The air is thus constrained to follow the shape of the mountain and the effect of the mountain on the flow above will therefore be greater and extend to greater heights than when there are no lee waves and separation takes place somewhere near the mountain crest. This is in addition to the reasons already given in Scorer II, Section 5 (*a*).

(*c*) *Periodic release of eddies in two-dimensional flow over a ridge*

The only cases of unsteady flow studied by perturbation theory have dealt with flow starting from rest either suddenly or gradually, or with flow steadily increasing or decreasing. The problem still remains of how the air flows at high levels in cases where the flow continues for a long time and the steady perturbation solution indicates that the disturbance increases indefinitely with height (for which there is no observational evidence). Periodic flow has been discussed by Förchtgott (O.S.T.I.V. 1952 and in a more recent unpublished communication) and Mr. J. W. S. Pringle has described to me how the air sweeps periodically down the lee slope of the Malvern hills often for hours on end with a period of a minute or two. The essential features of Förchtgott's observations are (*a*) that wave clouds seem to shift downwind in the course of minutes much more slowly than the wind, and then suddenly in the course of a few seconds, make an upwind leap to their original position, the process being repeated over and over again, (*b*) periodic variations of surface wind strength and direction accompanied by the passage of short periods of ' turbulence,' (*c*) similar periodic shifts in the position of the region of maximum updraught have been observed by soaring pilots and the passage of regions of ' turbulence ' has been likewise observed.

In his earlier descriptions Förchtgott likened the flow to that in a Kármán vortex street, but this presents the difficulty that in that case the eddies which are shed move away with the wind while the wave clouds, although undergoing the small periodic shifts just described, do remain in more or less the same position. The description now to be given is in many respects the same as Förchtgott's later description (presented at the Society's meeting in January 1955) but interprets the flow in terms of a lee eddy and waves, rather than in terms of vortices. Besides describing how waves may be modified by separation, it also describes a type of flow which may possibly be dealt with satisfactorily at high levels by perturbation theory because it corresponds to a periodic oscillation in the shape of the mountain. On this assumption it may be possible to find a satisfactory solution at high levels.

The air descending the lee side of a mountain is subject to gravitational forces producing rotation about a horizontal axis, when it is stably stratified, because the surfaces of constant density are tilted. Thus when it has reached a position a few kilometres downwind the tilt may have passed the equilibrium position and the air is now in the upwind side of the first lee wave. As it oscillates about the equilibrium position it produces the flow usually described as a train of lee waves.

If separation now occurs just to the lee of the mountain crest, the eddy that is built up gradually extends the effective size of the mountain and so the position of the first lee wave is slowly shifted downwind. Eventually the eddy builds up to such a size that it is shed and a rush of air down the lee slope takes place. Consequently the position of oscillation of the air as it passes through the wave system is suddenly shifted upwind on the passage of a region of confused motion produced by the eddy as it is carried along by the wind and gradually dissipated.

In steady flow through a lee-wave train we may follow a parcel of air in the manner st described, but in this case it follows a parcel following an identical course and so ere is no dispersion of its oscillatory motion. But when it is preceded and followed y parcels executing waves of slightly different phase (because they passed down the e side of the eddy at a different stage of its growth) the oscillatory motion is dispersed to the neighbouring parcels with the consequence that successive waves in the train re damped more than if the motion were steady, as indeed Förchtgott observes.

The mechanism inducing the separation may be thought of as follows : the motion n the lee side of the mountain consists of two components – the train of lee waves which largely characteristic of the airstream, and the additional *hill disturbance* which is of a milar type to the disturbance produced in an airstream that cannot contain lee waves. As explained in the previous section the phase of the lee waves is such as to inhibit separa- on. The hill disturbance may or may not do this, according to the structure of the irstream; if it *is* such as to produce a pressure perturbation maximum in the middle f the lee slope it will induce separation at the top when such a pressure field becomes stablished. It takes time however for the flow to become established because only by xecuting wavy motions at higher levels can the airflow produce the pressure fields at the

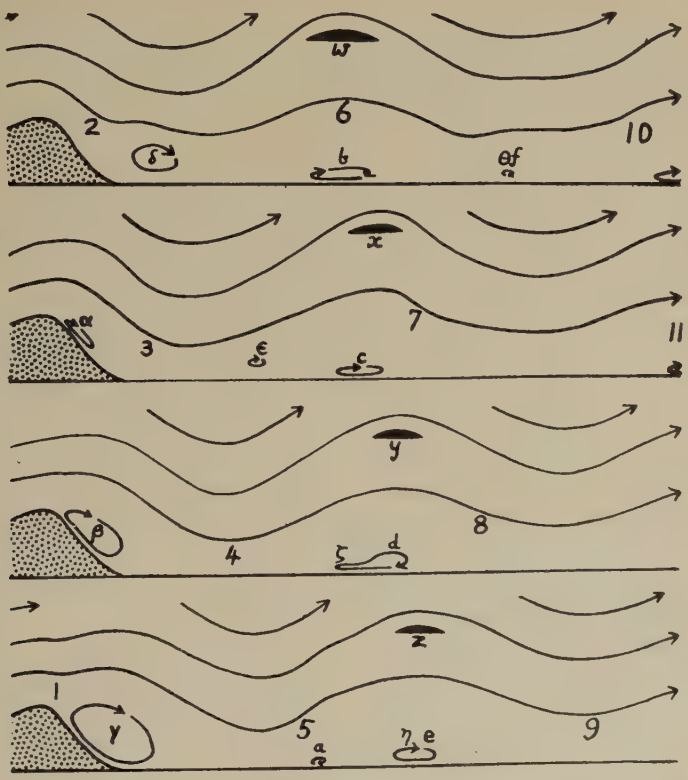

Figure 7. Sequence of streamline patterns when a lee eddy is periodically released. The sequence is repeated indefinitely. As the eddy grows (α, β, γ) it shifts the downflow on the lee side of the hill slowly downwind and the lenticular in the first lee wave follows it (w, x, y, z). When the eddy is shed it moves with the wind (δ, ϵ, ζ, η, θ) and amalgamates with the eddy under the lee wave (under the lenticular). When the lenticular makes its leap against the wind ($z \rightarrow w$) the eddy under it is shed (e, f) and a new one is formed (a). The region of greatest perturbation originates in the downslope wind which sheds the eddy, and this is advected with the wind (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). In this example the eddy is shed as the perturbation surge reaches the first wave, but it may happen at a different moment according to circumstances. Typical dimensions are : wavelength 5 km, interval between successive pictures 1 min, height of hill 300 m.

ground corresponding to steady flow. When the hill disturbance has established itse the separation sets in and the eddy grows by accumulation of air from the boundar layer coming over the hill. When the eddy has reached proportions comparable wit the size of the hill the wavy motion above has been modified so that there is no longe an adverse pressure gradient to the lee of the hill crest and the reverse flow ceases; flo down the slope is resumed discontinuously as the air of the boundary layer penetrate into the region formerly occupied by the eddy. The hill disturbance now begins to b modified again and soon separation is resumed, the eddy gradually grows and the whol performance is repeated. The sequence is illustrated in Fig. 7.

(*d*) *The effect of waves produced by mountains upstream*

If the airstream is already executing wave motions when it ' arrives ' at the mountai in question the phase of these waves may determine whether or not separation takes place and may therefore determine how great is the effect of the mountain in question. A Camphill the upcurrents produced by Bradwell Edge, the ridge normally used by sail plane pilots for soaring, often vary enormously in a time of the order of half an hou as the wavelength characteristic of the airstreams alters and the phase of waves produce upstream is such as to add to or subtract from the shape of the ridge. A fine exampl has been given by Roper in the paper already mentioned. At Camphill it may be impossibl for gliders to remain airborne though they had, an hour previously, been soaring t twice the height of the hill. With larger hills this effect may perhaps produce or destro up- and down-currents of proportions dangerous to powered aircraft.

(*e*) *Violent short-lived disturbances*

Many airstreams cannot execute steady wave motions on account of the unsuitability of their temperature and velocity profiles at high levels. The motion then becomes unsteady or separation may occur with eddies filling the valleys. No theory has yet been given to describe continued unsteady motion, though Förchtgott has recounted what patterns of flow he has observed in airstreams unsuitable for waves (Scorer, 1951 (a) or Corby 1954). It is reasonable, in view of what has been said, to suppose that any cir- cumstance which may cause the flow to follow the ground contour without separation will cause unsteady disturbances to grow to greater proportions if the circumstance prevails at the moment when separation would otherwise have set in.

Such a circumstance is a burst of heavy rain. I have observed small clouds which were slowly ascending a mountain side to be caused to descend by intermittent heavy rain (Scorer 1953 (a)), and it is well known that downdraughts in shower clouds are produced in this way. Whatever be the mechanism by which the density of the air is increased, such a downdraught impinging on a mountain slope will cause the air to flow down the slope, and if it be a lee slope separation of the flow may there be prevented. The most violent effects of a mountain on the airflow may be expected if intermittent heavy rain is occurring, and they may be much more violent than the motions normally associated with such rain.

4. The effect on separation of the condition of the air near the surface

(*a*) *Sloping ground*

Separation is inhibited if forward momentum is applied to the air close to the surface. This can be effected by cooling the air at the surface. A katabatic wind therefore prevents separation and causes the air to follow the ground shape. A striking example

of this has been observed at Dunstable where lee waves have been observed to appear at the time of the onset of the katabatic wind (Lee and Neumark 1952, see remarks by Doughty given there). Gliders have soared in the waves so produced to heights exceeding fifteen times that of the hill – probably the greatest ratio ever achieved.

When the air is so highly stable it behaves as if it were truly inviscid. I have observed (Scorer 1953 (b)) smoke generated at the top of the Long Mynd to pass down the west slope which has a gradient of about one in two, in an east wind of 20 mi/hr in the evening, without rising more than a few metres above the ground even after it had passed at least a mile across the valley at the foot of the mountain. Here the top of the hill is rounded with a radius of curvature of about 50 m so that there is no salient edge.

It may be that many strong so-called katabatic winds are in fact really strong down-slope winds of the helm-wind type, induced when katabatic flow prevents separation.

On the other hand when the surface is heated anabatic flow will be encouraged and it is concluded that separation is likely when the lee slope is facing the sunshine.

(b) *Level ground*

The problem of whether separation will take place at the top of a windward-facing slope is very different if the ground is level for some distance. Observations with smoke puffs at Camphill (Scorer 1951 (b) and Saunders 1955) in a westerly wind indicate that, even though the cliff top provides a salient edge, separation does not occur when the air is stirred by convection but that it does occur when there is not such stirring. In the latter case an eddy between two and ten metres deep, and extending downwind between 50 and 200 m from the cliff top, is formed : smoke puffs are slightly distorted by shear but are not enlarged by turbulence, but if they are released so that part of them is inside the eddy they may be drawn out into long filaments by the shear. The eddy is not strictly two-dimensional and steady and occasional smoke puffs pass across its boundary. Although the ground slopes slightly downwards, the eddy is very limited in extent and it is reasonable to suppose that if the ground were level the forward flow at the surface would be resumed at a shorter distance downwind from the cliff top.

When there is no standing eddy the smoke puffs are enlarged as they move with the wind, presumably by the turbulence. The turbulence is therefore on a scale small compared with the size of the possible eddy and the usual argument, that turbulence inhibits separation by causing forward gusts across the point where separation is otherwise about to occur, is not convincing. It seems likely that the velocity profile associated with an eddy viscosity whose coefficient increases with distance from the surface, having a thinner boundary layer, would be less subject to separation than one produced by a uniform viscosity. The eddies produced by convection, namely the bubbles, increase with height up to heights of several hundred metres, that is to say, to heights far above the boundary layer we are concerned with, whereas those found in neutral or stable air would not increase in size beyond a height comparable with the roughness dimension of the locality. The velocity profile produced when thermal convection is prevalent would thus be less susceptible to separation at the top of a wind-facing slope than that prevailing at other times.

5. General remarks

An interesting example of these ideas is provided by Ludlam's observations of orographic cirrus (Ludlam 1952 (a) and (b)). The very idea of orographic cirrus over the hills of England has met with considerable scepticism. The original explanation was that as the temperature profile changed in the evening the amplitude of the orographic

disturbance was increased, the mechanism being described by perturbation theory. To this we now add the effect of lee waves, which become possible as these temperature changes take place, and the replacement of anabatic by katabatic flow on the lee slopes, for both increase the effect of the hills on the airflow at high levels by inhibiting separation from the surface.

As in aerodynamics, small causes can have large results by altering the effective profile of a mountain, and the results of separation seem to be at least as important as, for instance, the second-order terms neglected in perturbation theory. The ideas here collected have grown mainly as by-products of other studies, and a field study of the particular problem of separation would be most rewarding.

The influences described here are not readily reproduced experimentally, and I think that further progress in their study is most likely to come from observations made out-of-doors. The most likely use of model experiments is to demonstrate the operation of a mechanism revealed by watching nature.

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The calculation of a sea-breeze circulation in terms of the differential heating across the coastline

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Summary

A method is derived for calculating the velocity, temperature and pressure distributions of the sea-breeze produced when an initially isothermal and static atmosphere is heated differentially across a long straight coastline. Heat is assumed to be distributed vertically from the land surface by convection currents; internal friction is included but surface drag is taken as zero. The motion is separated into two parts, a rotational non-divergent component near the coast and a mainly irrotational large-scale tidal motion transferring mass from land to sea. The former is considered first and a numerical method devised to enable the non-linear equations to be solved. The solutions are given as a set of diagrams each showing values of velocity, temperature and pressure calculated over a network of points, and each referring to a particular time in the development of the motion. In Section 5 the tidal motion is considered and an analytical solution obtained since the equations can be linearized. The rotational solution represents the spread inland of a tongue of cold air; well inland the wind rises some hours after heating has started, and is accompanied by a fall in temperature. An upper return current occurs in all the diagrams as well as subsidence over the sea and upper cooling inland. The vertical temperature profiles show a sharpening of the discontinuity in lapse rate at the top of the convection layer produced by the vertical velocity component. Conditions for convective instability are found to be produced dynamically in a shallow layer of the upper return current.

Comments are made on the formation of ' heat lows ' and ' cold highs ' over continents.

1. Introduction

The sea-breeze provides the simplest example of one of the most fundamental of atmospheric processes – the generation of motion by differential heating. It was in the hope that an investigation into this one particular example would throw some light on the whole question of the reaction of the atmosphere to heating and cooling processes, as well as help towards a deeper understanding of the sea-breeze itself, that this detailed study was made.

The sea-breeze has recently been studied theoretically by Haurwitz (1947), Schmidt (1947), Defant (1950) and Malkus and Stern (1953). Both Haurwitz and Schmidt calculate velocity fields corresponding to certain *assumed* temperature distributions. Defant, in his analysis, which goes rather further, linearizes the equations of heating and motion and derives periodic solutions. Malkus and Stern consider the effect of differential heating on a stable atmosphere flowing across an island and show that lee waves analogous to those produced in the lee of mountains can occur. This aspect does not arise here, where the effect studied is that associated with an initially static atmosphere.

In the first three of these approaches to the problem the desire to obtain analytical solutions of a broadly simple character has necessitated the introduction of seriously restrictive assumptions. For instance, as Defant writes in his summary, he must restrict his considerations to ' the normal sea-breeze in the sense of J. Hahn, and not to a sea-breeze of frontal character.' This restriction is necessary as a result of assuming the wind component, normal to the coast, to be proportional to $\cos lx$ where π/l is the width of strips of land and sea, and this expression smooths out any abrupt change of wind of the kind which is usually observed well inland on a sea-breeze occasion. It is precisely such a property as this which is important and requires explanation, but which must of necessity be suppressed at the outset in any theory which starts by linearizing the equations.

It is essential in making an *a priori* approach to the problem to postulate only the distribution in space and time of the heating and cooling sources (referred to as the *heating function*) and the initial and boundary conditions. Then the fields of wind, temperature and pressure must *all* be derived using the equations of heating and motion, i.e., they must all be regarded as strictly dependent variables. (This formulation of the problem should eventually be even further extended; for processes which define the heating function are themselves dependent on the temperature and velocity fields and sensitive, in varying degrees, to changes in these fields. For example, the formation of a cloud layer in a region of upward motion is accompanied by a release of latent heat of condensation).

If the equations of motion are not linearized, there immediately arise insuperable difficulties on attempting to obtain analytical solutions corresponding to given boundary conditions; the equations are non-linear, and periodic solutions, even assuming they can be obtained, cannot be superimposed to correspond to a prescribed heating function. Hence it is necessary to use a numerical technique.

The velocity field is both rotational and divergent as is seen from the vorticity and continuity equations (16) and (19). It is by analysing this property that the equations are broken down into a form adaptable to numerical solution. For it is shown that the velocity field resulting from differential heating consists, in the case of the sea-breeze, of two fundamentally different components; one is a tidal motion of small velocity which affects a region of continental dimensions, whereas the other component is of larger velocity and much more concentrated, affecting a small region in the neighbourhood of the coast; it is this latter component which corresponds to the observed sea-breeze. The tidal component is calculated by considering the hydrodynamic source distribution associated with the heating function, and it is found possible to obtain an approximate analytical solution in this case; since the velocities are small the Eulerian operator $\frac{d}{dt}$ can be approximated to by $\frac{\partial}{\partial t}$, i.e., the equations can be linearized. The main sea-breeze component is associated with the concentration of vorticity in the region of large horizontal temperature gradient near the coast and, when defined in this way, is non-divergent, since the compressibility is associated with the tide, and is calculated separately. In this case, however, linearization of the equations by replacing $\frac{d}{dt}$ by $\frac{\partial}{\partial t}$ is not possible without fundamentally affecting the properties of the solution, since the velocities are too large for advection to be neglected. The non-linear equations are therefore solved numerically. The solution consists of a set of space distributions (patterns) of velocity, temperature and pressure given by values at a number of discrete points forming a regular array. Each pattern refers to a particular time in the development of the sea-breeze.

2. Notation

Suffix 0 denotes the initial value, suffix 1 the value at the surface, and a prime ′ an increment.

p = pressure

ρ = density of air

T = temperature °K

Q = quantity of heat

θ = potential temperature, defined by $\theta = T\left(\frac{p_1}{p}\right)^{\frac{\kappa-1}{\kappa}}$

y, z = horizontal and vertical coordinates in plane of paper

x = horizontal coordinate normal to (out of) the plane of the paper

t = time

u, v, w = velocity components along Ox, Oy, Oz

ξ = $-$ x component of vorticity

c = velocity of sound in air

c_p = specific heat of air at constant pressure

κ = ratio of specific heats of air

g = acceleration due to gravity

H = height of layer in which heat is distributed by convection currents

$\frac{d}{dt}$ = the Eulerian operator $\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

γ = dry adiabatic lapse rate

l = the Coriolis parameter. $l = 2\omega \sin\phi$, ω the angular velocity of the earth, ϕ the latitude.

3. The equations of heating and motion

(a) *The coordinate system and initial conditions*

The land and sea are each assumed semi-infinite, and the problem is made two-dimensional in the sense that all the variables are independent of one space coordinate, x, by choosing axes Oy (y positive inland) and Oz normal to the coastline Ox. Since the equations of motion, neglecting the Coriolis term in the equation of vertical motion, are invariant with respect to orientation of the horizontal axes about Oz, the theory applies to any long straight coastline separating large areas of land and sea.

The atmosphere is taken as initially static and isothermal at 50°F. The initial distribution of θ is obtained from the equation

$$\frac{\partial\theta_0}{\partial z} = \gamma\left(\frac{p_1}{p}\right)^{\frac{\kappa-1}{\kappa}}$$

or neglecting terms $O(z^2)$

$$\frac{\partial\theta_0}{\partial z} = \gamma\left(1 + \frac{\gamma z}{T_0}\right)$$

Taking $\gamma = 9{\cdot}86°\text{C km}^{-1}$, $T_0 = 283°\text{K}$, the following values are obtained :

TABLE 1. Variation of $\frac{\partial\theta_0}{\partial z}$ with height

z (km)	0	1	2	5
$\frac{\partial\theta_0}{\partial z}$ (°C km^{-1})	9·86	10·20	10·56	11·60

In the calculation of the sea-breeze, a constant value $\Gamma = 10°\text{C km}^{-1}$ at all levels is used to simplify the arithmetic. Taking 50°F as a zero of temperature, i.e. putting $\theta_{01} = 0$ then gives

$$\theta_0 = \Gamma z \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

(*b*) *The heating equations*

When the atmosphere takes up heat from the land surface, two main dynamical processes occur. As a result of buoyancy forces, convection currents are created and, secondly, as a result of gravity acting on the horizontal density-gradient across the coast-line, the larger-scale sea-breeze circulation is set up. The details of the behaviour of convection currents and of the interaction between these two processes are in themselves complex problems which have not so far been fully investigated. It is therefore necessary here, where the sea-breeze circulation alone is being considered, to postulate the mode of action of the convection currents. In previous treatments of the sea-breeze, diffusion coefficients for the transfer of heat in both the horizontal and vertical have been used, but this seems to the author to define a process which is too unrealistic.

In the treatment below, heat is regarded as supplied to the atmosphere from the land surface at a given rate, being distributed vertically by convection currents; apart from this there are no heat sources or sinks. The temperature profile over the land near the coast is largely determined by the sea-breeze circulation which develops, but this circulation only extends to about 100 km inland as is later shown : well inland, the temperature profile is determined by convection only. It is postulated that the lapse rate is always dry adiabatic in the heated layer $0 \leqslant z \leqslant H(t)$, (see Fig. 1). Nearer the coast, the upper limit of convection H is also a function of y, depending on the sea-breeze velocity. This postulate seems fairly realistic as convection is observed to exercise a control over the vertical temperature distribution, tending to prevent the lapse rate from exceeding the dry adiabatic when other factors, such as heating over the land surface, would otherwise create super-adiabatic conditions. These latter conditions are often observed (they must occur while heating proceeds, since, otherwise, convection would need to take place instantaneously with infinite velocity) but, since the sea-breeze is generated by *horizontal* temperature gradients, the solution corresponding to the assumption of a small super-adiabatic lapse rate would differ little from the one obtained below. It is postulated that the convection currents affect the sea-breeze in no other way; they almost certainly predominate in determining eddy viscosity over the land, but no variation of eddy viscosity between land and sea is defined, and this simplification together with the neglect of surface friction are restrictions which are far more serious than the postulate that the lapse rate never exceeds dry adiabatic.

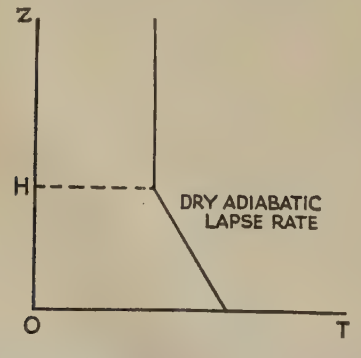


Figure 1. Temperature profile well inland.

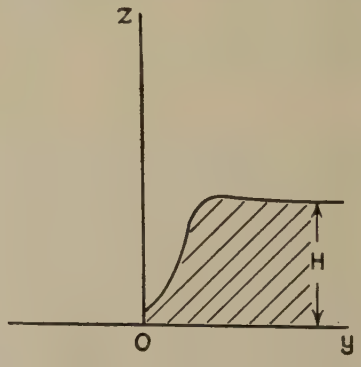


Figure 2. Variation of the upper limit of convection with distance inland.

For $z \geqslant H(t)$, well inland, T and θ are unchanged. Hence there is always a discontinuity in $\frac{\partial\theta}{\partial z}$ at the level $z = H(t)$. Nearer the coast, H is shown later to vary with t as indicated in Fig. 2. The shaded area represents the convection region.

The following equations express the conditions postulated above:

$$\frac{d\theta}{dt} = 0 \quad \text{where} \quad \frac{\partial\theta}{\partial z} > 0 \qquad . \quad . \quad . \quad (2)$$

$$\frac{d\theta_c}{dt} \geqslant 0 \qquad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{\partial\theta}{\partial z} \leqslant 0 \qquad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

θ_c denotes the value of θ in the region of convection, defined by $\frac{\partial\theta}{\partial z} = 0$, i.e., $\frac{\partial\theta_c}{\partial z} = 0$.

The change in heat content of unit vertical column of the atmosphere not affected by the sea-breeze circulation is given to a sufficient degree of approximation by

$$\frac{\partial Q}{\partial t} = c_p \frac{\partial\theta_c}{\partial t} H\rho_{H/2} \qquad . \quad . \quad . \quad . \quad (5)$$

$H\rho_{H/2}$ is the approximate mass of the part of the column being heated and $\frac{\partial\theta_c}{\partial t}$ is the same at all levels, $0 \leqslant z \leqslant H$. This is therefore the amount of heat passing upwards through the base of the column and will be regarded as a given function of t, the same at all points inland. Fig. 3 is a graph of the heating function used in the computation, showing the total heat acquired by the convection layer up to any time, t. The variation of θ_c well inland is also shown. The heating is assumed to have stopped after 6·2 hr, the motion becoming adiabatic everywhere after that time.

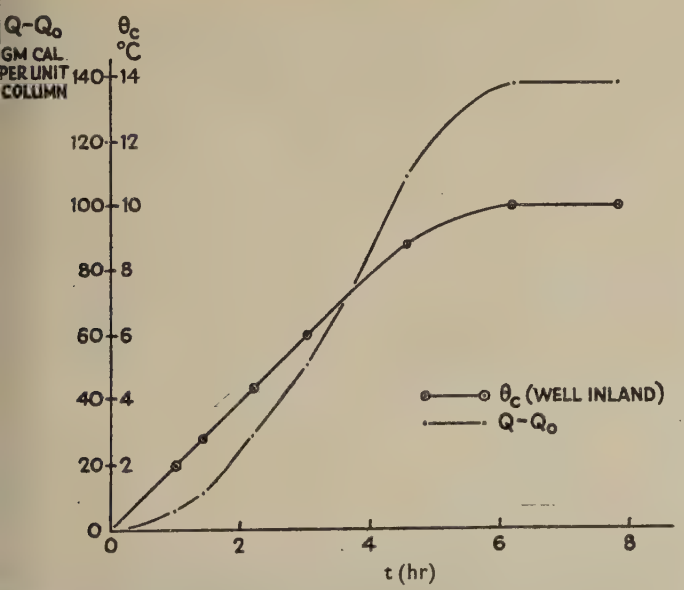


Figure 3. The heating function and θ_c well inland as functions of time.

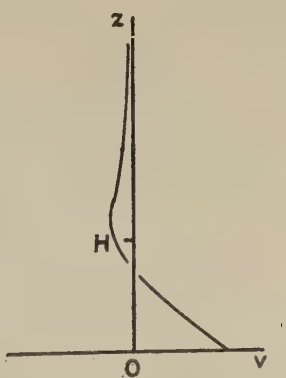


Figure 4. The vertical distribution of v.

Continuity of θ at $z = H$ gives the relation between H and θ_c.

$$\theta_c(t) = \{\theta_0\}_{z=H(t)} \quad . \quad . \quad . \quad . \quad (6)$$

i.e. from Eq. (1),

$$\theta_c(t) = \Gamma H \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Modifications of Eqs. (5) and (7) by the sea-breeze circulation arise in three ways

(i) θ is modified by advection in the region $z \geqslant H$, and θ_c is not given by (6) but by the equation

$$\theta_c(t) = \{\theta(t)\}_{z=H(t)} \quad . \quad . \quad . \quad . \quad (8)$$

the right-hand side being calculated using the equation $\dfrac{d\theta}{dt} = 0$.

(ii) Horizontal advection of heat takes place in the convection region. (Since $\dfrac{\partial\theta_c}{\partial z} = 0$) the vertical advection term $w\dfrac{\partial\theta_c}{\partial z}$ is zero.) A typical velocity distribution is shown in Fig. 4.

Since the heat Q in Eq. (5) is taken up by air particles which are moving, $H\dfrac{\partial\theta_c}{\partial t}$ must be replaced by $\displaystyle\int_0^H \left(\frac{\partial\theta_c}{\partial t} + v\frac{\partial\theta_c}{\partial y}\right) dz$; but $\dfrac{\partial\theta_c}{\partial t}$ and $\dfrac{\partial\theta_c}{\partial y}$ are independent of z since $\dfrac{\partial\theta_c}{\partial z} = 0$. Hence this expression may be written as $H\left(\dfrac{\partial\theta_c}{\partial t} + \bar{v}\dfrac{\partial\theta_c}{\partial y}\right)$ where

$$\bar{v} = \frac{1}{H}\int_0^H v\, dz \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This form of approximation, which neglects the variation of ρ with z, is found to be within 5 per cent if H is not taken greater than 1·5 km. Thus the heat entering the base of a moving column of air is given by

$$\frac{\partial Q}{\partial t} = c_p \rho_{H/2} H\left(\frac{\partial\theta_c}{\partial t} + \bar{v}\frac{\partial\theta_c}{\partial y}\right) \quad . \quad . \quad . \quad (10)$$

Q is a given function of time, and is taken to be independent of $y\ (> 0)$, so that Eqs. (10) and (8) suffice to determine H and θ_c at any time. A further complication is found to arise however when v has a large negative value near the top of the convection layer :

(iii) Eq. (3) may be written

$$\frac{\partial\theta_c}{\partial t} + v\frac{\partial\theta_c}{\partial y} \not< 0 \quad . \quad . \quad . \quad . \quad . \quad (3')$$

On calculating θ_c using Eqs. (10) and (8), Eq. (3′) may be violated above a certain level H_c. This means physically that, above this level, warm air is being advected from inland and is raising the temperature more rapidly than the convection currents from the surface. $H\,(H_c)$ is given in this case as the level at which v satisfies the equation

$$\frac{\partial\theta_c}{\partial t} + v\frac{\partial\theta_c}{\partial y} = 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In practice this condition is examined before Eqs. (8) and (10) are applied.

It also happens that just above the level H_c, v increases again with height sufficiently for the warm air advected above this level to be rendered unstable through a shallow

layer. In this case, indicated when the computation gives $\frac{\partial\theta}{\partial z} < 0$, $\frac{\partial\theta}{\partial z}$ is put equal to zero, and θ made continuous with values calculated using $\frac{d\theta}{dt} = 0$ at levels above.

Summarizing, $\frac{d\theta}{dt}$ is zero everywhere outside the convection region; inside the convection region θ_c is calculated from Eqs. (8) and (10) except that H is given by (11), and if necessary (4) is satisfied by putting $\frac{\partial\theta}{\partial z} = 0$, when warm air is advected in the upper return current of the circulation.

(c) *The equations of motion*

These are taken as

$$0 = \frac{du}{dt} - lv - X \quad . \quad . \quad . \quad . \quad (12)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt} + lu - Y \quad . \quad . \quad . \quad . \quad (13)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} + g \quad . \quad . \quad . \quad . \quad (14)$$

The friction terms X and Y are functions of u and v defined below. Operating on Eq. (13) by $\frac{\partial}{\partial z}$ and Eq. (14) by $\frac{\partial}{\partial y}$, subtracting and resubstituting Eqs. (13) (neglecting Y) and (14) $\left(\text{neglecting } \frac{dw}{dt}\right)$ gives the equation for the x-component of vorticity

$$\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{d}{dt}\right)\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) = -\frac{g}{T}\frac{\partial T}{\partial y} + \left(\frac{dv}{dt} + lu\right)\frac{1}{T}\frac{\partial T}{\partial z} - l\frac{\partial u}{\partial z} + \frac{\partial Y}{\partial z} \quad . \quad (15)$$

ρ has been eliminated using the gas equation $p = R\rho T$.

Taking $\frac{\partial T}{\partial y} = 5°\text{C (50 km)}^{-1}$, $g = 10^5 \text{ km hr}^{-2}$ gives $\frac{g}{T}\frac{\partial T}{\partial y} = \frac{10^4}{T} hr^{-2}$; $l = 0{\cdot}4 \text{ hr}^{-1}$, $u = 25 \text{ km hr}^{-1}$, $\frac{\partial T}{\partial z} = 10°\text{C km}^{-1}$, $\frac{dv}{dt} = 0$ gives $\left(\frac{dv}{dt} + lu\right)\frac{1}{T}\frac{\partial T}{\partial z} = \frac{10^2}{T} \text{ hr}^{-2}$; $\frac{d\theta}{dt} = 2°\text{C hr}^{-1}$, $\frac{\partial v}{\partial z} = -50 \text{ hr}^{-1}$ gives $\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\left(\frac{dv}{dz} - \frac{\partial w}{\partial y}\right) = -\frac{10^2}{T} \text{ hr}^{-2}$. The terms $\left(\frac{dv}{dt} + lu\right)\frac{1}{T}\frac{\partial T}{\partial z}$ and $\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)$ in Eq. (15) can thus be neglected in comparison with $\frac{g}{T}\frac{\partial T}{\partial y}$. These typical values, like those used to justify other approximations, are taken from the computed sea-breeze and not from observations. The neglected terms could be computed as a further approximation if such accuracy was required.

Putting $\frac{\partial T}{\partial y} = \frac{\partial\theta}{\partial y}$, since $\frac{1}{p}\frac{\partial p}{\partial y} \ll \frac{1}{T}\frac{\partial T}{\partial y}$, $\left(\frac{\partial p}{\partial y} \text{ is of order 1 mb } (50 \text{ km})^{-1}\right)$, reduces Eq. (15) to

$$\frac{\partial\xi}{\partial t} = -v\frac{\partial\xi}{\partial y} - w\frac{\partial\xi}{\partial z} - \frac{g}{T}\frac{\partial\theta}{\partial y} - l\frac{\partial u}{\partial z} + \frac{\partial Y}{\partial z} \quad . \quad . \quad (16)$$

where

$$\xi = \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The negative value of the vorticity is used so as to give positive values of the stream function defined below.

The continuity equation is

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{1}{\kappa p}\frac{dp}{dt} + \frac{1}{\theta}\frac{d\theta}{dt}$$

Neglecting $\frac{\partial p}{\partial t}$ and $v\frac{\partial p}{\partial y}$ in comparison with $w\frac{\partial p}{\partial z}$, and putting

$$\frac{1}{\theta}\frac{d\theta}{dt} = G \qquad . \quad . \quad . \quad . \quad (18)$$

(i.e. $G = 0$ outside the convection region) gives

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{gw}{c^2} + G \qquad . \quad . \quad . \quad (19)$$

Eqs. (12) and (18) for u and θ may be written in the form

$$\frac{\partial u}{\partial t} = -v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + lv + X \qquad . \quad . \quad . \quad (20)$$

$$\frac{\partial \theta}{\partial t} = -v\frac{\partial \theta}{\partial y} - w\frac{\partial \theta}{\partial z} + \theta G \qquad . \quad . \quad . \quad . \quad (21)$$

The boundary conditions are

$$w = 0 \text{ at } z = 0 \text{ for all } y \text{ and } t \qquad . \quad . \quad . \quad . \quad (22)$$

$$u \text{ and } v \to 0 \text{ as } y \to \pm\infty \text{ and } z \to \infty \text{ for all } t \qquad . \quad . \quad (23)$$

The initial conditions are

$$u_0, v_0, w_0 = 0;\ \theta_0 = \Gamma z;\ p_{10} = 1{,}000 \text{ mb} \qquad . \quad . \quad (24)$$

The amount of heat passing into the atmosphere from the land surface is given as a function of t, and G can be calculated in terms of this heating function and the velocity field using Eq. (10) in the form

$$\frac{d\theta_c}{dt} = \frac{1}{c_p \rho_{H/2} H}\frac{\partial Q}{\partial t} + (v - \bar{v})\frac{\partial \theta_c}{\partial y} \qquad . \quad . \quad . \quad (10')$$

The five Eqs. (16) (including (17)), (19), (20), (21) and (10′), with the boundary and initial conditions (22), (23) and (24), are solved for the five dependent variables u, v, w, θ and G.

If v and θ are regarded as known functions of y, z and t, Eqs. (10′) and (18) determine $G(y, z, t)$ inside the convection region.

If v, w and G are regarded as known functions of y, z and t, Eqs. (20) and (21), which become linear, with the conditions (23) and (24) suffice to determine u and θ everywhere at any time t.

If u, θ and G are regarded as known functions of y, z and t, Eqs. (16), (17) and (19) with the conditions (23) and (24) can be used to determine v and w, but the Eqs. (16) and (19) are non-linear and neither gives v or w explicitly. A method for their solution is now derived.

Integration of the right-hand side of Eq. (16) with respect to t gives the space distribution of vorticity (x-component) at any time t; the right-hand side of Eq. (19) gives the distribution of sources and sinks at any time t. Expressed vectorially,

$$\text{curl } \mathbf{v} = \mathbf{A}(y, z, t) \qquad . \quad . \quad . \quad . \quad (25)$$

$$\text{div } \mathbf{v} = B(y, z, t) \qquad . \quad . \quad . \quad . \quad (26)$$

here $\mathbf{v} = v\mathbf{j} + w\mathbf{k}$ (u being considered separately in (20)). $\mathbf{A}$ is a vector distribution of constant direction $\mathbf{i}$ (along Ox) obtained by integrating Eq. (16) and B is a scalar distribution given by Eq. (19).

A theorem due to Helmholtz (see e.g. Abraham and Becker 1937) shows that if a vector field $\mathbf{v}$ contains a source distribution ρ and vorticity distribution $\mathbf{c}$, so that simultaneously

$$\text{div } \mathbf{v} = 4\pi\rho \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$\text{curl } \mathbf{v} = 4\pi\mathbf{c} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

hen the solution, which is unique, is found by splitting $\mathbf{v}$ into two parts, $\mathbf{v}_1$, $\mathbf{v}_2$, i.e.,

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

nd, solving the separate pairs of equations,

$$\text{curl } \mathbf{v}_1 = 0, \quad \text{div } \mathbf{v}_1 = 4\pi\rho \quad . \quad . \quad . \quad (30)$$

$$\text{curl } \mathbf{v}_2 = 4\pi\mathbf{c}, \quad \text{div } \mathbf{v}_2 = 0 \quad . \quad . \quad . \quad (31)$$

The conditions to be satisfied are that div $\mathbf{c} = 0$, i.e., $\mathbf{c}$ cannot be chosen arbitrarily, and all the sources and vorticity must be at a finite distance.

In the example considered div $\xi\mathbf{i} = \dfrac{\partial\xi}{\partial x} = 0$, and, in the earth's atmosphere, the sources and vorticity are in a finite region, so that the theorem can be applied to Eqs. (16) and (19). Putting

$$v = v_I + v_R \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$w = w_I + w_R \quad . \quad . \quad . \quad . \quad . \quad (33)$$

these become

$$\frac{\partial\xi_R}{\partial t} = -v\frac{\partial\xi_R}{\partial y} - w\frac{\partial\xi_R}{\partial z} - \frac{g}{T}\frac{\partial\theta}{\partial y} - l\frac{\partial u}{\partial z} + \frac{\partial Y}{\partial z} \quad . \quad . \quad (34)$$

where $$\xi_R = \frac{\partial v_R}{\partial z} - \frac{\partial w_R}{\partial y}$$

$$\frac{\partial v_R}{\partial y} + \frac{\partial w_R}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$\frac{\partial v_I}{\partial y} + \frac{\partial w_I}{\partial z} = \frac{gw}{c^2} + G \quad . \quad . \quad . \quad . \quad (36)$$

$$\frac{\partial v_I}{\partial z} - \frac{\partial w_I}{\partial y} = 0 \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Integration of Eq. (34) with respect to t gives the distribution at any time of the vorticity generated by the heating function, and Eq. (35) and conditions (22) and (23) then enable the associated velocity field to be calculated. Eq. (34) shows that this vorticity must be produced where $\dfrac{\partial\theta}{\partial y}$ is different from zero, and will be mainly concentrated where this term is largest, i.e., near the coast where $\dfrac{\partial G}{\partial y}$ is largest. The computation below shows this region to have horizontal dimensions of order 100 km and the velocities in it to be of order 10 km hr^{-1}. This therefore corresponds to the observed sea-breeze,

and the incompressible part of the motion given by Eqs. (34) and (35) is henceforth referred to as the sea-breeze component, or just the sea-breeze. The region it affects is referred to as the region S.

Unlike the vorticity distribution, G in Eq. (36) extends over an infinite region in the yz plane, and it is shown in Section 5 that the large-scale effect of this source distribution is to transfer mass from land to sea in a tidal motion with dimensions of order 5,000 km and velocity of order 1 km hr^{-1}. Outside S the tidal solution alone applies, while inside S the tide is superposed on the non-divergent sea-breeze circulation.

The pairs of Eqs. (34), (35) and Eqs. (36), (37) do not separate the tide from the sea-breeze completely since v_I and w_I occur in the advection terms in (34) and w_R occurs in (36); also the tide is found to have a small rotational component (compared with the sea-breeze rotation) arising from horizontal temperature gradients associated with adiabatic motion. These complications are found however to be small effects. It is justifiable, to the order of accuracy required, to proceed with the solution of Eqs. (34) and (35) in the region S neglecting v_I and w_I as small compared with v_R and w_R. The rotational part of the tide in this region is also quite negligible.

Eq. (35) enables v_R and w_R (referred to in future as v and w where there is no ambiguity) to be expressed in terms of a stream function ψ i.e.

$$v = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial y} \qquad . \qquad . \qquad . \qquad . \qquad (38)$$

and Eqs. (34) and (35) combine to give

$$\frac{\partial}{\partial t}(\nabla^2 \psi) = -v\frac{\partial}{\partial y}(\nabla^2 \psi) - w\frac{\partial}{\partial z}(\nabla^2 \psi) - \frac{g}{T}\frac{\partial \theta}{\partial y} - l\frac{\partial u}{\partial z} + \frac{\partial Y}{\partial z} \qquad . \qquad . \qquad (39)$$

u and θ are given by Eqs. (20) and (21).

The boundary conditions may now be expressed as

$$\theta = \theta_0, \quad u = 0 \quad \text{and} \quad \psi = 0 \quad \text{round } S \qquad . \qquad . \qquad . \qquad (40)$$

These follow from the continuity of p, u, v, w and θ with the values calculated from the tidal field round the boundary of S since v_I and w_I are continuous through this boundary, and v_R and w_R are zero on it. u is actually equal to the tidal value on S, but this is small; it could be superposed on the whole region for greater accuracy.

X and Y are defined in terms of u and v by considering, separately, the frictional effects at the surface and those due to eddy viscosity within the atmosphere. The surface drag is usually taken to be proportional to v^2, but to simplify the computation is here taken to be zero, i.e. $\frac{\partial v_1}{\partial z} = 0$, or, in terms of ψ,

$$\frac{\partial^2 \psi}{\partial z^2} = 0 \quad \text{at} \quad z = 0 \qquad . \qquad . \qquad . \qquad . \qquad (41)$$

Also

$$\frac{\partial u_1}{\partial z} = 0 \quad \text{at} \quad z = 0 \qquad . \qquad . \qquad . \qquad . \qquad (42)$$

These equations are used only to calculate u_1 and v_1, the frictional force above the surface being taken as $K\frac{\partial^2 \mathbf{v}}{\partial z^2}$. K is made to vary with time.

It was felt at the start of the computation that, in view of the complex nature of frictional parameters and their large observed variations, the above crude but convenient assumption of no surface drag could justifiably be used in a first application of this approach to the sea-breeze problem, which could in fact be regarded as an investigation into surface and lower-layer friction, since the other parameters involved, g and l, are accurately known. Later it was felt that it would have been worth while using a square-law assumption in spite of the extra arithmetic involved.

Regarding G as known, Eqs. (39), (20) and (21) constitute a set of non-linear simultaneous equations to be solved for u, θ and ψ. Analytical solutions cannot be obtained without neglecting the advection terms which are of fundamental importance, and hence a numerical method for their solution must be devised. The physical interpretation of these equations points the way towards the technique to be used. The term $\frac{g}{T}\frac{\partial\theta}{\partial y}$, Eq. (39) expresses the fact that vorticity is generated in a region of horizontal temperature gradient. $\frac{\partial\theta}{\partial y}$ is positive for $z < H$, and hence negative vorticity (x-component) is generated in this region. A circulation of the form shown in Fig. 5 develops, and H is modified by this circulation. The size of the region S affected is determined by the velocity of propagation of the sea-breeze. Starting from static conditions it increases in size more or less symmetrically about the coast as the circulation develops, but at a finite rate, so that this region is always finite. The advection terms in Eq. (39) express the property that the velocity field continually redistributes the vorticity field; but the velocity field is the space integral of this changing vorticity field. Thus a fundamental property of the motion is that the velocity structure continually changes. Vorticity is first generated on the coast, but is soon advected inland, together with the region of horizontal temperature gradient in which it is created.

Since numerical methods must be used, the solution will need to consist of a series of numerical values of u, θ and ψ at lattice points in space, each lattice corresponding to a different time in the development of the motion. Values of ψ, u and θ on such a network of points were actually computed at $t = 0{\cdot}2$, $0{\cdot}4$, $0{\cdot}6$, $1{\cdot}0$, $1{\cdot}4$, $2{\cdot}2$, $3{\cdot}0$, $4{\cdot}6$, $6{\cdot}2$, $7{\cdot}8$, $9{\cdot}4$ and $12{\cdot}2$ hr. The details of the rather involved computation are not given here but may be found in the author's Ph.D. Thesis (Pearce 1953). The usual finite-difference formulae were used for the space derivatives in Eqs. (20) and (21), and Simpson's Rule was used for the time integration. It was found more convenient to calculate the pressure field in two ways by integrating Eqs. (13) and (14) with respect to y and z respectively (by the trapezium rule), using the tidal pressures round S as boundary conditions (see below) and making the two answers agree as nearly as possible, rather than to calculate the changes in vorticity distribution.

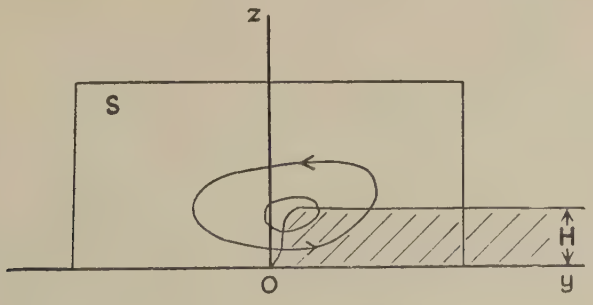


Figure 5. The sea breeze circulation.

The variation of ψ with z is found to be of the form shown in Fig. 6. $\frac{\partial^2 \psi}{\partial z^2}$ is large and negative just above the surface; thus for ψ to tend to zero at the upper boundary of S, $\frac{\partial^2 \psi}{\partial z^2}$ must be positive at higher levels, i.e. $\frac{\partial \theta}{\partial y}$ must be negative. This corresponds to the temperature gradient between the ascending air ahead of the circulation, and the subsiding air behind. Since ψ has one maximum value in S and is zero on the boundary, the streamlines always form a closed circulation as shown in Fig. 5.

Km-hr units and °C are used throughout. Since ψ varies over a wide range of values as the sea-breeze develops, it is multiplied at each stage by some factor N chosen such that $N\psi$ takes values up to about 100 on each pattern. Also values of $100\,\theta$ are computed.

Two difficulties arise which are not capable of being resolved without decreasing the mesh lengths and time intervals. At one or two mesh points near the centre of the disturbance where $\frac{\partial \theta}{\partial t}$ changes rapidly over the interval of integration, the assessment of θ using Simpson's Rule may be widely in error. In this case θ is interpolated. The second difficulty arises in the computation of u_1 near the coast where a maximum and minimum value of $u\,(y)$ as shown in Fig. 7 occur, the minimum being caused by the frictional drag of the returning air above. u is computed on a finer mesh in this region.

(*d*) *The results of the computation*

These are presented as follows: One pattern, that computed for $t = 4{\cdot}6$ hr is reproduced in full (Figs. 8 (a) and (b)) with values of ψ, u, θ and $\nabla^2 \psi$ at each mesh point, together with the two pressure curves computed from Eqs. (13) and (14), and the distribution of $\frac{\partial Q}{\partial t}$ given by Eq. (10). The eight patterns from 1·4 hr to 12·2 hr are all drawn on the same scale in Figs. 10 (a) and (b), but without numerical values at each mesh point. Fig. 11 shows the variation of temperature with height at various distances from the coast at 7·8 hr, after heating has ceased, and the onset of this sea-breeze as it would be recorded at a point 20 km inland is shown in Fig. 12.

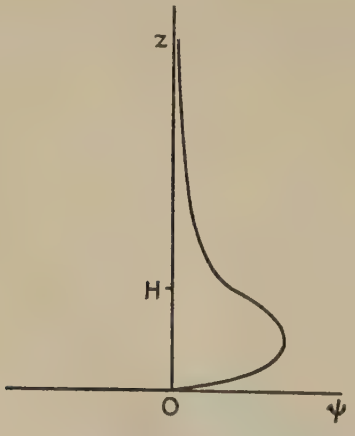


Figure 6. Vertical distribution of the stream function.

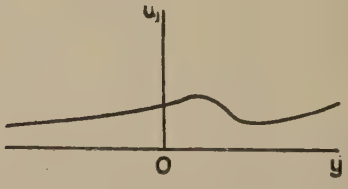


Figure 7. Variation of u_1 near the origin.

Fig. 8 (a) is a typical pattern of the series. The maximum velocity is at the surface, just inland, near the region where the horizontal temperature gradient becomes large. The streamlines represent the spread inland of a tongue of cold air, as is clearly illustrated when all the patterns are considered together in Figs. 10 (a) and (b). Fig. 8 (a) shows that the maximum return (negative) velocity occurs just below the level $z = H$ at about 0·7 km (2,000 ft) and just above the maximum surface velocity. H changes rapidly in this region. Ahead of the advancing tongue, H is slightly larger than under static conditions well inland, since the air above this level is ascending and is slightly cooled; but the cold air, behind, has only just started to warm up over the land and H is small. Subsidence over the sea makes the horizontal temperature gradient decrease upwards from the surface and above $z = H$ this gradient is reversed by the adiabatic warming over the sea and cooling over the land. It is in the region just above the centre of the circulation that the warm air advected from inland tends to become unstable. A persistent feature of the velocity component parallel to the coast is the small surface value just inland, caused by the frictional drag of the upper return current. This has the effect of maintaining the wind almost normal to the coast for a time.

The form of the pressure curves in Fig. 8 (b) and the agreement between them are typical of all the patterns. In Fig. 9 the heating function near the centre of the circulation is seen to differ considerably from the value well inland. This is, in fact, the worst pattern in this respect, and to obtain a better approximation to the constant value it would be necessary to use a smaller mesh for the calculation of $\frac{\partial \theta_c}{\partial y}$.

Figs. 10 (a) and (b) show how the cold air spreads inland, slowly at first, then with increasing speed up to a maximum after about four hours, and thereafter more slowly as the wind turns to blow more nearly parallel to the coast. In the final pattern there is only slight overturning in the yz-plane, resulting mainly from internal friction, and the wind is nearly geostrophic. In each of the patterns up to 4·6 hr, heating is taking place over the land and the temperature profiles in Fig. 10 (b) are all of the same form. After 6·2 hr, the heating is assumed to have stopped and the motion is adiabatic everywhere; the patterns show the tongue of cold air spreading inland under the warmer air above. Convection from the surface is no longer taking place but the discontinuity in $\frac{\partial \theta}{\partial z}$, formed during the heating, persists and is defined by the level $z = H$. Since the pressure curves are all of the same form as at 4·6 hr, only surface pressures are reproduced.

An interesting feature of the vertical temperature distributions in Fig. 11 is the sharpness of the discontinuity at the top of the convection layer, an effect which is advected seawards in the upper flow. This sharp discontinuity arises because w decreases fairly rapidly with height above $z = H$, producing no change of θ in the convection region where $\frac{\partial \theta}{\partial z} = 0$, but a large amount of cooling in the region just above $z = H$, and progressively less cooling above this level. The inversion of temperature above $z = H$, in which $\frac{\partial \theta}{\partial z}$ is larger than in an isothermal layer, further accentuates the effect.

Fig. 12 emphasizes an important property of the sea-breeze, that its onset, inland, occurs a few hours after heating has started. It is likely that the abruptness of the onset has been rather smoothed out in the computation as a result of using finite difference approximations to $\frac{\partial \theta}{\partial y}$, especially near the surface.

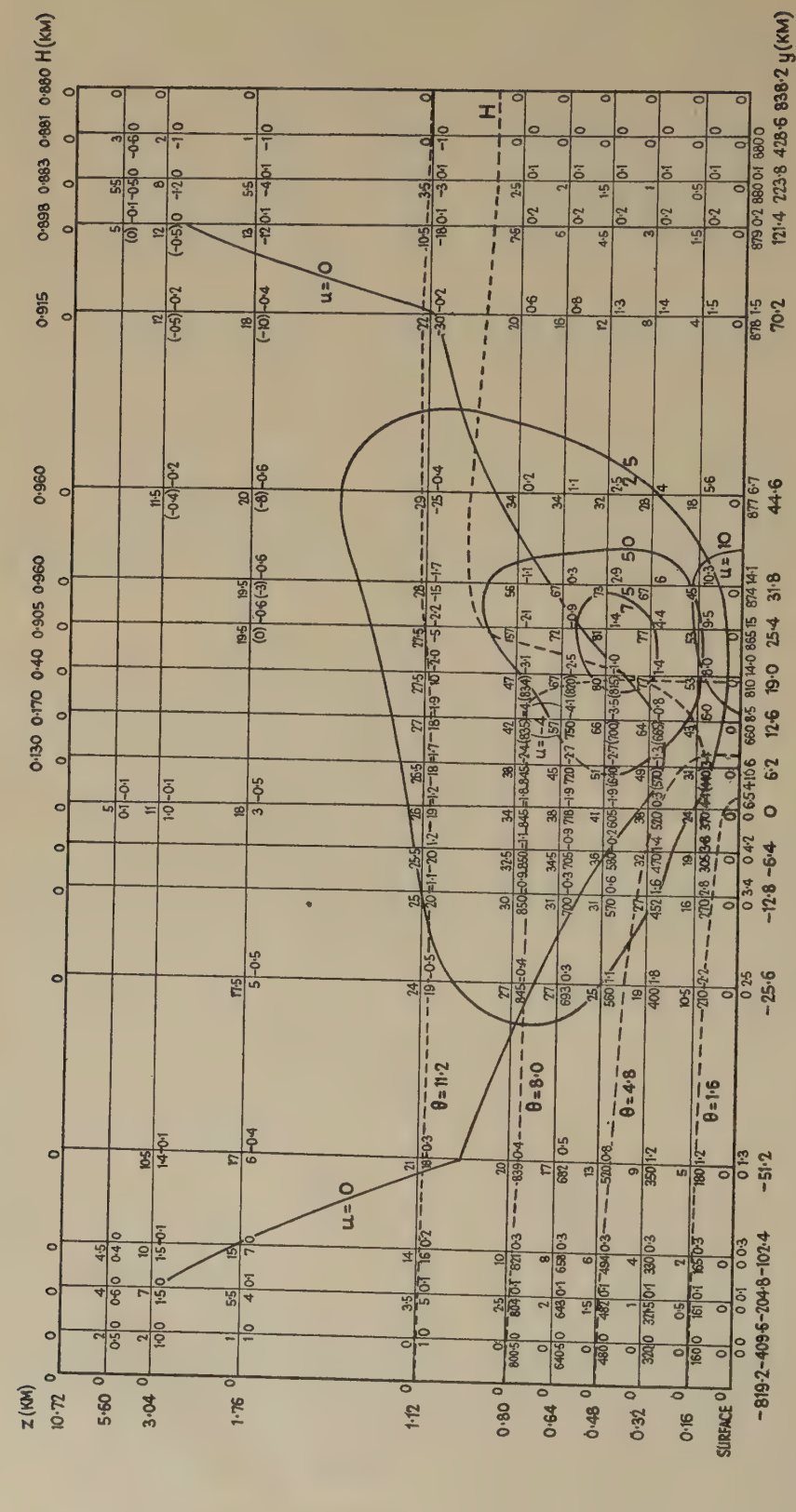


Figure 8. (a) Values of 10 ψ, 100 θ (or 100 θ'), u and H at $t = 4{\cdot}6$ hr. Values of 10 ψ are entered above each mesh point and 100 θ (or 100 θ') and u in that order below each mesh point. Brackets denote interpolated values. The full curves are streamlines and isopleths of the wind component parallel to the coast, and the pecked curves are isopleths of θ. H is also shown as a pecked line. Surface velocity at coast = 16 km hr^{-1}, maximum surface velocity = 36·0 km hr^{-1} at 19·0 km, minimum $v = -10{\cdot}3$ km hr^{-1}, maximum $w = 0{\cdot}24$ km hr^{-1}, minimum $w = -0{\cdot}23$ km hr^{-1}. The diagram is drawn to scale in the region $-51{\cdot}2$ km $\leqslant y \leqslant 70{\cdot}2$ km, $0 \leqslant z \leqslant 1{\cdot}76$ km.

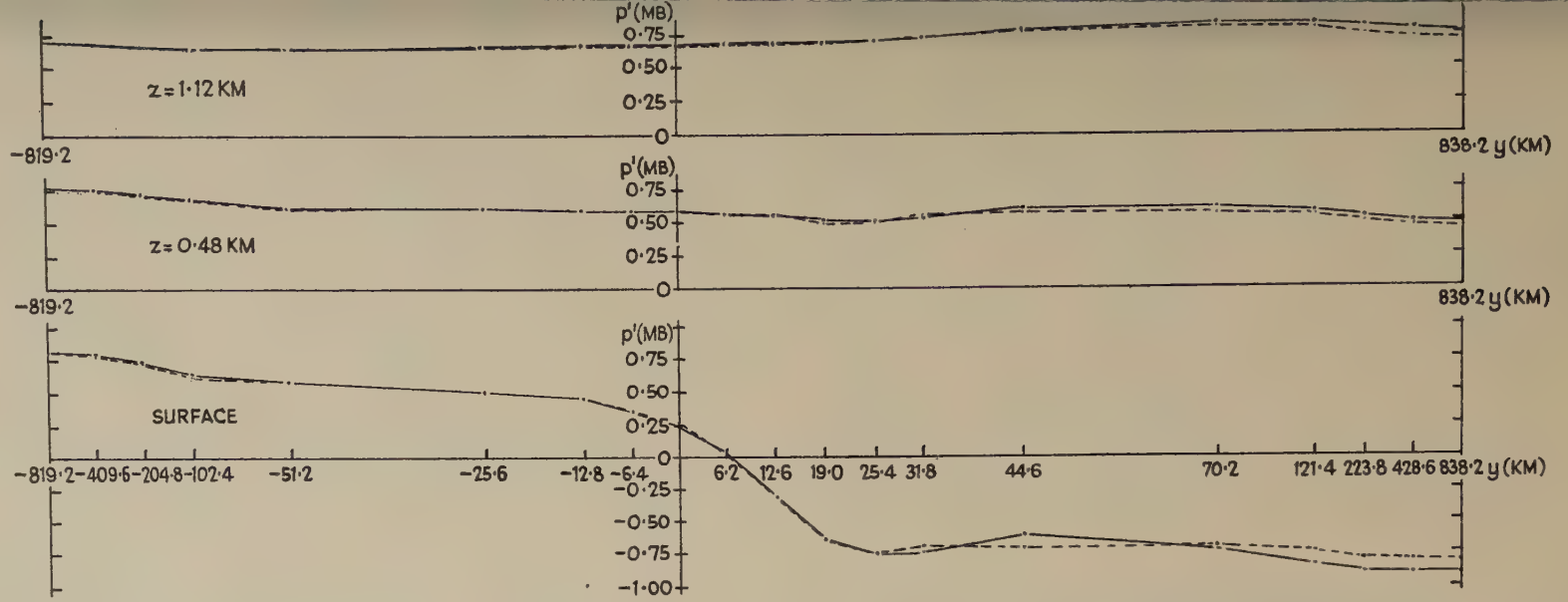


Figure 8 (b). The pressure distribution at $t = 4{\cdot}6$ hr. The full line represents the pressure distribution obtained by integration of the equation of horizontal motion, and the pecked line that obtained from the hydrostatic equation. The pressures at each of the three levels are continuous with the tidal pressure field at the extremes of the pattern (see Fig. 16).

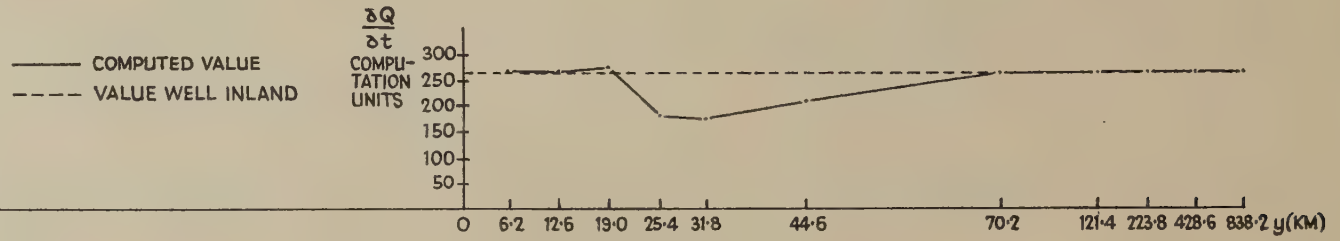


Figure 9. The heating function at $t = 4{\cdot}6$ hr.

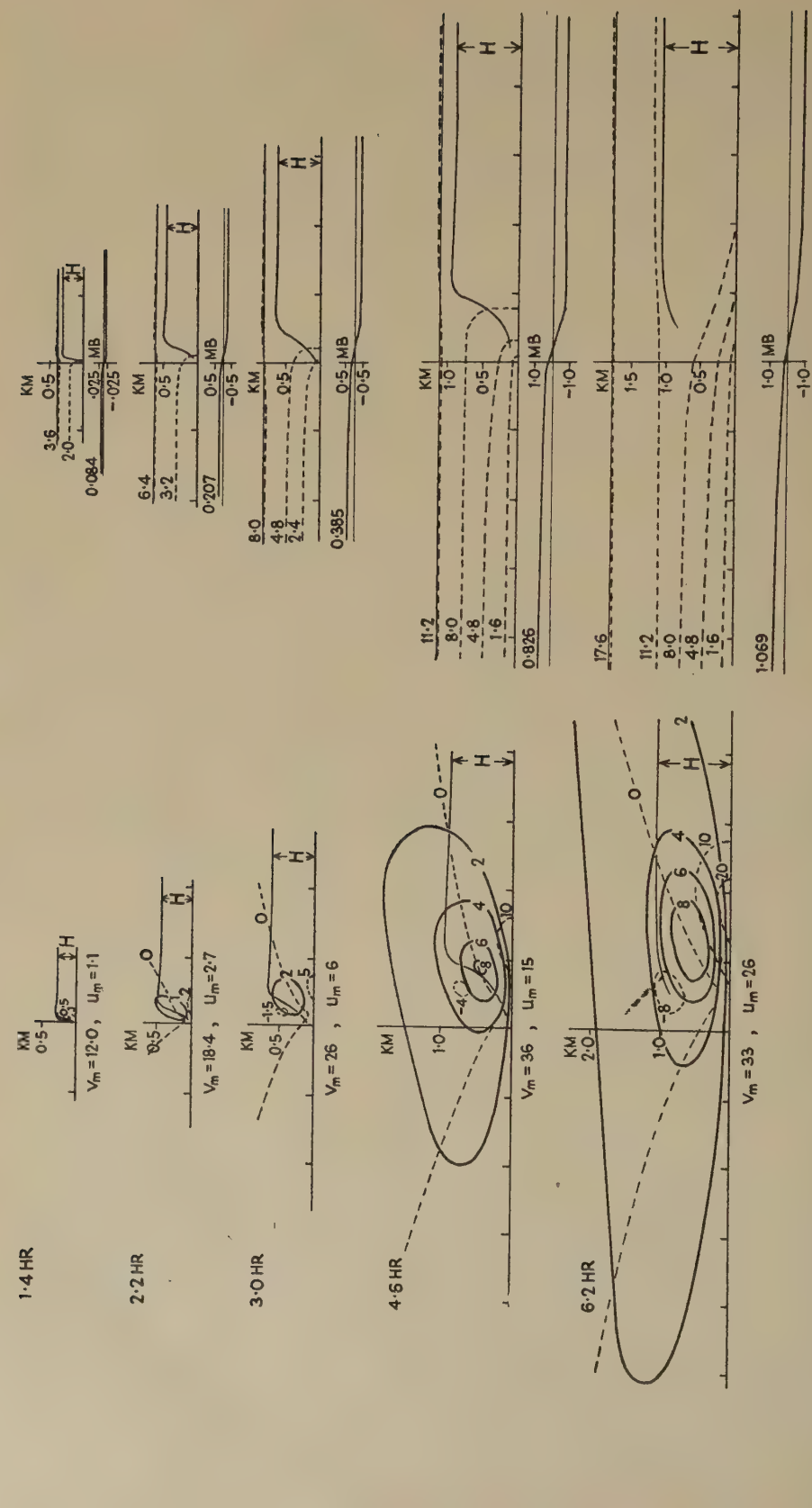
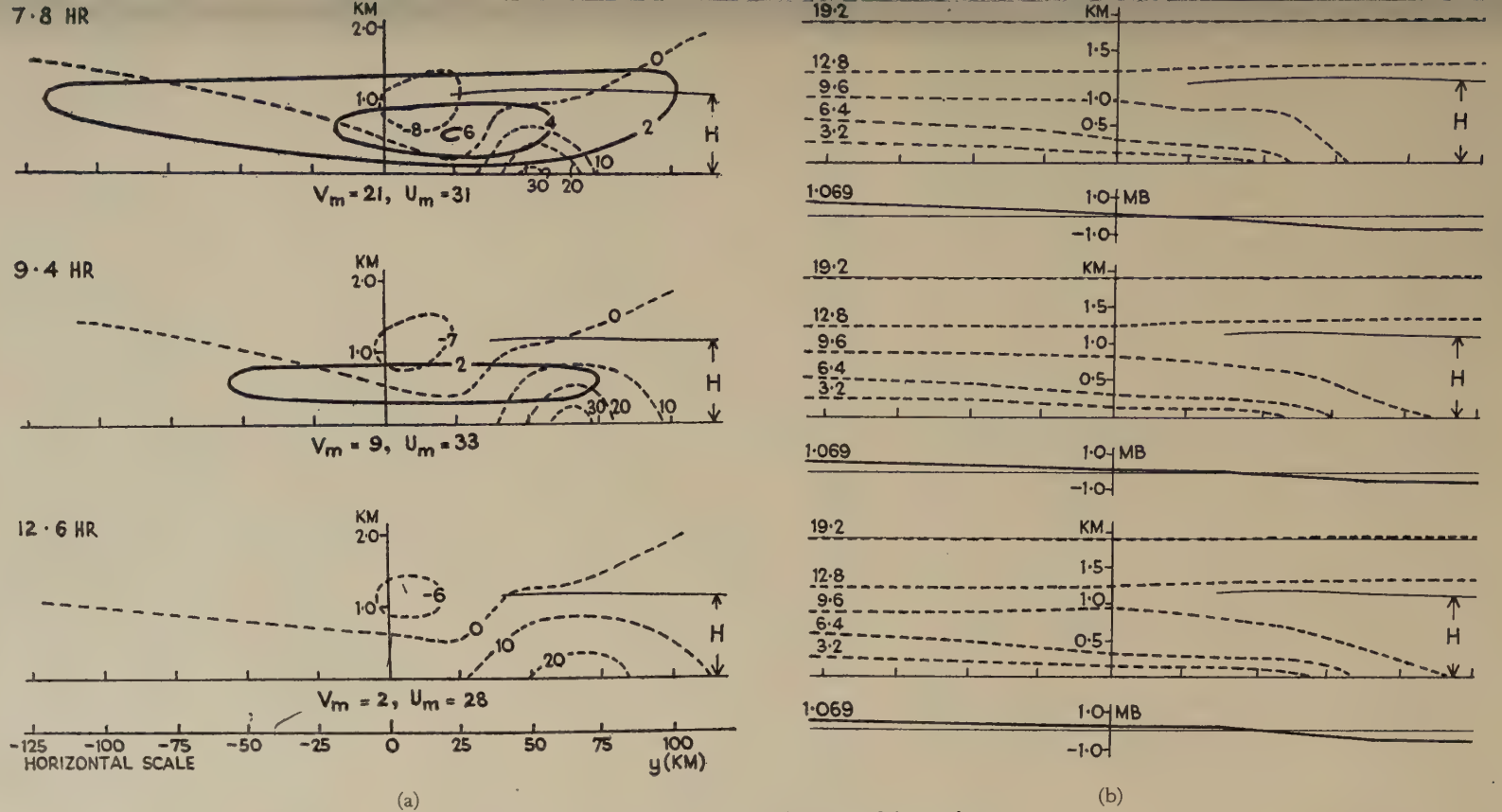


Figure 10. The development of the sea-breeze (continued below).



(a) (b)

Figure 10 (continued). The development of the sea-breeze.

(a) Velocity field. Full lines are streamlines of the motion in the plane of the diagram with values of ψ in km^2 hr^{-1}. Pecked lines are isopleths of the velocity component u in km hr^{-1} (positive out of diagram). Maximum values of the horizontal velocity components u_m and v_m are given in km hr^{-1} for each pattern.

(b) Temperature and pressure fields. The pecked lines are of constant potential temperature θ. Full lines indicate the top of the convection region. The lower diagrams give the distribution of surface pressure p_1, with the tidal boundary pressure in mb indicated on the left.

All the diagrams are drawn to the same scale. Times are indicated on the left of each group.

The distributions of the other two components of vorticity can be computed directly from the patterns. The y-component is in general negative in the lower layers and positive above, being of the same order of magnitude as ξ. The general distribution of the z-component is as shown in Fig. 13. Its magnitude is of order one-tenth the magnitudes of the horizontal components.

A more general discussion follows in Section 6.

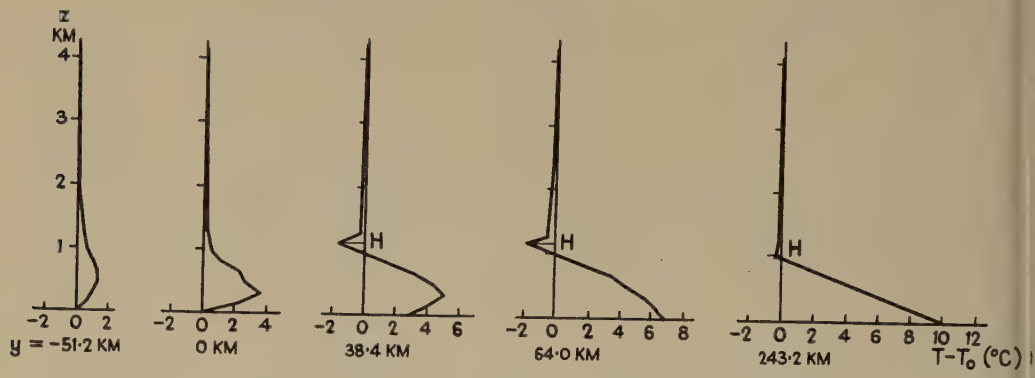


Figure 11. Vertical temperature distributions at $t = 7{\cdot}8$ hr.

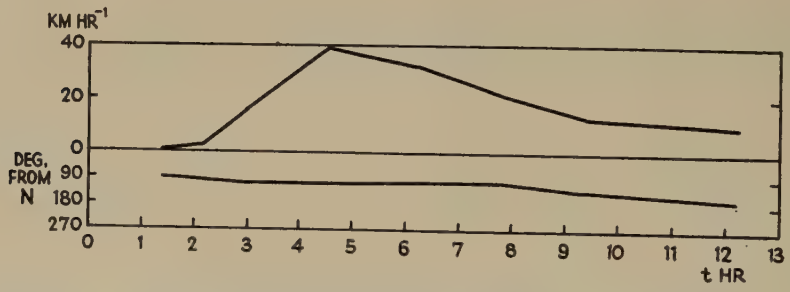


Figure 12. The onset of the sea-breeze 20 km inland. Coastline N-S. Sea to E.

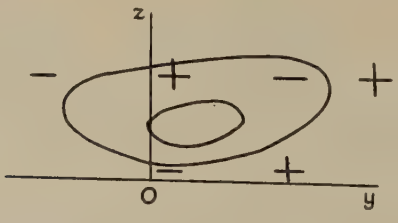


Figure 13. Distribution of the sign of the vertical component of vorticity $\left(-\frac{\partial u}{\partial y}\right)$.

5. The tidal component of the solution

In this section the divergent component of the velocity field is calculated (Eqs. (36) and (37)). This is caused by the velocity source resulting from the heating and consequent expansion of the layer of air in contact with the land surface, and its effect is to produce a tide which transfers mass from land to sea. A solution is obtained for a heated layer extending uniformly inland from the coastline, neglecting modifications of the convection region by the sea-breeze. The effect of the latter in the case of a large land mass is very

small since the tide extends for distances of order of thousands of km while the sea-breeze is mainly concentrated within 100 km of the coastline. First a simpler problem, that of calculating the tide produced by slowly raising half the bed of an ocean of uniform depth, is considered. Although gravity is solely responsible for the propagation in this case, while compressibility is responsible in the atmosphere, the form of the solution is found to be the same in both cases.

(a) *The atmosphere replaced by an incompressible fluid*

Let the depth of ocean be D, its density be ρ,and the surface elevation be $\zeta(y, t)$. The excess of pressure at any level is $g\rho\zeta$. Thus v is independent of z and continuity of mass for a vertical column is expressed by

$$D\frac{\partial v}{\partial y} = \frac{\partial}{\partial t}(\zeta_1 - \zeta) \qquad (43)$$

The equations of horizontal motion, neglecting second order terms, are

$$-g\frac{\partial \zeta}{\partial y} = \frac{\partial v}{\partial t} + lu \qquad (44)$$

$$0 = \frac{\partial u}{\partial t} - lv \qquad (45)$$

The boundary condition is taken as $\zeta_1 = 0$ for $y < 0$, and ζ_1 is a function of t only for $y > 0$. Thus, except at the origin, $\frac{\partial \zeta_1}{\partial y} = 0$, and Eq. (44) may be written as

$$-g\frac{\partial}{\partial y}(\zeta - \zeta_1) = \frac{\partial v}{\partial t} + lu \qquad (46)$$

Eqs. (43), (45) and (46) combine to give the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - gD\frac{\partial^2}{\partial y^2} + l^2\right)(\zeta - \zeta_1) = 0 \qquad (47)$$

This equation, which is also satisfied by u and v, has a general solution of the form

$$\zeta - \zeta_1 = A\,e^{\alpha y} + B\,e^{-\alpha y}$$

for waves of period $\frac{2\pi}{\sigma}$, where

$$\alpha = \sqrt{l^2 - \sigma^2}/V \quad \text{and} \quad V^2 = gD \qquad (48)$$

Continuity of ζ and v at $y = 0$ gives the following solution which is finite at $y = \pm\infty$:

$$\left.\begin{aligned} \zeta &= \left(Z_1 - \frac{Z_1}{2}e^{-\alpha y}\right)e^{i\sigma t}, \quad y > 0 \\ \zeta &= \frac{Z_1}{2}e^{\alpha y}\,e^{i\sigma t}, \quad y < 0 \end{aligned}\right\} \qquad (49)$$

where

$$\zeta_1 = Z_1\,e^{i\sigma t}, \quad y > 0$$

$$\zeta_1 = 0, \quad y < 0$$

and Z_1 is a constant.

The pressure distribution is simply given by

$$p' = g\rho\zeta \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (50)$$

and the velocity distribution is given by

$$\left.\begin{aligned} \frac{v}{i\sigma} = \frac{u}{l} &= -\frac{(\zeta_1 - \zeta)}{\alpha D} \quad , \quad y > 0 \\ \frac{v}{i\sigma} = \frac{u}{l} &= -\frac{\zeta}{\alpha D} \quad , \quad y < 0 \end{aligned}\right\} \qquad . \qquad . \qquad (51)$$

For $\sigma > l$, α is imaginary and wave propagation occurs with p' and v in phase, and u differing in phase from p' by $\frac{1}{2}\pi$. As $l \to \sigma$ the velocity of propagation increases, until when $l = \sigma$ a state of resonance occurs with $\frac{\partial \zeta}{\partial y}$ equal to zero at all times. Where $\sigma < l$, ζ falls off exponentially on either side of the origin, p' and $-u$ being in phase, and v differing in phase from p' by $\frac{1}{2}\pi$. It must be borne in mind when interpreting these results that their application on a global scale is not justified since on this scale a spherical polar coordinate system should be used.

Since v and $\frac{\partial v}{\partial y}$ are independent of z and the fluid is incompressible, w is linear in z and is given by

$$w = w_1 + \frac{z}{D}(w_s - w_1) \qquad . \qquad . \qquad . \qquad . \qquad (52)$$

where

$$w_s = \frac{\partial \zeta}{\partial t}$$

w_1 is defined as discontinuous at $y = 0$, and Eq. (52) thus implies w in the fluid to be discontinuous at $y = 0$. This impossible result arises from the neglect of the term $\frac{dw}{dt}$, which includes $v\frac{\partial w}{\partial y}$, in using the hydrostatic equation. $\frac{\partial w}{\partial t}$ associated with the expansion of the heated strip of the atmosphere can certainly be neglected in comparison with g, and $v\frac{\partial w}{\partial y}$ is of the same order as this term if the change in w_1 across the coastline takes place through a horizontal distance of order 10 km (Pearce 1953). This complication can therefore be disregarded since in the atmosphere the discontinuity in the boundary condition is smoothed out to this extent by the sea-breeze. Figs. 14 (a), (b) and (c) illustrate the solution for $\sigma < l$.

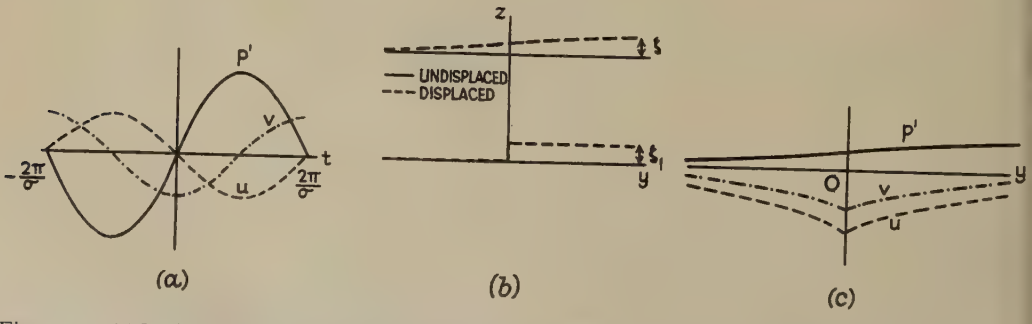


Figure 14. (a) Variation of p', u and v with time. (b) Displacement of the ocean surface produced by varying periodically the level of half the ocean bed. (c) p', u and v as functions of y.

(b) *The tide produced in an isothermal atmosphere by varying the land level*

The equations of motion, continuity and adiabatic motion, are taken as

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial v}{\partial t} + lu \quad . \quad . \quad . \quad . \quad (53)$$

$$0 = \frac{\partial u}{\partial t} - lv \quad . \quad . \quad . \quad . \quad (54)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g + \frac{\partial w}{\partial t} \quad . \quad . \quad . \quad . \quad (55)$$

$$\frac{\partial w}{\partial z} + \frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial \rho}{\partial t} \quad . \quad . \quad . \quad . \quad (56)$$

$$\frac{dp}{dt} = c^2\frac{d\rho}{dt} \quad . \quad . \quad . \quad . \quad (57)$$

Linearization of the equations of motion by replacing $\frac{d}{dt}$ by $\frac{\partial}{\partial t}$ is possible here since the advection terms are very small compared with local rates of change. Following the method given by Lamb (1932), these equations give

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial y^2} - l^2 u + c^2 l\frac{\partial}{\partial y}\left(\frac{\partial \zeta}{\partial z} - \frac{g\zeta}{c^2}\right) \quad . \quad . \quad . \quad . \quad (58)$$

$$\frac{\partial^2 v}{\partial t^2} = c^2\frac{\partial^2 v}{\partial y^2} - l^2 v + c^2\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial z} - \frac{gw}{c^2}\right) \quad . \quad . \quad . \quad . \quad (59)$$

$$\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial}{\partial z}\left(\frac{\partial w}{\partial z} - \frac{\kappa g w}{c^2}\right) + c^2\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z} - \frac{(\kappa-1)gv}{c^2}\right) \quad . \quad . \quad . \quad (60)$$

$$\frac{\partial^2 p'}{\partial t^2} = c^2\frac{\partial^2 p'}{\partial y^2} - l^2 p' - c^2\rho_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial \zeta}{\partial z} - \frac{g\zeta}{c^2}\right) - c^2\rho_0 l^2\left(\frac{\partial \zeta}{\partial z} - \frac{g\zeta}{c^2}\right) \quad . \quad (61)$$

noting $\zeta = \int w dt$. Putting $w = 0$ gives the solution for horizontal waves; each of Eqs. (58), (59) and (61) reduces to the same form as Eq. (47) and u, v and p' are the same functions of y as in Eqs. (50) and (51). But to satisfy Eq. (60) v must in this case be a function of z given by

$$v = v_1 \exp\left\{\frac{(\kappa-1)gz}{c^2}\right\} \quad . \quad . \quad . \quad . \quad (62)$$

and thus from Eq. (53), u and p'/ρ_0 must be similar functions of z. Physically this is because the atmosphere is compressible and the wave is propagated not as a gravity wave as in the ocean but as a compression wave, in which horizontal density gradients are produced. Gravity acting on these gives rise to a horizontal component of vorticity.

Since in the problem considered w_1 is not zero, the horizontal free-wave solution cannot be used and it is necessary to obtain the differential equation containing w explicitly. This is found from Eqs. (59) and (60). Operating on Eq. (60) by $\frac{\partial}{\partial y}$ and taking a time factor $e^{i\sigma t}$ gives

$$\frac{\sigma^2}{c^2}\frac{\partial w}{\partial y} + \frac{\partial^2}{\partial z\,\partial y}\left(\frac{\partial w}{\partial z} - \frac{\kappa g w}{c^2}\right) + \left(\frac{\partial}{\partial z} - \frac{(\kappa-1)g}{c^2}\right)\frac{\partial^2 v}{\partial y^2} = 0$$

$\frac{\partial^2 v}{\partial y^2}$ is next substituted from Eq. (59), then the resulting equation is differentiated partially with respect to y and v finally eliminated by substituting for $\left(\frac{\partial}{\partial z} - \frac{(\kappa - 1)g}{c^2}\right)\frac{\partial v}{\partial y}$ from Eq. (60). The resulting equation for w is

$$\frac{\partial^2 w}{\partial z^2} + \left\{\frac{(\kappa - 1)g^2}{(l^2 - \sigma^2)c^2} - \frac{\sigma^2}{l^2 - \sigma^2}\right\}\frac{\partial^2 w}{\partial y^2} - \frac{\kappa g}{c^2}\frac{\partial w}{\partial z} + \frac{\sigma^2}{c^2}w = 0$$

For periods of order a day the term $\frac{\sigma^2 w}{c^2}$ is of order $10^{-7}\ w\ \mathrm{km^{-1}\ hr^{-1}}$, while the other terms are of order $10^{-3}\ w\ \mathrm{km^{-1}\ hr^{-1}}$ or larger over the main part of the region considered. Neglecting this term, changing the scale of y and writing $\frac{\kappa g}{c^2} = 2r$ then gives as the differential equation for w

$$\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial y^2} - 2r\frac{\partial w}{\partial z} = 0 \qquad . \qquad . \qquad . \qquad . \qquad (63)$$

The general solution which is finite as $z \to +\infty$ is

$$w = \int_{-\infty}^{+\infty} A_k\, e^{iky} \exp\left\{(r - \sqrt{r^2 + k^2})\, z\right\} dk$$

The boundary condition at $z = 0$ is $w = w_1$, $y > 0$, and $w = 0$, $y < 0$, which may be written

$$w = \frac{w_1}{2} + \frac{w_1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin ky}{k}\, dk$$

Thus the solution required is

$$w\,(y, z) = \frac{w_1}{2} + \frac{w_1}{\pi}\int_0^{\infty} \frac{\sin ky}{k} \exp\left\{(r - \sqrt{r^2 + k^2})\, z\right\} dk \qquad . \qquad (64)$$

The vertical displacement is given by a similar expression :

$$\zeta\,(y, z) = \frac{\zeta_1}{2} + \frac{\zeta_1}{\pi}\int_0^{\infty} \frac{\sin ky}{k} \exp\left\{(r - \sqrt{r^2 + k^2})\, z\right\} dk \qquad . \qquad (65)$$

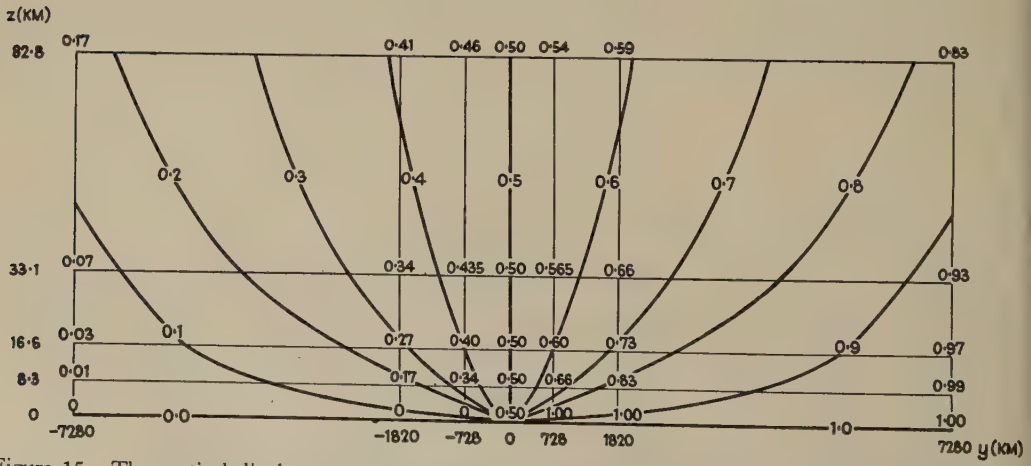


Figure 15. The vertical displacement of an isothermal atmosphere when the land level is slowly varied. The isopleths are of $\frac{w}{w_1}$ or $\frac{\zeta}{\zeta_1}$.

Because of the surd in the exponential the integral on the right-hand side of these quations cannot be evaluated in terms of simple functions and a numerical method ust be used. A few values have been computed and the results are shown in Fig. 15. he Euler-Maclaurin formula, which reduces to the simple trapezium rule in this case, as used for each integration and the accuracy of the results checked by recomputing ith twice the interval size. The constants are taken as $l = 0{\cdot}4\ \text{hr}^{-1}$ ($\phi = 49°49'$), $\frac{\pi}{\sigma} = 24$ hr, $c = 1{,}212\ \text{km hr}^{-1}$. By changing k to k/y and writing the integrand as $\frac{\sin k}{k} \exp\left[rz\left\{1 - \left(1 + \frac{k^2}{r^2 y^2}\right)^{\frac{1}{2}}\right\}\right]$ the integral is seen to tend to $\frac{1}{2}\pi$ as y tends to $\pm\infty$ or all finite z. Hence $w \to w_1$ as $y \to +\infty$ and $w \to 0$ as $y \to -\infty$. The convergence o these values becomes slower as z increases. The distributions of ζ and w are broadly he same as for an incompressible fluid, though they are smoother functions of y near $= 0$ in the atmosphere.

p', u and v are now found by substituting Eqs. (64) and (65) into Eqs. (61), (58), 59) and (60). These substitutions require the partial derivatives with respect to y and of the infinite integral. These are respectively $\int_0^\infty \cos ky \exp\{(r - \sqrt{r^2 + k^2})\, z\}\, dk$ and $\int_0^\infty \frac{\sin ky}{k}(r - \sqrt{r^2 + k^2}) \exp\{(r - \sqrt{r^2 + k^2})\, z\}\, dk$. Both these integrals converge for inite values of y and z except at $z = 0$, though the first is known to be zero at $z = 0$ except at $y = 0$. The integrand of the second behaves like $\sin ky$ at $z = 0$ as $k \to \infty$.

Eq. (61) which gives the horizontal distribution of p' at any level z may be written

$$\frac{\partial^2 p'}{\partial y^2} - \alpha^2 p' = \alpha^2 \rho_0 \left(c^2 \frac{\partial \zeta}{\partial z} - g\zeta\right) \quad . \quad . \quad . \quad (66)$$

where

$$\alpha^2 = \frac{l^2 - \sigma^2}{c^2} \quad . \quad . \quad . \quad . \quad . \quad (67)$$

The right-hand side of this equation becomes a function of y and z on substituting Eq. (65). Again a solution in terms of simple functions cannot be obtained, and p' can only be expressed in terms of y and z by integrating numerically with respect to y at each level. The solution obtained for ζ shows however that in the lowest 10 km of the atmosphere $\frac{\partial \zeta}{\partial z}$ is very small compared with $\frac{g\zeta}{c^2}$ and ζ itself, except near $y = 0$, is very nearly equal to its value at the surface; i.e. $\frac{\partial^2 \zeta}{\partial y^2} \simeq 0$ except at $y = 0$.

Thus putting

$$\delta p\,(z, t) = g\rho_0\,(z)\,\zeta_1\,(t) \quad . \quad . \quad . \quad . \quad (68)$$

Eq. (66) becomes approximately

$$\left(\frac{\partial^2}{\partial y^2} - \alpha^2\right)(p' - \delta p) = 0 \quad . \quad . \quad . \quad . \quad (69)$$

in the lower layers. The solution of this equation which remains finite as $y \to \pm\infty$ and satisfies continuity of p' and v at $y = 0$ is

$$\left.\begin{aligned} p' &= \delta p - \frac{\delta p}{2} e^{-\alpha y}, \quad y > 0 \\ p' &= \frac{\delta p}{2} e^{\alpha y}, \quad y < 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (70)$$

and the corresponding values of u and v satisfying Eqs. (65) and (66) are

$$\left.\begin{aligned} \frac{v}{i\sigma} = \frac{u}{l} &= \frac{p' - \delta p}{\rho_0 \alpha c^2}, \quad y > 0 \\ \frac{v}{i\sigma} = \frac{u}{l} &= -\frac{p'}{\rho_0 \alpha c^2}, \quad y < 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (71)$$

Comparison with Eqs. (49), (50) and (51) therefore shows that the lower layers of the atmosphere, disturbed in this way, behave in very nearly the same way as would an equivalent incompressible fluid. The distributions of u, v and p' are as shown in Fig. 14 (c).

The solution of Eq. (66) for large values of z has not been computed. It differs mainly from the incompressible case in having larger values of u and v at higher levels, and since $\frac{\partial^2 \zeta}{\partial y^2}$ is not zero at these levels, u and v, though tending to zero as $y \to \pm \infty$ do not do so exponentially.

(c) The tide produced in an isothermal atmosphere by heating the lower layers

$\frac{d\theta}{dt}$ is no longer zero everywhere and Eq. (57) becomes

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\kappa p}\frac{dp}{dt} - G \quad . \quad . \quad . \quad . \quad (57')$$

where G is zero everywhere except in the region $0 \leqslant z < H$, $y > 0$. Carrying out the analysis as before we obtain the following equation for w instead of Eq. (63).

$$\frac{\partial}{\partial z}\left(\frac{\partial w}{\partial z} - G\right) + \frac{\partial^2 w}{\partial y^2} - 2r\left(\frac{\partial w}{\partial z} - G\right) = \frac{c^2}{(\kappa - 1)g}\frac{\partial^2 G}{\partial y^2} \quad . \quad . \quad (63')$$

and the boundary condition at $z = 0$ becomes

$$w = 0 \text{ for all } y$$

The equations for u, v and p' also contain the combination $\frac{\partial w}{\partial z} - G$ where only $\frac{\partial w}{\partial z}$ occurs in Eqs. (58) to (61). To solve Eq. (63'), a function w^* is defined by the equation

$$w^* = w + \int_z^H G\, dz \quad . \quad . \quad . \quad . \quad (72)$$

Then since w satisfies Eq. (63'), w^* must satisfy

$$\frac{\partial^2 w^*}{\partial z^2} + \frac{\partial^2 w^*}{\partial y^2} - 2r\frac{\partial w^*}{\partial z} = \frac{c^2}{(\kappa - 1)g}\frac{\partial^2 G}{\partial y^2} + \int_z^H \frac{\partial^2 G}{\partial y^2}\, dz \quad . \quad . \quad (73)$$

and the boundary condition at $z = 0$

$$w^* = 0, \quad y < 0\,; \qquad w^* = \int_0^H G\, dz, \quad y > 0.$$

Eq. (73) has been derived from the five equations (53) to (57), and, apart from the neglect of advection terms, must therefore when integrated give the complete vertical velocity field including that associated with the sea-breeze circulation. $\frac{\partial G}{\partial y}$ and $\frac{\partial^2 G}{\partial y^2}$ are non-zero only in the region S since the boundary of S is defined to be where the vorticity and therefore $\frac{\partial G}{\partial y}$ become zero. The component of w required here is the one which

plies everywhere outside S and is continuous through this region, but does not depend n the generation of vorticity inside S, i.e. on $\frac{\partial G}{\partial y}$ or $\frac{\partial^2 G}{\partial y^2}$. Thus the tidal component f w^* satisfies the equation

$$\frac{\partial^2 w^*}{\partial z^2} + \frac{\partial^2 w^*}{\partial y^2} - 2r\frac{\partial w^*}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad (74)$$

e., w^* satisfies the same differential equation and boundary conditions as w in sub-section p), and is given by Eq. (64). Similarly $\zeta^* \left(= \int w^* \, dt\right)$ is given by Eq. (65). w_1 in Eq. (64) $\int_0^H G \, dz$, which is equal to GH outside S, and ζ_1 in Eq. (65) is $\int_0^H \frac{T'}{T_0} \, dz$ outside S. ince w^* and ζ^* are nearly constant at these surface values in the lower layers, the solution n the convection region is thus simply

$$w = G\,z, \quad \zeta = \int_0^z \frac{T'}{T_0} \, dz\,; \qquad y > 0, \quad 0 \leqslant z \leqslant H \quad . \quad . \quad (75)$$

Thus for all levels above $z = H$, outside S where $G = 0$, ζ and hence p', u and v ave the same values as in the problem where the land level was varied. Between the urface and $z = H$, p', in order to satisfy the hydrostatic equation, has the same value or $y < 0$, but for $y > 0$, $(p' - g\rho_0 \zeta)$ must here replace $(p' - \delta p)$ obtained in the former ase. Inside S the integral for ζ in Eq. (75) extends up to the upper boundary. u and have the same values as in the former problem since $\frac{\partial G}{\partial y} = 0$ and $\frac{\partial p'}{\partial y}$ is the same. These olutions are shown in Fig. 16 and may be expressed analytically as

$$\left.\begin{aligned} p' - g\rho_0 \zeta &= -\frac{\delta p}{2} e^{-\alpha y} \quad , \quad y > 0 \\ p' &= \frac{\delta p}{2} e^{\alpha y} \quad , \quad y < 0 \\ \frac{u}{l} = \frac{v}{i\sigma} &= -\frac{1}{\alpha \rho_0 c^2} \frac{\delta p}{2} e^{\mp \alpha y} \quad , \quad y \gtrless 0 \end{aligned}\right\} \quad . \quad . \quad (76)$$

^{2} has been taken as constant everywhere, though actually it changes with T where G is ot zero. The effect of this on the above results is very small and has not been calculated.

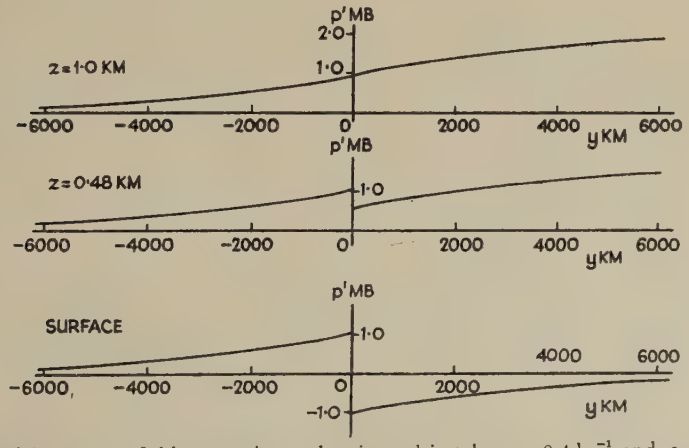


Figure 16. The tidal pressure field at maximum heating. l is taken as 0·4 hr^{-1} and c as 1,212 km hr^{-1}, $\sigma = \frac{\pi}{12}$ hr^{-1}.

(d) *Application to a finite land mass*

The land is regarded as being of finite width $2L$ in the direction of the y-axis, and bounded on both sides by very large oceans. The motion produced by heating the land surface must be symmetrical about a vertical axis Oz at the centre of the strip. The boundary conditions to be satisfied are thus that u and v tend to zero as $y \to \pm\infty$ and $\frac{\partial p}{\partial y}$, u and v are all zero at $y = 0$. Approximate solutions for the lower layers, satisfying these conditions, can be written down by regarding the motion as compounded of two solutions of the kind already obtained diverging from $y = +L$ and $y = -L$. These solutions are

$$\left.\begin{aligned} p' &= \pm\frac{\delta p}{2}\left[\exp\left\{\mp\alpha(|y| - L)\right\} \mp \exp\left\{-\alpha(|y| + L)\right\}\right], \; |y| \gtrless L \\ \frac{u}{l} &= \frac{v}{i\sigma} = -\frac{1}{\alpha\rho_0 c^2}\frac{\delta p}{2}\left[\exp\left\{-\alpha|y + L|\right\} - \exp\left\{-\alpha|y - L|\right\}\right] \end{aligned}\right\} \quad . \quad (77)$$

Graphs are drawn in Fig. 17 showing the values of p' and u for a land strip of width 1,212 km when the heating function is at its maximum value. Since this occurs after a quarter period, v is everywhere zero. The constant values are taken as above.

The smaller the width of land the nearer does p_H' tend to zero, and p_1' to $-\delta p$, although, before the limit is reached, the sea-breezes from each coast meet and considerably modify the heating region. It is also seen from Fig. 16 that for a very large land mass p_H' is nearly equal to $\frac{1}{2}\delta p$ for the whole width of the sea-breeze region and p_1' is equal to $\mp\frac{1}{2}\delta p$ for $y \gtrless 0$. Thus the above theory may be summed up for the purpose of giving pressure boundary-values round the region S by saying that for a small land area, i.e., of dimensions less than 1,000 km, $p' \simeq 0$ at the upper boundary for all y, while for a continent $p' \simeq \frac{1}{2}\delta p$ on this boundary. In the former case all the air displaced vertically over the heated land may be regarded as distributed by the tide over the very large sea area.

(e) *The modification of the tide by the sea-breeze*

The actual source distribution is given by $\frac{gw_R}{c^2} + G$, the right-hand side of Eq. (36), G being modified by advection in the sea-breeze region as given by Eq. (10′). However, the latter has very little effect on the tide since the boundary condition for w^* is calculated using $\int_0^H G\,dz$, i.e. from the total amount of heat entering a vertical column, $\frac{\partial Q}{\partial t}$, which is taken as constant in y for $y > 0$. Thus only the distribution of $\frac{gw_R}{c^2}$ need be considered. Where w_R is negative, i.e. subsidence occurs, this term produces the same

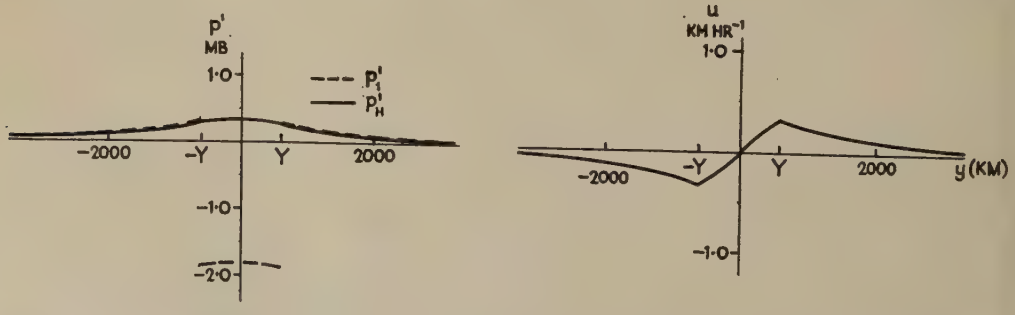


Figure 17. The tidal pressure and velocity distribution across a long strip of land about 1,200 km wide.

ct as would cooling of an atmosphere assumed to remain static. This is because air is compressed in its descent and the volume of a given mass decreases. Although abatic warming takes place, it is because it is produced dynamically that the region s as a sink and not as a source. Where w_R is positive, the converse occurs.

w^* is found by evaluating $\int_0^q \frac{g w_R}{c^2} dz$, $z = q$ being the upper boundary of S. Its gnitude is found to be of the same order as GH, e.g., at $t = 4{\cdot}6$ hr, $\frac{H}{\theta}\frac{d\theta}{dt} \simeq \frac{1}{200}$ km hr^{-1}. $y = 12{\cdot}6$ km, taking the maximum value of w_R as $0{\cdot}25$ km hr^{-1} at $z = 0{\cdot}5$ km, and a linear tribution of w_R on either side of $z = 0{\cdot}5$ km, w_R being zero at $z = 1$ km and the surface, ves $\int_0^q \frac{g w_R}{c^2} dz \simeq \frac{1}{80}$ km hr^{-1}. This is the maximum value on the pattern. As w_R decreases ay from the centre of the circulation, so does the integral. On the boundary of S value is zero. Thus superposed on the source distribution, G, is a source-sink distbution of the same order of magnitude, localized near O. The effect on the tide already rived is found by superposing the correction to w^*, but, when the region affected by e sea-breeze is small compared with the total region heated, the correction to the tide also small.

6. Discussion of the results

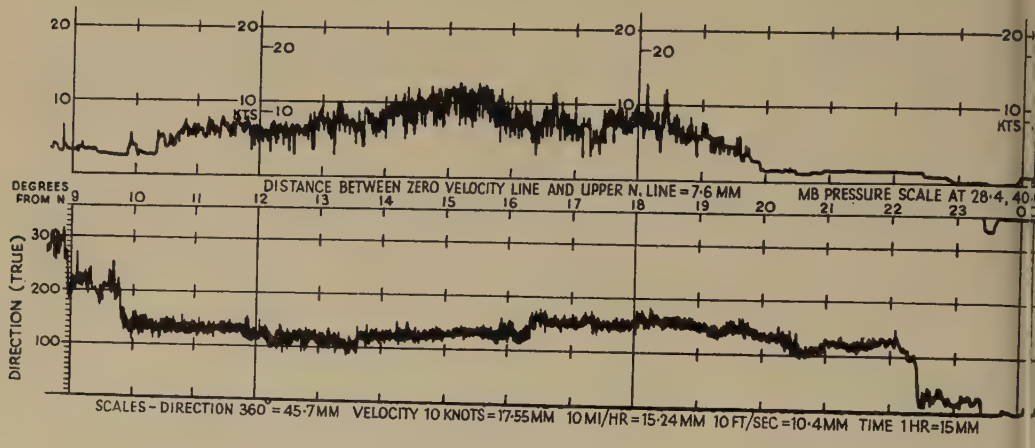
(*a*) *Comparison with observations*

The first impression received on examining wind records at a coastal station is that ere are very few occasions when the atmosphere is sufficiently calm for the sea-breeze be the only disturbance. There are plenty of occasions on which sea-breezes occur, ut there is nearly always a general wind superposed. For instance, examination of the cords for Leuchars, Fife, for April to September 1948 (the other months of the year hen the diurnal heating is small not being considered) revealed about 55 sea-breeze ccasions, only one of which could be regarded as superposed on an initially static atmosphere. Two noticeable features of these records are the sudden onset of the sea-breeze hen a light off-shore wind is superposed and the effect of cloudiness, which is to diminish onsiderably the sea-breeze velocities.

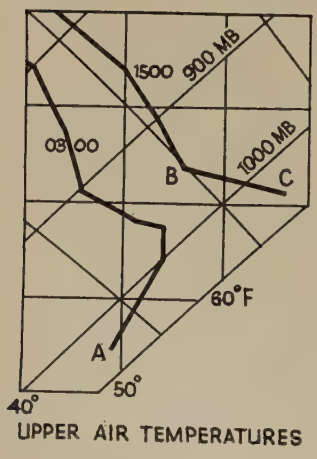
Below are reproduced records of wind and temperature for an occasion at Leuchars hen the atmosphere was almost calm before and after the sea-breeze, and a wind record or Pendennis Castle made on a similar occasion. These are compared with the above eoretical results. Although this comparison is only relevant in broad detail it serves illustrate the complexity of the practical problem. The main complication at Leuchars the effect of the Fife Ness peninsula which breaks up a coastline lying approximately 10°-30° from Kirkcaldy to beyond Arbroath, neglecting the Firth of Tay and the land n the south side of the Firth of Forth. The heating functions in the two cases are omparable. In units of large squares on the tephigram, the maximum theoretical heating unction is 1·62 units. From the soundings made at Leuchars, the triangle ABC on the ephigram, which represents heating modified by advection, is 0·96 units.

The initial inversion to 1,500 ft (03 GMT ascent), by restricting the sea-breeze to a allower layer than would an isothermal atmosphere in the early stages of the heating, probably the main reason for the observed smaller velocities in the early stages; iction has larger control over a shallow sea-breeze than over a deeper one. The rather ore pronounced increase from 12 GMT may be due to the more rapid increase in depth f the sea-breeze as the top of the convection layer extends above the top of the inversion. n considering the upper return flow, it is interesting that the winds between 1,000 ft nd 6,000 ft at 15 GMT all show a larger westerly component than at either 09 or 21 GMT.

The maximum speed obtained in the computation is 40 km hr^{-1} at 6·2 hr at a distance 38·4 km inland, and the maximum penetration about 80 km inland. These values are rather high because surface friction was neglected. The introduction of a square-law assumption into the computation has been found to be a much simpler matter than was at first thought when the above computation was started, and it is hoped later to publish results for this more realistic case.


KTS
DEGREES FROM N
DISTANCE BETWEEN ZERO VELOCITY LINE AND UPPER N. LINE = 7·6 MM
MB PRESSURE SCALE AT 28·4, 40
DIRECTION (TRUE)
SCALES – DIRECTION 360° = 45·7 MM VELOCITY 10 KNOTS = 17·55 MM 10 MI/HR = 15·24 MM 10 FT/SEC = 10·4 MM TIME 1 HR = 15 MM

(i) Leuchars anemogram.

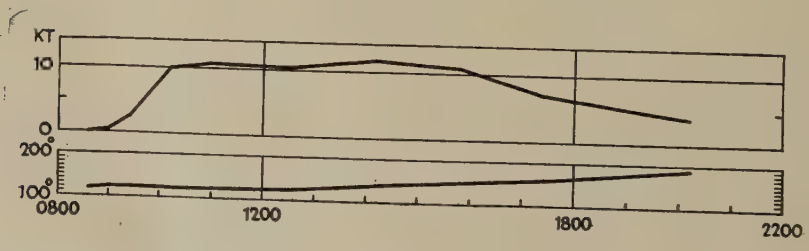


Upper winds

	0900		2100			1500	
Ft	Dirn	Kt	Dirn	Kt	Ft	Dirn	Kt
Surf	280	04	140	04		120	08
1,000	210	04	140	06		258	07
2,000	223	03	155	03	2,500	343	07
3,000	063	05	130	03			
4,000	017	07	103	05		210	07
5,000	006	03	106	10	5,500	158	04
6,000	026	04	117	13	7,100	135	04
8,000	323	07	137	07	8,900	149	03

(ii) Upper-air temperatures and winds.

(a)



(b)

Figure 18. Sea-breeze recorded at Leuchars, 4 July 1952. (a) Leuchars anemogram, upper temperatures and winds. (b) Anemogram constructed from patterns taking t_0 = 08 GMT, y = 5 km and coastline 210°-030°.

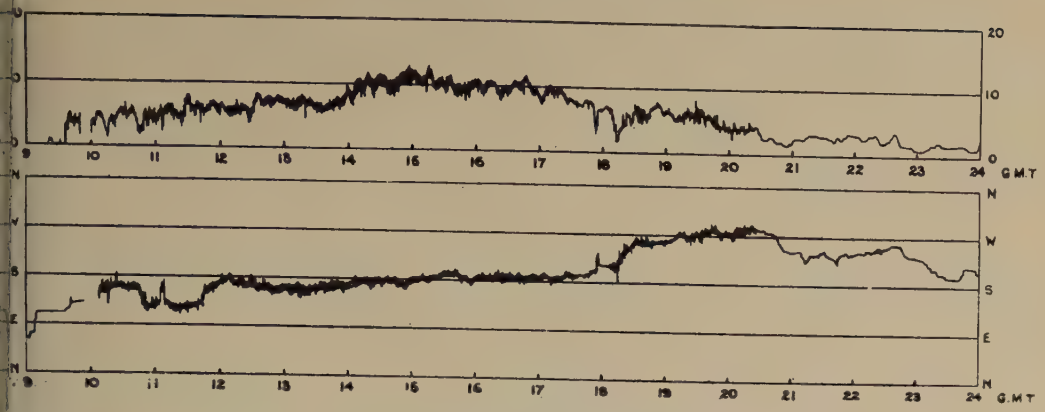


Figure 19. Sea-breeze recorded at Pendennis Castle, 31 Aug. 1941.

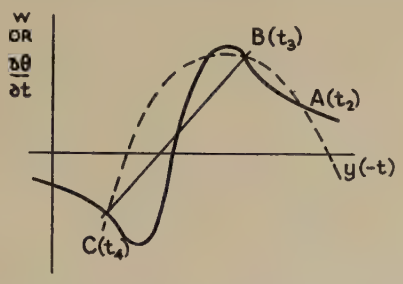


Figure 20. Form of $w\,(y)$ and $\frac{\partial\theta}{\partial t}\,(t)$ at a mesh point near the circulation centre (full curve). Pecked line shows Simpson's Rule approximation.

(*b*) *The accuracy of the method*

Over most of the region S, the first order differential coefficients in Eqs. (13), (14), 20) and (21) are smooth and higher-order terms small in comparison; the accuracy of he computation as measured by the general agreement in all the finite difference equations s satisfactory, and could, if required, be progressively improved by decreasing the space neshes and time intervals. The main difficulties arise near the centre of the circulation vhere, for instance, w is of the form shown in Fig. 20. This is also the general form f the time change in w at a mesh point through which the centre of the circulation passes, nd hence, also of $\frac{\partial\theta}{\partial t}\left(\text{neglecting } v\,\frac{\partial\theta}{\partial y}\right)$. It is seen that applying Simpson's Rule to the alues A, B and C gives quite a false result; the trapezium rule through B and C is etter. Thus near the centre of the disturbance it is necessary to interpolate values of θ. The variations in $\frac{\partial Q}{\partial t}$ near the centre also arise from a similar effect. Only by reducing he time intervals and mesh sizes considerably can this region of interpolation be narrowed lown. The results obtained, however, suggest that the mesh sizes and time intervals hosen are quite small enough, and in the right proportion, to determine the main features f the sea-breeze.

It should be noted that no stability difficulties arise in the computation; the com- arison of the pressure distributions, calculated in two independent ways and satisfying he boundary conditions round S at each stage, prevents instability arising. An error in he velocity field tends to alter the temperature distribution in such a way as to require he velocity error to be removed for consistency of the pressure solutions.

(c) *General properties of the solution*

The patterns represent the spread inland of a tongue of colder air from over th sea. Its movement is at first slow, becomes more rapid as heating continues, but eventuall slows down as the wind blows more nearly parallel with the coast. The onset of the sea breeze well inland does not occur until a few hours after heating commences and, when does occur, the wind increases fairly rapidly and is accompanied by a falling off of tempera ture. The circulation also spreads out to sea, the velocities over the sea being smaller tha those inland near the centre of the circulation which is situated near the tip of the advancin tongue of colder air.

A persistent feature of the wind component parallel to the coast is the surfac minimum just inland, which results from the frictional drag of the returning air above This has the effect of maintaining the wind almost normal to the coast, for a time, jus inland. It may be the explanation of the reluctance of the wind to veer as shown b the above anemograms for Leuchars and Pendennis Castle, but the shape of the coastlin may be the determining feature.

A feature of the patterns which has not been confirmed by direct observation is th upper reverse flow. There can be no doubt that this flow exists, but it is of small velocit (5 km hr^{-1} at most), and mainly concentrated near the centre of the disturbance, so tha it is a difficult effect to deduce with certainty from pilot-balloon observations. It ma be that the tendency for the wind direction to remain steady for a period, just inland fron the coast, provides the best means of verification.

Observations of the fine structure of the pressure field and of the finer details o wind and temperature are not available, so that it is not possible to look for such feature as the dynamical production of a shallow layer of instability in the upper return flow, an the sharpening of the discontinuity in lapse rate at the top of the convection layer, which is also produced dynamically. The difficulty of making such observations is a seriou disadvantage in choosing to study the sea-breeze as an example of motion produced by differential heating. It might be possible by the kind of analysis used here to show tha such properties as these should occur on the larger scale of the depression or genera circulation, and are in fact general properties of a differentially heated atmosphere; the differential equations and boundary conditions are much the same. On this scale many more observations become available.

The separation of the motion into the tide and sea-breeze components has an important large-scale application over a continent surrounded by sea. Because it is a concentrated disturbance of comparatively large velocity, the sea-breeze extends only a small distance inland, being restricted by the Coriolis effect, whereas the tide extends over the whole continent. Thus if the land is progressively heated over a period of days, i.e., nocturnal cooling is less than daytime heating, a deficit of mass results inland where the sea-breeze does not penetrate, and a ' continental low ' forms. The opposite effect occurs in early winter, the obvious example being the establishment of the Siberian anticyclone. A full treatment of these effects must, however, take account of the variation of the Coriolis parameter with latitude.

Acknowledgment

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e work to be undertaken, and to the Meteorological Office, London, for making available
ie meteorological records which are here reproduced, and granting permission to publish
iem. The anemogram for Pendennis Castle, which is taken from a Meteorological
)ffice publication, is Crown copyright and is reproduced by permission of the Controller
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Theory of air flow over a heated land mass

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SUMMARY

The velocity field of a stable atmosphere over and around a heated land mass is found, as a function of both time and distance, by consideration of the equations of motion, continuity and added heat. A heat function is introduced, in the last of these, which is not only fairly simple mathematically but also bears a good resemblance to the atmospheric picture. Certain terms are neglected initially to simplify the mathematics of the problem but they are used later to check the validity of the results and it is found that, in general, the solution would have been negligibly affected by the inclusion of these terms. There is difficulty in using the boundary condition at infinite height but this is overcome by evaluating the problem by two methods.

The numerical work was done on the EDSAC at Cambridge and some graphs of these results, using a normal type of ground temperature distribution and lapse rate, are shown.

1. INTRODUCTION

This paper describes a mathematical treatment of the effects of diurnal variation of heating of a land mass, 200 km wide, which is taken as a horizontal strip, assumed infinite in the y-direction, and whose surface is flat, i.e., the contours of the ground are assumed to have no effect on the resultant motion.

Malkus and Stern (1953) have attempted to find the velocity field over an island 10 km in width. They are concerned with a specified time and use a Fourier series instead of a Fourier integral to obtain a single isolated pulse. They find that the upper boundary condition is a major stumbling-block and a ' lid ' is introduced in one case to get the sea breeze in the correct sense. In Part II of the same work they introduce an ' equivalent mountain function,' arrived at by the form of the equations, to try to get a closer approximation. However, in the present paper the extent of the land mass is an order of magnitude greater so that the release of bubbles of air of cumulus size, which would play such an important part over an island 10 km wide, becomes much less significant, as will be shown when the heating function is discussed.

We shall first of all solve the initial equations and then discuss the form of the heating function to be used. This will then be used to solve the differential equation.

2. LIST OF SYMBOLS USED

b	' half-width ' of land mass
B	defined by Eq. (19)
C, D	arbitrary constants
f	twice the vertical component of the earth's rotation
g	gravity
H_0, H_1	defined by Eq. (15).
σ, k	wave-numbers of time and distance
n	constant determining the rate of exponential decrease of heating in the vertical
p_0	pressure
T	' half time ' (equivalent to ' half-width ') of heating

U horizontal velocity perpendicular to the coast
W vertical velocity
x, y, z horizontal coordinates perpendicular and parallel to the coast, and vertical coordinate
β stability; $\beta \times$ temperature = adiabatic lapse rate − actual lapse rate
τ_0, Γ reciprocal of potential temperature and disturbance thereof
ρ_0 density
λ defined by Eq. (19)
$'$ a prime represents $\frac{\partial}{\partial z}$
$\rightleftharpoons$ real part of

The following symbols are those used for the variable when they refer to one pair of values of σ, k :

$H(x, z, t)$ heating function
H_{1A}, H_{1B} defined by Eqs. (15a) and (15b)
p disturbance of pressure
u, v, w horizontal velocities perpendicular and parallel to the coast and vertical velocity
τ disturbance of reciprocal of potential temperature
ρ disturbance of density

3. The initial equations

Since the mathematics is dealing with a strip sensibly infinite in the y-direction we may put

$$\frac{\partial}{\partial y} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

wherever it may appear in the initial equations.

The viscous terms are neglected and the variations of velocity, temperature, etc., are taken as perturbation quantities. Thus the equations are linearized and the three equations of motion become :

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0 + \rho}\frac{\partial p}{\partial x} + f v \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{\partial v}{\partial t} = -f u \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_0 + \rho}\frac{\partial (p_0 + p)}{\partial z} - g \quad . \quad . \quad . \quad (4)$$

The neglect of $u\frac{\partial u}{\partial x}$ in the first equation in comparison with $\frac{\partial u}{\partial t}$ is equivalent to saying that the advection of momentum plays no great part in determining the value of u in ensuing time. The comparison of these two terms will serve as a basis for acceptance or rejection of the calculated results.

Neglecting the variation of density in the equation of continuity, we write :

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The neglecting of the $\frac{1}{\rho}\frac{d\rho}{dt}$ term from Eq. (5) requires some explanation. The author (1954) included this term in his treatment of the problem and found that it affected the

result negligibly. The extra mathematics showing this is of considerable length, and for that reason has been omitted from the present paper.

We now assume that the heat is added in the form of a function, $H(x, z, t)$, of horizontal distance, height and time, which is as yet undefined, and is connected with the inverse of potential temperature by the relation

$$\frac{D}{Dt}(\tau_0 + \tau) = H(x, z, t)$$

or, neglecting the smaller terms,

$$\frac{\partial \tau}{\partial t} + \tau_0' w = H(x, z, t) \quad . \quad . \quad . \quad . \quad (6)$$

When H is zero we are left with an equation for adiabatic motion.

The modified pressure $(\varpi_0 + \varpi)$ is introduced and defined (cf. Scorer 1951) in the equation

$$\frac{1}{\rho_0 + \rho}\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right\}(p_0 + p) = \frac{1}{\tau_0 + \tau}\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right\}(\varpi_0 + \varpi) \quad . \quad . \quad (7)$$

The assumption is now made that the solution of the problem can be represented by a Fourier integral and that u, v, w, τ, ϖ can be taken as proportional to $\exp\{-i(kx + \sigma t)\}$ where σ and k are the wave numbers for the time and distance respectively. These are now substituted in Eqs. (2) to (6) and the resulting equation in w is found to be

$$\frac{\partial^2 w}{\partial z^2} + \left(\frac{g\beta - \sigma^2}{\sigma^2 - f^2}\right) k^2 w + \frac{g H k^2}{\tau_0(\sigma^2 - f^2)} = 0 \quad . \quad . \quad . \quad . \quad (8)$$

The term in $\frac{\partial \dot{w}}{\partial z}$ is omitted as it can be shown to be negligible compared with the other terms.

We now proceed to discuss the form chosen for the heating function.

4. The heating function

On a normal sunny day during the period of heating, we should expect the lines of equal potential temperature for an originally stationary air mass to be somewhat similar to those in Fig. 1.

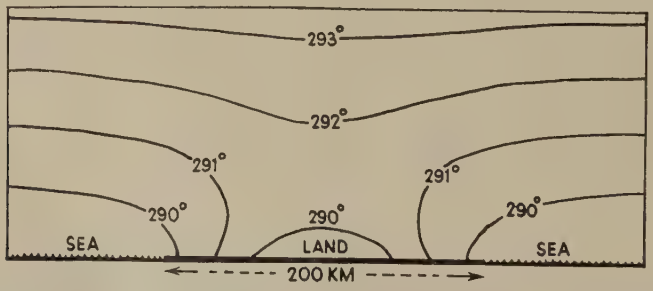


Figure 1. The diagram shows what might be a typical distribution of potential temperature over the land mass on a day when there is no overall velocity.

This situation is arrived at mainly by the upward transport of heat by bubbles of arying sizes rising over the land mass to a level which is approximately determined by eir original potential temperature and size (Malkus 1954). The larger ones, covering n area of a few km^2, may have a vertical velocity of the order of several metres per cond, while, nearer the ground, small-scale eddying is the major factor in this vertical ansport (Scorer 1954).

However, this paper does not pretend to evaluate the vagaries of localized vertical elocities over and around a land mass but attempts to find the velocity field, on a larger cale, resulting from the levelling by gravity of the surfaces of constant potential temperature Fig. 1. Thus the motions described above produce the time and space distribution of ur heating function and are not the solution to our problem. For this reason the final valuation is computed over steps of 20 km.

(*a*) *Horizontal variation of heating*

To be representative of a typical case we must take a heating function which increases irly rapidly at the coast to a maximum over the land and is negligible over the sea. It ust also be symmetrical about $x = 0$, the centre of the land mass.

Satisfying these conditions there are two well-known functions :

$$\int_0^\infty e^{-kb} \cos(k\,x)\, dk = \frac{b}{b^2 + x^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

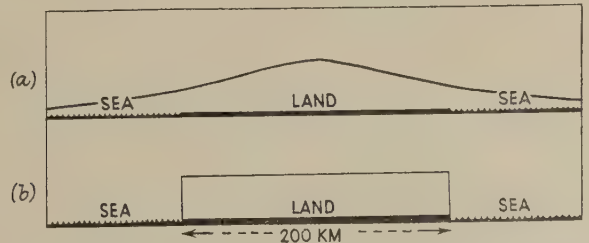
$$\int_0^\infty \frac{\sin\{k\,(x+b)\} - \sin\{k\,(x-b)\}}{k}\, dk = \begin{cases} \pi & -b < x < b \\ 0 & |x| > b \end{cases} \quad . \quad (10)$$

The first (9), is the ' mountain function ' shown by Fig. 2 (a) while the second is a tep function with discontinuities of heating at the coast, Fig. 2 (b). Neither of these is uite correct for there is no actual discontinuity at the coast, with no heating over the ea, as in Fig. 2 (b), nor is the variation of heating so unmarked at the coast as in Fig. 2 (a). o save time in the numerical work, the simpler of these numerically – the step unction – was chosen for the integration. As far as the mathematics of the problem, s opposed to numerical computation, is concerned, we may abbreviate (10) to :

$$\int_0^\infty \frac{\sin(kx)}{k}\, dk \quad . \quad . \quad . \quad . \quad . \quad (11)$$

(*b*) *Time variation of heating*

The heating is a measure of the rate of change of the inverse of potential temperature nd will thus have zeros at sunrise and mid-afternoon, and be negative in the morning



igure 2. Two possible mathematical representations of the distribution of heating in the x-direction. The vertical coordinate gives a measure of the rate of heating.

and positive in the evening. A suitable representation of this is given by the sum of three mountain functions :

$$-\frac{T}{T^2+t^2}+\frac{3}{4}\frac{T}{(t-4T)^2+T^2}+\frac{1}{4}\frac{T}{(t+2T)^2+T^2} \quad . \quad . \quad (12)$$

as shown in Fig. 3 where $t = 0$ is 09 hr. This gives the variation over one day as a single pulse tending rapidly towards zero as the next or previous day is reached. This is a suitable assumption since there is evidence that the heating of one day has little or no effect on the motions during the following day (Scorer 1952). Once again as far as the mathematics of the problem is concerned we will take the function

$$\frac{T}{t^2+T^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

since the actual numerical calculations may be adjusted afterwards. This may be rewritten :

$$\text{(a)} \quad \int_0^\infty \cos(\sigma t)\, e^{-\sigma T}\, d\sigma \qquad \text{or} \qquad \text{(b)} \quad \int_0^\infty \sin(\sigma T)\, e^{-\sigma t}\, d\sigma \; . \quad . \quad (14)$$

the second one only being applicable when $t < 0$.

We now define H_1, the time and horizontal-distance variations of H, as :

$$H_1 = i H_0 \int\int_0^\infty \frac{e^{-\sigma T}(e^{i\sigma t}+e^{-i\sigma t})\, e^{ikx}}{k}\, d\sigma\, dk \qquad \left[= i\int\int_0^\infty H_{1A}\, d\sigma\, dk\right] \quad . \quad (15a)$$

$$= -i H_0 \int\int_0^\infty \frac{e^{-\sigma T - i\sigma t}(e^{ikx}-e^{-ikx})}{k}\, d\sigma\, dk \qquad \left[= -i\int\int_0^\infty H_{1B}\, d\sigma\, dk\right] \quad (15b)$$

where H_0 is a numerical constant and H_{1A} and H_{1B} are defined.

(c) *Vertical variation of heating*

The heating is assumed to have an exponential decay, $\exp(-nz)$, in the vertical. n is dependent to some extent on H_0 in practice. H_0 will determine the maximum temperature-excess of land over water. This, in turn, from Fig. 1, will give the height at which the potential temperature is equivalent to this land temperature and we should not expect the bubbles, which cause the heating, to rise above this level, i.e., most of the heat is put in below this level. In the example calculated, the value for H_0 gives a maximum temperature-excess of land over sea of approximately 4·5°C.

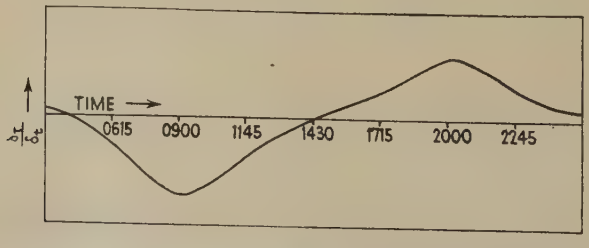


Figure 3. The rate of change with time of the inverse of potential temperature is plotted against time. This is a mathematical representation for the present work chosen to simulate, as nearly as possible, the atmospheric case. T is taken as 10^4 sec.

The value assumed for the static stability, β, is equivalent to a lapse rate which is C/km less than the dry adiabatic in the lowest kilometre and thus most of the heat ould be put in below this level. This will be achieved by a value for n of 2 km^{-1} and is gives a heating function which inserts $\frac{6}{7}$ of the heat below 1 km and $\frac{3}{5}$ below the 5 km level.

It must be noted that in this work the depth of the heated layer has been taken to the same as that of the cooled layer whereas, as is well known, the latter is much allower. To change the value for n when heat is being removed would double the nount of calculation; this has therefore been left for the present. The approximation not too poor, however, for it applies during most of the day and the initial cooling, as vening approaches, can be considered to be caused at the higher level by the dissipation clouds although these must be insufficient in extent to make much difference to the nount of sunshine at the ground.

This function for the heating will now be introduced into the equations.

5. Mathematical evaluation of w and u

We now make some approximations in Eq. (8). If the static stability corresponds a lapse rate 3°C/km less than the dry adiabatic in the lowest kilometre, the value of β is 10^{-4} sec^{-2}. Now for σ^2 to be comparable with this, σ must exceed 10^{-3} sec^{-1}. But he resultant velocities, like the original heating function, will have a factor $\exp(-\sigma T)$, here $T = 10^4$ sec, and thus will become negligible when σ is greater than 10^{-3} sec^{-1}. Hence we may neglect σ^2 with respect to $g\beta$. This is in agreement with practice, for $\geqslant 10^{-3}$ sec^{-1} corresponds to a period of heating of less than $\frac{1}{2}$ hr. These are the eriods associated with the bubbles of warmer air which serve to give us our initial eating distribution and can thus be ignored.

To simplify the expression still further we wish to neglect f in comparison with . This is obviously permissible in equatorial regions but, in our latitudes, f is equivalent a period of 18 hr and might easily affect the result. However, we will put $f = 0$ at resent, bearing in mind that, for application to these latitudes, the criterion :

$$\frac{\partial^2 u}{\partial t^2} \geqslant f^2 u \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (16)$$

eeds to be satisfied. This is equivalent to saying that situations in which the wind lows for a long time in any one direction cannot be included as the Coriolis forces will egin to have an appreciable effect.

With these two omissions Eq. (8) becomes

$$\frac{\partial^2 w}{\partial z^2} + \frac{g\beta k^2 w}{\sigma^2} + \frac{g H_{1\theta} k^2 e^{-nz}}{\tau_0 \sigma^2} = 0 \qquad . \qquad . \qquad . \qquad (17)$$

here $H_{1\theta}$ is H_{1A} or H_{1B} as defined in (15).

The solution of (17) is

$$w = \frac{B e^{-nz}}{n^2 + \lambda^2} + C e^{i\lambda z} + D e^{-i\lambda z} \qquad . \qquad . \qquad . \qquad (18)$$

here

$$B = \frac{-g H_{1\theta} k^2}{\tau_0 \sigma^2}, \qquad \lambda = \left| \frac{(g\beta)^{1/2} k}{\sigma} \right| \qquad . \qquad . \qquad . \qquad (19)$$

nd C, D are arbitrary constants.

6. Boundary conditions

On applying the boundary condition $w = 0$ at $z = 0$ we find :

$$C + D = \frac{-B}{n^2 + \lambda^2} \quad . \quad . \quad . \quad . \quad . \quad (20$$

but the other condition, $w = 0$ at $z = \infty$, fails to give any further information.

Throughout the next few paragraphs several assumptions are made; although at this particular stage no proofs can be given that they are in fact correct, a mathematical verification will be given later by evaluating w by another method and showing that the two methods give the same result.

Assume at first that $C = 0$. This gives for the solution for w :

$$w = \frac{B}{n^2 + \lambda^2}\left(e^{-nz} - e^{-i\lambda z}\right) \quad . \quad . \quad . \quad . \quad (21$$

Now $t = 0$ is the time of maximum *rate* of heating at the ground for a single pulse as is seen from Eqs. (6) and (13), and we would certainly not expect the resultant vertical velocities to be symmetrical about this point in time. The resultant vertical velocity, however, will be symmetrical about the centre of the land mass, i.e. antisymmetrical about the part, $\sin(kx)/k$, we are dealing with, and this gives us a method of choosing between H_{1A} and H_{1B} for $H_{1\theta}$. As far as the e^{-nz} term is concerned both these expressions are identical but, for the $e^{-i\lambda z}$ term, we see that only H_{1B} gives a solution which is satisfactory, i.e. we get a term $e^{-i(\lambda z + \sigma t)}$ which is not symmetrical with respect to t leaving the whole expression antisymmetrical with respect to x. Thus we replace $H_{1\theta}$ by H_{1B}.

It must be pointed out here that the assumption is made that the value of w found for each pair of values of the wave numbers σ and k can be integrated, as for the heating, to give the final W corresponding to that heating.

Hence we get the final expression for W :

$$W = \frac{i\,g\,H_0}{\tau_0}\int\int_0^\infty \frac{k\,e^{-\sigma T - i\sigma t}\,(e^{ikx} - e^{-ikx})\,(e^{-nz} - e^{-i\lambda z})}{\sigma^2\,(n^2 + \lambda^2)}\,d\sigma\,dk \quad . \quad . \quad (22)$$

By substituting for k in terms of λ from equation (19) we may integrate directly over σ.

By using the theory of residues we may split the λ-integration into two parts to make the numerical work simpler; the first part is zero for $t < 0$, W_1 for $t > 0$, and $\frac{W_1\,(t=0)}{2}$ for $t = 0$ where

$$W_1 = \frac{gH_0\,\pi}{(g\beta)^{\frac{1}{2}}\,\tau_0} \times$$

$$\times\left[\frac{e^{-nz}T_1(T_1^2+t^2-n^2x^2) - e^{-\frac{zT_1}{x}}\left\{T_1(T_1^2+t^2-n^2x^2)\cos\left(\frac{zt_1}{x}\right) - t_1(T_1^2+t_1^2+n^2x^2)\sin\left(\frac{zt_1}{x}\right)\right\}}{(T_1^2 - t_1^2 - n^2x^2)^2 + 4\,t_1^2\,T^2}\right] \quad (23)$$

where $T_1 = (g\beta)^{1/2}\,T$ and $t_1 = (g\beta)^{1/2}\,t$. The second part W_2 is given by

$$W_2 = \frac{g\,H_0\,t}{\tau_0\,(g\beta)}\int_0^\infty \frac{\lambda\,(e^{-nz} - e^{-\lambda z})}{n^2 - \lambda^2}\left\{\frac{1}{t^2 + \left(T + \frac{\lambda x}{(g\beta)^{1/2}}\right)^2} - \frac{1}{t^2 + \left(T - \frac{\lambda x}{(g\beta)^{1/2}}\right)^2}\right\}d\lambda \quad . \quad (24)$$

with the final vertical velocity W given by

$$W = W_1 + W_2 \quad . \quad . \quad . \quad . \quad (25)$$

The horizontal velocity is found by using the equation of continuity (5) :

$$\frac{\partial U}{\partial x} = -\frac{\partial W}{\partial z} \quad . \quad . \quad . \quad . \quad (26)$$

and finding $\partial W/\partial z$ at the same point as W. U is then roughly found by a step-by-step summation in the x-direction. The constant of integration in evaluating U in this way is immaterial since we must finally replace Eq. (11) by (12); thus this constant may be taken as zero.

We wish now to find a solution of a new Eq. (14) which, when the boundary condition $w = 0$ at $z = \infty$ is applied, gives an equation enabling us to find the new arbitrary constant. For this return to Eq. (14b) which we may use instead of Eq. (14a). This equation is applicable only when $t < 0$ but gives a heating function identical with that used previously in this part of the domain. It may be argued that when $t > 0$ the expression becomes infinite, thus destroying our model, but *since we can use the fact that time is uni-directional and what happens afterwards does not affect that which goes before*, the new W found by this method should be identical with the one previously found. Thus we have a new form of the heating function.

We see that, in the original equations, the σ's are only introduced by derivatives with respect to t. Therefore, for the original work leading up to the differential equation (17), we shall now get (σ) instead of $(-i\sigma)$ for each derivative, giving us for the new Eq. (17) :

$$\frac{\partial^2 w}{\partial z^2} - \frac{g\beta k^2}{\sigma^2} w - \frac{g H k^2}{\tau_0 \sigma^2} = 0 \quad . \quad . \quad . \quad . \quad (27)$$

Since our steps of t are to be calculated in multiples of 10^4 sec, all that was said earlier concerning the magnitude of the various terms still applies and we find, using now both boundary conditions,

$$w = \frac{-B(e^{-nz} - e^{-\lambda z})}{n^2 - \lambda^2} \quad . \quad . \quad . \quad . \quad (28)$$

This eventually leads us to the expression

$$W = \frac{g H_0 t}{\tau_0 (g\beta)} \int_0^\infty \frac{\lambda (e^{-nz} - e^{-\lambda z})}{n^2 - \lambda^2} \left\{ \frac{1}{t^2 + \left(T + \frac{\lambda x}{(g\beta)^{1/2}}\right)^2} - \frac{1}{t^2 + \left(T - \frac{\lambda x}{(g\beta)^{1/2}}\right)^2} \right\} d\lambda \quad . \quad (29)$$

which is seen to be the same expression as that for $W (= W_2)$ when $t < 0$.

Therefore the assumptions made in the first evaluation were in fact correct, and we may now continue with the actual numerical work.

7. Methods of numerical evaluation

Since there were a large number of evaluations of a similar nature to be carried out, the EDSAC, at the Mathematical Laboratory, Cambridge, was used.

The values of the variables chosen were as follows :

$z = 0,\ 0{\cdot}5,\ 1,\ 2,\ 3,\ 4,\ 5$, km

$x = 0,\ 20$, then in steps of 20 km to 360 km

$t = 0,\ 10^4,\ 2 \times 10^4, \ldots\ldots 8 \times 10^4$ sec

For the constants, we have :

$n = 2\ \text{km}^{-1}$

$g\beta = 10^{-4}\ \text{sec}^{-2}$ corresponding to a lapse rate of approximately $7°\text{C km}^{-1}$
$\tau_0 = 1/290$
$T = 10^4$ sec
$b = 100$ km
$H_0 = -1{\cdot}05136 \times 10^{-5}$

The last constant is found by using the expression,

$$H(z, x, t) = \frac{\partial \Gamma}{\partial t}$$

at the ground and assuming the rate of increase of temperature at 0900 hr, is $1°\text{C hr}^{-1}$, which is typical of the kind of situation imagined.

The numerical integration was performed by applying Gauss's 6-pt formula to integral (24). The values were then subtracted in accordance with Eq. (10) and the results were printed out.

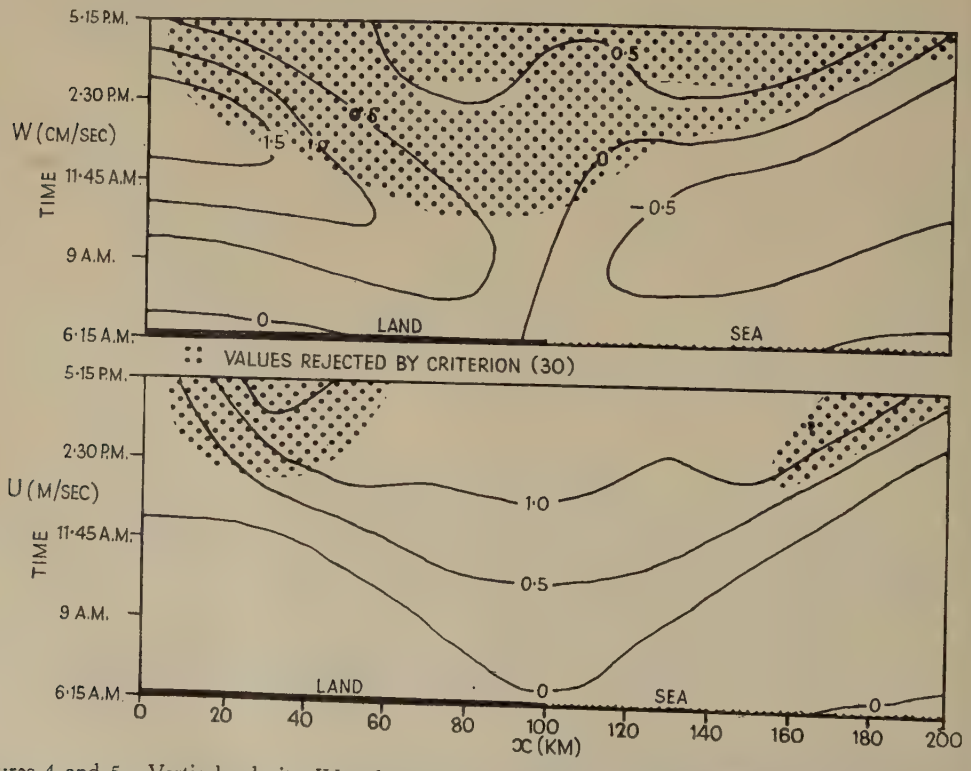
The horizontal velocities were found by using :

$$U_{(x+20)} - U_x = -\left\{20\,\frac{\partial W}{\partial z}\right\}_{(x+10)}$$

where U_x is the horizontal velocity at x km.

Thus the velocity field for a single heating pulse is found and if we add the values in the proportions indicated by Eq. (12) we shall obtain the final velocity field at various times during the day.

Some graphs of these results are shown in Figs. 4 to 10.



Figures 4 and 5. Vertical velocity W and horizontal velocity U are plotted against time and horizontal distance for a vertical height of 1 km. Inserted numbers indicate the magnitude of the different velocity curves.

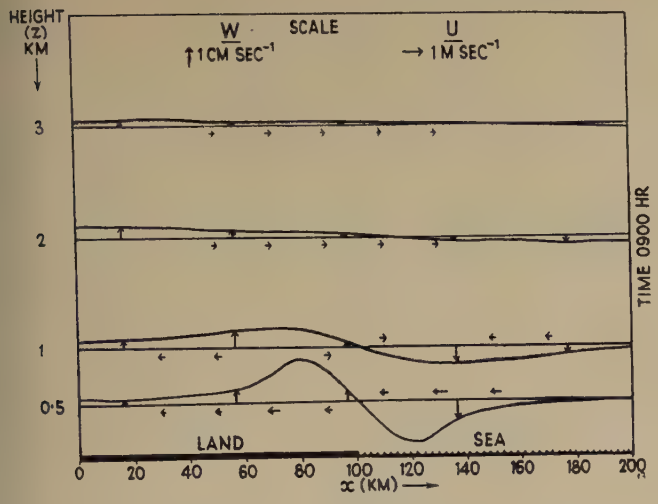


Fig. 6

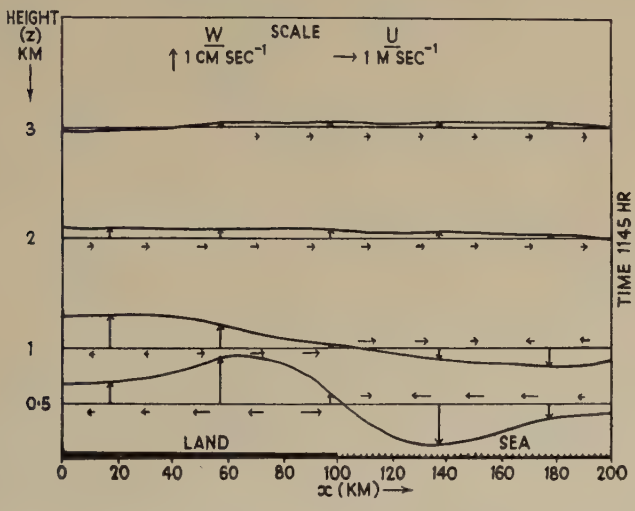


Fig. 7

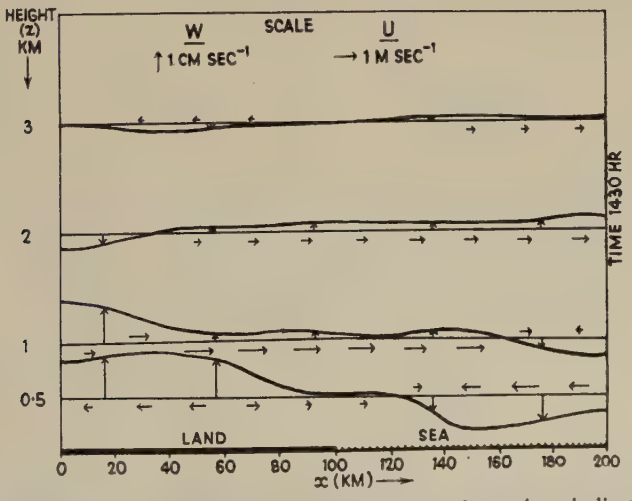


Fig. 8

Figures 6 to 10. Vertical and horizontal velocities at heights of 0·5, 1, 2, 3 km, at the various indicated times. The arrows indicate the magnitude and direction of the velocity field with the ratio,

$$\frac{\text{vertical scale}}{\text{horizontal scale}} = \frac{100}{1}.$$

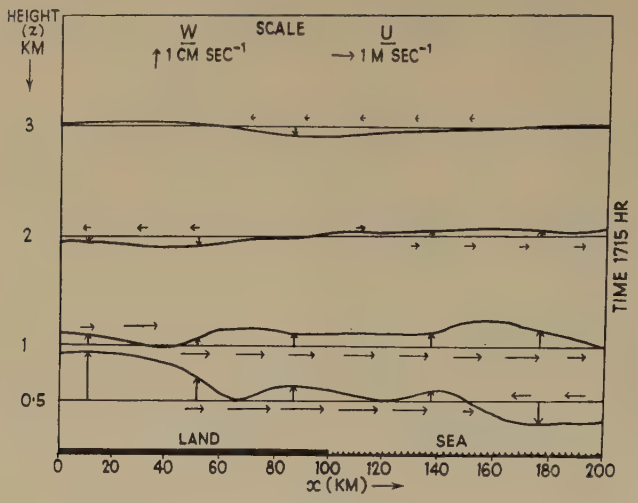


Fig. 9

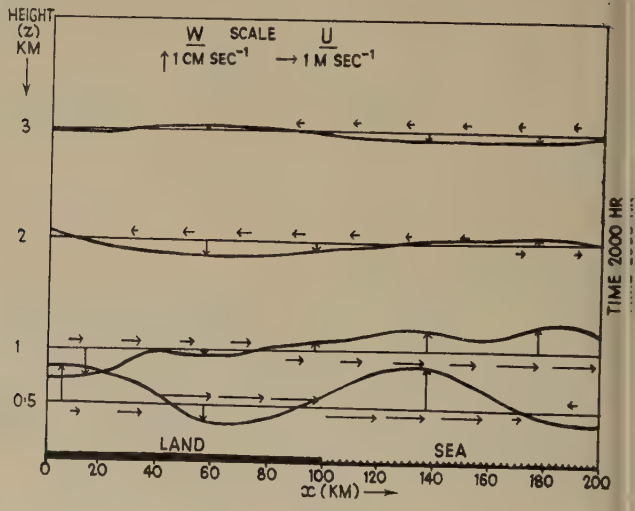


Fig. 10

Figures 9 and 10. See previous page for legends.

8. Discussion of results

We must now take the values we have found for the velocity field and examine the effect of our neglecting $U\frac{\partial U}{\partial x}$ with respect to $\frac{\partial U}{\partial t}$ and $U\frac{\partial W}{\partial x}$ with respect to $\frac{\partial W}{\partial t}$. We can find the numerical values of these terms and, to illustrate the areas in which the final values are least appropriate, we must choose some criterion which will reject the values in these regions. Criterion (30) :

$$U\frac{\partial (U, W)}{\partial x} < \frac{1}{4}\frac{\partial (U, W)}{\partial t} \quad . \quad . \quad . \quad . \quad (30)$$

was chosen arbitrarily although the rigidity of its application is governed by factors outlined below.

The values of U at $z = 0$ become completely meaningless when the criterion above is applied but elsewhere it is, in the main, satisfied and even where it is not, provided

$U\frac{\partial U}{\partial x}$ does not become large compared with $\frac{\partial U}{\partial t}$, we would not expect the actual velocity to be vastly different from that found. To put this another way – if the numerical constant determining the temperature excess is decreased by a factor r then the ratio $U\frac{\partial U}{\partial x}\Big/\frac{\partial U}{\partial t}$ also decreases by a factor r since the velocities are directly proportional to the temperature excess. Thus we could always make our results valid in a particular domain by introducing a scale factor r. Thus, for example, if (a) for a temperature excess of 10°C we have at a point P,

$$U = 2 \text{ m sec}^{-1},\ \frac{\partial U}{\partial x} = 5 \times 10^{-5} \text{ sec}^{-1},\ \frac{\partial U}{\partial t} = 10^{-4} \text{ m sec}^{-2}$$

then

$$\left(\frac{\partial U}{\partial t}\right)_P = \left(U\frac{\partial U}{\partial x}\right)_P$$

but if (b) the temperature excess was 1°C then,

$$U = 0{\cdot}2 \text{ m sec}^{-1},\ \frac{\partial U}{\partial x} = 5 \times 10^{-6} \text{ sec}^{-1},\ \frac{\partial U}{\partial t} = 10^{-5} \text{ m sec}^{-2}$$

with

$$\left(\frac{\partial U}{\partial t}\right)_P = 10\left(U\frac{\partial U}{\partial x}\right)_P$$

The situation (b) is a valid result from criterion (30), but (a) is not. However, the configuration is a similar one and we would not expect the resultant velocity field to be a vastly different one, although we might perhaps expect U to be 1·8 m sec^{-1} instead of 2 m sec^{-1}.

The values at $z = 0{\cdot}5$ km have been drawn up with this in mind, for criterion (30) is not satisfied at most points although nowhere does the term $U\frac{\partial U}{\partial x}$ outweigh the term $\frac{\partial U}{\partial t}$ as it does at $z = 0$.

For the height $z = 1$ km the two graphs of W, U, plotted against time and distance have been drawn (Figs. 4 and 5). On these two graphs the slopes of the curves are equivalent to $\frac{\partial W}{\partial x}\Big/\frac{\partial W}{\partial t}$ and $\frac{\partial U}{\partial x}\Big/\frac{\partial U}{\partial t}$ respectively. If we multiply these two terms by U we see that we have, in effect, the relation which determines the feasibilities of our results, and thus we would wish the slopes of these curves to be as small as possible. As can be seen, particularly in the graph of U, these lines do have a horizontal aspect and the regions which do not comply with criterion (30) are marked. At these we may appeal to intuition, as in the case $z = 0{\cdot}5$ km, to say that, even there, the error is not a very large one and to take our final results as the correct ones.

In the layer below 1 km, over the centre of the land mass, the term $W\frac{\partial W}{\partial z}$ also assumes a magnitude comparable with $\frac{\partial W}{\partial t}$ and some of the values of W should be modified in this region also.

However Figs. 6 to 10 have been drawn without these modifications in the lower layers and it should be borne in mind that, at points which have a large vertical or horizontal change of W or U associated with a fairly large value of W or U, the real picture is actually more smoothed out although the general motion will be the same.

At the higher levels almost all the results are acceptable ones as the velocities become much smaller in magnitude.

In Fig. 4, the graph of vertical velocities at 1 km, we see an example of the tendency of vertical motions in the lowest kilometre to start at the coast when heating begins and to move inwards and outwards, increasing in intensity as the heating proceeds. The maximum over the land at 1 km occurs over the centre of the land mass at approximately mid-afternoon. There is no sign of this tendency at higher levels; in fact, by mid-afternoon the air is beginning to descend at these levels as is shown in Figs. 6 to 10. These give the distribution of velocities at specified times.

The first, Fig. 6, shows the distribution at 09 hr. Then we have air rising at all levels over the land and descending over the sea except that, at higher levels, there is some ascent over the sea near the coast. The air flows out at the higher levels with an inward flow over the lowest kilometre. At 1145 hr the ' sea breeze,' as it will henceforth be called is shallower, and therefore stronger, particularly over the coast. The rising air is now seen to be spread over a much larger distance at the higher levels and extends well out over the sea. An important new feature is seen at this time – the descent of air over the centre of the land mass at 3 km – and by the time of the next diagram at 1430 hr we see that there is considerable subsidence at this level along with an inflow of air or a ' reversed circulation.' The sea breeze has become shallower still and the regions of maximum vertical velocity in the lowest kilometre have now moved well away from the coast.

By 1715 hr and 2000 hr we are encountering the difficulty that the depth of the cooling layer is in fact not as large as the depth of the heated layer. This is especially true in the latter case but both of these graphs have been drawn to show the sort of thing the theory gives. It will also be seen that, at 2000 hr, $W\frac{\partial W}{\partial z}$, at about 0·75 km height over the centre of the land mass, becomes large along with the horizontal velocities at this level, giving an even more unrealistic picture.

To check these results by observations is very difficult indeed, for, to begin with, an initially stationary air mass is a rare phenomenon in the free atmosphere. The magnitudes of the velocities are too small to be measured by any present means and the only pointers we can get are the formation and dissipation of cloud forms. According to our results there should be preferential areas for the formation of cumulus clouds, i.e., nearer the coasts in the morning and further inland in the afternoon. Subsidence in the afternoons might tend to stop any small-scale shower production although we must remember that an appreciable formation of clouds will destroy the initial heating function. Dissipation of medium-level clouds might also indicate subsidence at that level.

9. Conclusion

The present paper has been an attempt to put into a mathematical theory numerical values which bear some resemblance to those found in the similar atmospheric case.

On introducing these values we find that the results we obtain are not inconsistent with what we might expect and also that they exhibit several interesting features. One of these is the evidence of subsidence, with its associated inflow of air, over the land while the heating is still taking place; and another is the movement of the wave-like perturbations of velocity outward from the centre of the land mass. The latter are probably due to a combination of the inertia of the circulation and the static stability of the air mass : the air over the heated land heaves up and then falls back again, producing waves in the air similar to those seen when a stone is dropped into water.

Acknowledgments

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Further computations of 24-hr pressure changes based on a two-parameter model

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SUMMARY

The results of 13 computed 24-hr forecasts, using the Sawyer-Bushby (1953) two-parameter model atmosphere, are discussed, and further computations to investigate the apparent limitations of the model are described. The effects of the neglect of heating of the atmosphere over the sea, of the use of incorrect boundary conditions and of the use of 500-mb rather than 600-mb data are investigated. It is concluded that the imposition of artificial boundary conditions can cause considerable errors, although these will not be serious in the region of the British Isles, and that the effects of non-adiabatic heating of the air over the sea should be included in the computations.

1. INTRODUCTION

In order that a preliminary estimate could be made of the effects of the limitations of the model used, 24-hr forecasts were computed for 11 synoptic situations in addition to those reported by Bushby and Hinds (1954a), and the results are described below. A comparison is given between these results and the results obtained by conventional forecasting methods.

The discrepancies between the actual and computed charts were examined to decide if any aspect was common to more than one of the computations. It was immediately apparent that, in winter months, the computed thickness over the sea was often much less than the actual thickness and this was ascribed to the neglect of the heating of cold air masses over the sea. In certain of the situations where the displacement of features of the contour map was computed to be more than actually occurred, this was thought to have been due to the use of the 500-mb contour heights and 1,000-500 mb thicknesses instead of the 600-mb contour heights and the 1,000-600 mb thicknesses to which the theoretical equations were designed to apply. It was also apparent that, with the present size of the 18 × 14 grid, the assumption of zero change along the boundary was sometimes causing error in the computed forecast charts over a considerable area. Further investigations have been made into these three limitations on the model and the results are discussed below.

Another common discrepancy was the over-estimation of the contour heights in anticyclonic regions. This is probably due to the use of the geostrophic approximation to obtain a contour height from the computed vorticity field, but this has not yet been specifically investigated.

2. SOLUTIONS OF THE BASIC EQUATIONS USING 500-MB DATA

The first two cases dealt with were described in detail by Bushby and Hinds (1954a); subsequently a further 11 computations were carried out following the same method. Full details of the later results are given in an unpublished paper (Bushby and Hinds 1954b). Table 1 gives correlation coefficients, for all 13 integrations, between computed 24-hr

nges and actual changes for a 12 × 8 grid centrally situated inside the 18 × 14 grid. rrelation coefficients are also given using the 24-hr changes forecast by conventional thods in place of the computed changes.

In examining Table 1 it must be remembered that correlation coefficients do not e a good measure of the success of the forecast where the actual changes are small. Also, computed results for points on the north-western corner of the 12 × 8 grid are often cted by the boundary conditions and this region (south of Iceland) is a region where ssure changes are often much larger than those in the vicinity of the British Isles. This l obviously have an adverse effect on the correlation coefficients.

Nevertheless Table 1 does indicate that, even with this crude basic two-parameter del, a degree of success only slightly less than that obtained by the conventional ecaster has been obtained in twelve out of the thirteen situations. This is confirmed by visual inspection of the computed charts (Bushby and Hinds 1954b). The case in ich a complete failure was recorded was that of 8 January 1951, when a failure for British Isles was also recorded by the conventional forecaster. However, subsequent vestigation has shown that both the conventional forecast and the computed forecast re seriously affected by a wrong analysis of the initial data in Mid-Atlantic.

ABLE 1. CORRELATION AND REGRESSION COEFFICIENTS FOR COMPUTED FORECASTS USING BASIC METHOD AND CONVENTIONAL FORECASTS

recast period mmencing GMT	Method	500 mb			1,000-500 mb			1,000 mb (A) and surface pressure (B)		
		r	a	b	r	a	b	r	a	b
Mar. 49	A	0·88	0·64	− 13·2	0·84	0·49	− 2·6	0·83	0·49	− 5·4
	B	0·75	0·70	12·2	No data available			0·82	0·61	0·9
Jan. 51	A	0·68	0·63	13·2	0·59	0·81	19·8	− 0·27	− 0·18	− 2·2
	B	0·61	0·71	2·3	0·82	1·02	− 0·1	0·59	0·75	− 1·5
Aug. 51	A	0·90	0·69	− 10·2	0·84	0·75	− 5·4	0·85	0·58	− 0·7
	B	0·87	1·23	− 4·2	0·91	1·32	− 4·4	0·87	1·15	− 1·5
Sept. 51	A	0·89	0·83	− 12·5	0·47	0·43	− 3·3	0·72	0·53	− 5·6
	B	0·74	0·89	− 2·9	0·22	0·21	− 0·4	0·86	0·81	− 0·6
Jan. 52	A	0·43	0·26	5·6	0·76	0·35	7·5	0·22	0·23	− 1·8
	B	0·74	0·77	− 0·1	0·80	0·66	6·6	0·62	0·70	− 3·2
Jan. 52	A	0·61	0·36	25·3	0·58	0·34	− 3·3	0·53	0·54	− 22·9
	B	0·67	0·62	− 16·7	0·86	0·69	− 4·6	0·79	0·92	− 1·6
May 52	A	0·87	0·56	− 8·8	0·85	0·62	3·7	0·35	0·15	− 5·3
	B	0·94	0·85	− 2·1	0·92	1·02	1·5	0·67	0·66	− 2·2
July 52	A	0·48	0·87	− 8·3	0·46	0·68	− 7·7	0·27	0·19	0·2
	B	0·87	0·99	− 7·0	0·74	1·04	3·9	0·36	0·31	1·7
Dec. 52	A	0·46	0·43	− 5·9	0·08	0·06	− 2·6	0·55	0·51	− 6·5
	B	0·79	1·00	− 9·0	0·57	0·66	0·2	0·79	1·09	− 5·8
Jan. 53	A	0·89	1·20	− 30·9	0·63	0·65	− 8·5	0·90	0·85	− 12·7
	B	0·92	0·90	9·6	0·74	0·63	3·4	0·90	0·87	− 1·3
Jan. 53	A	0·96	0·75	− 12·3	0·89	0·56	1·2	0·88	0·61	− 13·8
	B	0·90	1·10	21·3	0·86	1·03	7·5	0·91	1·09	− 1·2
July 53	A	0·78	0·79	− 3·9	0·93	1·01	0·7	0·67	0·55	− 3·6
	B	0·91	1·04	5·9	0·89	1·14	7·7	0·89	0·79	1·4
Oct. 53	A	0·64	0·62	24·1	0·75	0·81	2·1	0·73	0·39	22·4
	B	0·90	1·10	9·1	0·85	1·38	− 5·4	0·73	0·59	6·5

r = correlation coefficient; A = computed forecast, B = conventional forecast; a, b are the regression coefficients of the best-ng straight line giving the actual change (Y tens of feet, or millibars) in terms of the forecast change (X tens of feet, or millibars) he form $Y = aX + b$.

3. The effect of heating over the sea

The effect of non-adiabatic heating of cold air masses over the sea can be include in the calculations by the introduction of an additional term Q in Eq. (2) of Bushb and Hinds (1954a), the equation then becoming

$$\nabla^2 \partial h'/\partial t + \frac{4g\,(\nabla^2 h_m + l^2 \beta^{-2} g^{-1})\,\{\beta^2 g l^{-1} J(h_m, h') - Q + \partial h'/\partial t\}}{R A \Gamma_p (p_0 - p_1)} + J(h_m, \beta^2 g l^{-1} \nabla^2 h') + J(h', \beta^2 g l^{-1} \nabla^2 h_m + l) = 0 \quad . \quad . \quad (1$$

An empirical value for Q is given by

$$Q\,(\text{ft/hr}) = 2{\cdot}068 \,.\, 10^{-2}\,(h_s' - h') \text{ for } h_s' > h', \text{ otherwise } Q = 0 \quad . \quad (2$$

and

$$h_s'\,(\text{tens of ft}) = 53{\cdot}19\ (T_s + 282{\cdot}5) \quad . \quad . \quad . \quad . \quad . \quad (3$$

T_s being the monthly mean sea-surface temperature (°F). Eqs. (2) and (3) are based o the work of Craddock (1951) and on a statistical survey of the relationship between sea surface temperature and 1,000-500 mb thickness at Ocean Weather Stations I and J. N attempt was made to take into account the cooling of air masses which are warmer tha the sea nor of heating or cooling over land.

Three forecasts were recomputed taking into account the heating over the sea, an the correlation and regression coefficients for these forecasts are shown in Table 2. Th resulting 1,000-500 mb and 1,000-mb charts were slightly improved, but there was litt change at 500 mb. There was some indication that the effect of the heating of the ai by the sea had been somewhat underestimated.

TABLE 2. Correlation and regression coefficients for computed forecasts including effect c heating over the sea

Forecast period commencing 15 GMT	500 mb			1,000-500 mb			1,000 mb		
	r	a	b	r	a	b	r	a	b
8 Jan. 51	0·70 (0·68)	0·66	13·2	0·66 (0·59)	1·16	7·5	− 0·25 (− 0·27)	− 0·17	− 4
30 Jan. 52	0·62 (0·61)	0·37	− 26·5	0·59 (0·58)	0·41	− 6·5	0·59 (0·53)	0·53	− 20
30 Jan. 53	0·96 (0·96)	0·74	− 12·9	0·90 (0·89)	0·62	− 19·6	0·90 (0·88)	0·63	− 11

For the purposes of comparison the correlation coefficients shown in Table 1 for the basic computed forecasts are repeate in parenthesis after the new values in Tables 2-5.

4. The use of 600-mb data

The Sawyer-Bushby equations (1953) refer to motion between two pressure level p_1 and p_0 corresponding to the upper and lower limits of the troposphere. In computation using this model, p_1 has been taken as 200 mb and p_0 as 1,000 mb. The two variable h_m and h' are then the height of the 600-mb surface and the thickness of the 1,000-600 mb layer. Since 600-mb data are not readily available the earlier computed forecasts were made using the corresponding 500-mb data. This caused greater changes to be com puted than actually occurred. Three computed forecasts were therefore repeated using estimates of the 600-mb data derived as follows :

$$(h_m)_{600} = (h_m)_{500} - \tfrac{1}{5}\,(h')_{500} \quad . \quad . \quad . \quad . \quad . \quad (4$$

$$(h')_{600} = \tfrac{4}{5}\,(h')_{500} \quad . \quad . \quad . \quad . \quad . \quad (5$$

t the end of the integration process these equations are used to obtain computed values $(h_m)_{500}$ and $(h')_{500}$ from the corresponding 600-mb data.

Correlation and regression coefficients for the results are shown in Table 3. The se of 600 mb for the calculations appears to give a general, although small, improvement n the movement of the main features on the computed charts, especially the 1,000-mb harts. This improvement is not fully reflected in the correlation coefficients.

TABLE 3. CORRELATION AND REGRESSION COEFFICIENTS FOR COMPUTED FORECASTS USING PSEUDO-600-MB DATA

Forecast period commencing 15 GMT	500 mb			1,000-500 mb			1,000 mb		
	r	a	b	r	a	b	r	a	b
3 Jan. 51*	0·85 (0·68)	0·81	6·3	0·84 (0·59)	0·72	8·5	0·25 (− 0·27)	0·25	− 2·5
9 Sept. 51	0·88 (0·89)	1·17	− 14·8	0·30 (0·47)	0·30	− 1·5	0·75 (0·72)	0·70	− 6·6
0 Jan. 52	0·59 (0·61)	0·36	− 27·3	0·59 (0·58)	0·34	− 2·7	0·63 (0·53)	0·60	− 27·7

* Using actual boundary conditions.

5. USE OF CORRECT BOUNDARY CONDITIONS

When computing forecast charts using the Sawyer-Bushby model it is necessary to ssume the value of two parameters along the boundary. In the earlier computations h_m and h' were assumed to remain constant over the whole twenty-four hours at the oundary points of the 18 × 14 grid. At the ring of points immediately interior to this, h_m and h' were assumed constant during the solution of the differential equations, but for he purposes of time integration they were assumed to change by half the amount of the earest point inside the grid. In order to estimate what part of the error in the computed orecasts was due to this cause, three forecasts were re-computed using the actual changes n h_m and h' at the two other rings of points of the 18 × 14 grid. Correlation and regression oefficients for these computations are given in Table 4.

It appears that a considerable part of the error (as represented by the correlation oefficients) in the first series of computed forecasts was due to the use of constant boundary alues for h_m and h' rather than to any inadequacy in the Sawyer-Bushby equations or he method of computation. It is impossible to say at this stage, however, how much f this error would be removed if some estimate of the change along the boundary were o be made in advance and incorporated into the computations.

TABLE 4. CORRELATION AND REGRESSION COEFFICIENTS FOR COMPUTED FORECASTS USING ACTUAL BOUNDARY VALUES

Forecast period commencing 15 GMT	500 mb			1,000-500 mb			1,000 mb		
	r	a	b	r	a	b	r	a	b
8 Jan. 51	0·75 (0·68)	0·60	4·6	0·87 (0·59)	0·67	7·9	0·08 (− 0·27)	0·06	− 2·8
8 Jan. 51*	0·85 (0·68)	0·81	6·3	0·84 (0·59)	0·72	8·5	0·25 (− 0·27)	0·25	− 2·5
7 Jan. 52	0·90 (0·43)	0·75	− 2·3	0·88 (0·76)	0·59	4·0	0·83 (0·22)	0·71	− 5·2

* Using pseudo-600-mb data.

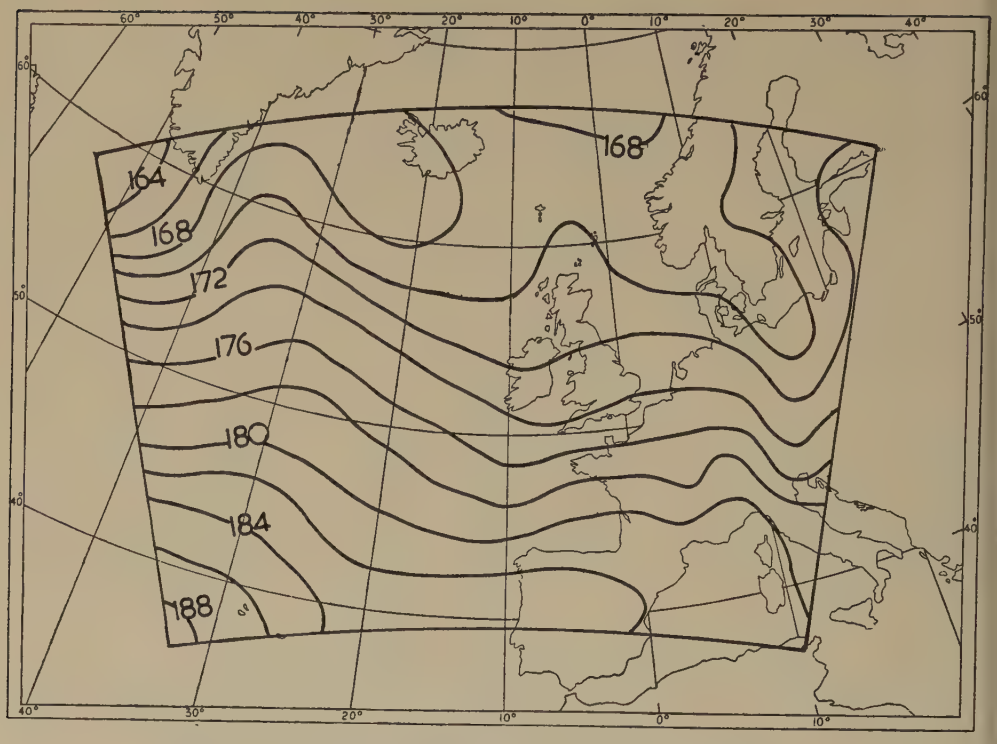

Figure 1. Computed 500-mb contours for 15 GMT 28 Jan. 1952.

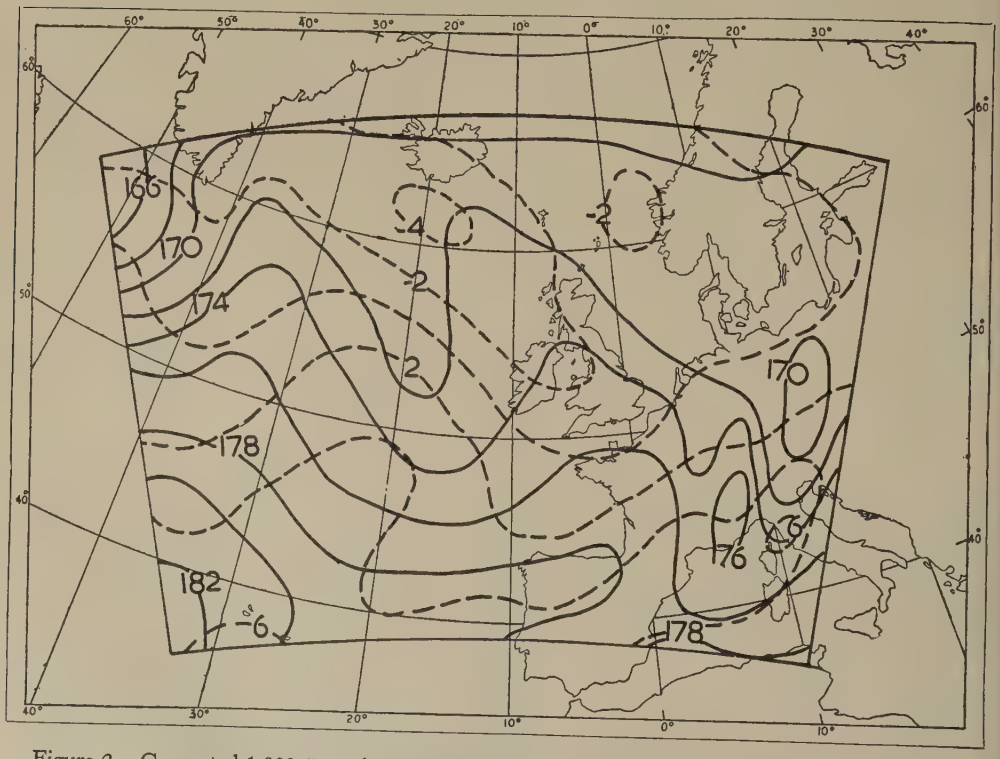

Figure 2. Computed 1,000-500 mb thicknesses and 1,000-mb contours for 15 GMT 28 Jan. 1952.
——— 1,000-500 mb thickness lines. - - - 1,000 mb contours.

6. Time integration based on initial data for 15 GMT 27 January 1952

As it appeared evident that the factors mentioned in Sections 3 to 5 all cause slight improvements to the calculations a programme was prepared in which the three factors were combined.

A series of actual and computed 500-mb contour, 1,000-500 mb thickness and 1,000-mb contour charts at 12-hr intervals covering the 24-hr period commencing at 15 GMT 27 Jan. 1952 are shown as Figs. 1 (a)-3 (d) of Bushby and Hinds (1954a). This computation has now been repeated using the correct boundary conditions and pseudo-600-mb data and taking into account the effect of heating over the sea. The resulting 24-hr forecast charts are shown in Figs. 1 and 2, and the correlation and regression coefficients in Table 5.

TABLE 5. Correlation and regression coefficients for computed forecast using effect of heating over sea, correct boundary values and pseudo-600-mb data

Forecast period commencing	500 mb			1,000-500 mb			1,000 mb		
15 GMT	r	a	b	r	a	b	r	a	b
27 Jan. 52	0·93 (0·43)	0·87	− 1·7	0·91 (0·76)	0·74	− 3·4	0·83 (0·22)	0·89	3·6

The original computation, using constant boundary values, was considerably in error over the western Atlantic owing to the fact that, during the forecast period, a cold trough moved from eastern Canada into the area covered by the computations. The use of correct boundary conditions gives computed forecasts which are very similar to actuality in this area. The earlier set of calculations also forecast a deep cold trough west of Ireland. This was partly due to the downstream effect of the computed warm ridge south of Greenland and partly due to the neglect of heating of cold air masses over the sea. Both these effects have been removed in the later computation and the 1,000-500 mb thickness is computed to be 200-300 ft higher than in the previous computation for this area. Other features are broadly similar on both sets of computed charts, but it will be noted that both correlation and regression coefficients have been greatly improved by the new method and are now better than those given by the conventional forecast. It is evident from Tables 4 and 5 that most but not all of the improvement is due to the use of the correct boundary values.

7. Conclusions

Whilst one must not give too much weight to results based on the small sample of cases so far investigated, it appears from a comparison of the 13 original computed forecasts with those more recently calculated, using a number of different variations, that the following conclusions can be drawn :

(i) A large amount of the error in the original computed forecasts was due to the imposition of artificial boundary conditions and no real assessment of the validity of the theoretical model can be made unless the actual boundary conditions are used. It is recognised that, if current data were used for the computations, it would be necessary to use forecast instead of actual boundary conditions, but this fact is not relevant while the model is being tested. By extending the area under consideration, the effect of erroneous boundary values on the computations in the region of the British Isles can be reduced.

(ii) In many synoptic situations non-adiabatic heating of the air over the sea has an important effect on the 1,000-500 mb thickness and should be included in the computations. Over a period as short as twenty-four hours there is little change in the 500-mb flow from this cause.

(iii) It is probably better to use pseudo-600-mb data rather than 500-mb data, as a basis for calculation.

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Geostrophic and ageostrophic wind analysis

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Summary

Aerological data for the British Isles, 1953, at nine pressure levels, are analysed, to give the geostrophic and ageostrophic components of the wind and the radius of curvature of the isobars. The mean total energy over unit surface area is found to be $1{\cdot}71 \times 10^9$ erg cm^{-2}.

The equation of energy associated with the ageostrophic component is obtained, and the work done by the flow down the pressure gradient is identified by means of this equation with a Coriolis term and a centrifugal term. These are estimated from the data, giving a value for the rate of generation of kinetic energy of $8{\cdot}8 \times 10^3$ erg cm^{-2} sec^{-1}. The mean net mass-flux across unit length of isobar is found to be 25 kg cm^{-1} sec^{-1}. The order of magnitude of this quantity is checked in the final section in which a mathematical analysis is applied to a model depression filling by the influx of mass across its perimeter.

1. Introduction

The paper is chiefly concerned with estimating the work done, over the British Isles, by the atmospheric pressure field which is set up by differential solar heating of the earth's surface. The estimate has been obtained by analysing radiosonde and other data for a number of stations at three-day intervals during 1953. The data are also employed to determine the mean kinetic energy, mean geostrophic and ageostrophic winds, and means of other quantities over the area in question.

Work is done on the motion when there exists a component of flow in the negative direction of the pressure gradient. In this case the potential energy of the field of pressure is transformed into kinetic energy. Kinetic energy, in turn, is being constantly dissipated by viscous forces back into heat. Since, in the mean, a steady kinetic-energy density can be assumed, the generation and dissipation of kinetic energy must balance.

To measure the work done by the pressure field directly would be difficult because it necessitates the determination of the pressure gradient. The simple directness of this method however is certainly appealing and it may seem worth while in the future to make an investigation along this line. In the analysis below, the difficulties have to a large extent been avoided by making certain simplifying assumptions about the nature of the flow. These assumptions (see § 2) reduce the equation of energy connected with the ageostrophic component to the same equation as would be obtained if that component were that of a corresponding gradient wind. It should be noted however that these assumptions are not so restrictive as the alternative assumption that the real wind and the gradient wind are at all times identical. With this equation the pressure term may be identified with functions of the ageostrophic component, the geostrophic component and the radius of curvature of the isobars. These three factors have been determined from the original data.

Solar radiation falling on the earth is, of course, a function only of latitude and time and thus, if the reflected radiation from the earth's surface also depends only on these variables, it might be expected that pressure would be approximately independent of longitude. If this were so, then not only the mean flow, but also the real flow, would be nearly zonal. However, owing to the varied nature of the earth's surface, the outgoing radiation has an important dependence on the longitude, giving rise to variations in the wind direction which cannot be neglected. Although, even as high as the ionosphere,

the winds vary appreciably from the zonal, the variations which are so prevalent at the ground tend to die away with height over the range which is of importance in this discussion (i.e. up to about 25 km). This, and the fact that above the 200-mb level the wind strength decreases with height within the range, are used to extend the graphs up to $p = 0$. This neglects the dynamic activity caused by direct radiation absorption above 25 km but, since the region contains only about 1 per cent of the total mass, the error involved is not likely to be appreciable. Eady (1949) and others have given mathematical models showing that zonal flow is unstable, the perturbed flow forming long waves and cyclone waves in which a preferred wavelength dominates the character of the disturbance.

It is natural to wonder how important these variations are. A relevant test is to compare the mean energy associated with the real flow with that of the mean flow. If u is the geostrophic component and v the ageostrophic component of the flow, of magnitude V, then the mean total energy of the real flow over unit surface area is calculated as :

$$\frac{1}{2g}\int_0^{10^4} \overline{u^2 + v^2}\, dp = 1{\cdot}71 \times 10^9 \text{ erg cm}^{-2}$$

This is obtained by finding the area under the curve for $\overline{V}^2$ against pressure p (see Fig. 1 and Table 1). The hydrostatic equation $dp = -\rho g\, dz$ is used to express the integral in terms of pressure.

Although the energy of the mean flow has not been found from the British Isles data, a useful order of magnitude can be determined if we compare it with the energy of the mean flow for North America (Starr 1951) at the same latitude, namely $0{\cdot}71 \times 10^9$ erg cm^{-2}.

An inspection of the maps suggests that the energy associated with the mean flow is probably considerably greater over North America than over the British Isles. Thus the energy associated with the differences from the mean is at least of the same order of magnitude as that of the mean, and the variations can therefore not be neglected.

Similarly the energy inputs are different in the two cases. The mean flow receives energy from two sources. Kuo (1951) has estimated the magnitude of the conversion of eddy kinetic energy into that of the mean flow at about 10^3 erg cm^{-2} sec^{-1} and a rough calculation shows that the input due to the meridional flow down the interzonal pressure gradient must be of a similar order of magnitude. Hence the energy input of the mean flow is not likely greatly to exceed 2×10^3 erg cm^{-2} sec^{-1}.

By comparison, the analysis below yields an input of $8{\cdot}8 \times 10^3$ erg cm^{-2} sec^{-1} for the whole flow. Furthermore, if the mean flux of energy away from the British Isles is ignored, these values indicate in what proportions the mean flow and the variations from it are losing energy by dissipation. The estimated total dissipation can also be compared with Brunt's value (1926) of 5×10^3 erg cm^{-2} sec^{-1} (see § 3).

The final section deals with one of the consequences of the data; a hypothetical mathematical model of a depression is allowed to fill by the influx of mass across its perimeter under conditions similar to those of the mean geostrophic and ageostrophic flow over the British Isles. The time-scale of the model's life is compared with observation.

2. Equations of energy

If the variations are to be taken into account, and if the equations of motion and energy are to be simplified at all, then it is necessary to choose the coordinate system so that the pressure is independent of one of the coordinates. This has the advantage that our interest need be focused on only one of the equations of motion, that of the

omponent of flow across the isobars. One of the coordinates will be height above round level, and the third will be orthogonal to the other two. As can be seen by comaring Figs. 2 and 3, showing the variations with pressure of the mean ageostrophic nd geostrophic components respectively, the former component is much smaller than he latter (except very near the ground). This fact enables us to simplify the resulting quation of energy to a tractable form.

The chief difficulty of choosing coordinates in this way, is that the coordinate netork is very complex. It is useful to approximate to the network in the neighbourhood f the point by a known coordinate system for which at the point, at least, the directions nd the radii of curvature of the two networks are identical.

The simplest network is a set of concentric arcs and corresponding radii, where he arcs approximate to the isobars at the particular point in question and the radii to he lines of steepest pressure-descent. This network does not entirely satisfy the conitions since it neglects the curvature of the last-named lines. The terms involving the urvature, in the energy equation, are small however and the equation reduces to the ame form as that of more complex networks.

The network which will be used below is the double system of confocal parabolae Jeffreys and Jeffreys 1946). If the coordinates in this system are (ξ_1, ξ_2, z) where $\xi_1 =$ contant, and $\xi_2 =$ constant approximate to the isobars and to the orthogonal system at he point respectively, then the following relationship holds :

$$(\xi_1 + i\xi_2) = (x_1 + ix_2)^{1/2},$$

where (x_1, x_2) are the Cartesian coordinates of the point; the origin is at the mutual focus nd $x_2 = 0$ is the axis of the parabolae.

If r and s are the radii of curvature of the isobar and the orthogonal line, respectively, hrough the point, it can be easily shown that

$$r = \frac{2(\xi_1^2 + \xi_2^2)^{3/2}}{\xi_1}, \qquad s = \frac{2(\xi_1^2 + \xi_2^2)^{3/2}}{\xi_2},$$

nd the fundamental elements are of length $h_1\, d\xi_1$ and $h_2\, d\xi_2$ where

$$h_1 = h_2 = 2(\xi_1^2 + \xi_2^2)^{1/2} = 2\,|\xi|.$$

t should be noted, firstly, that the transformation has a branch point at the focus and two Riemann sheets are required for a full interpretation. With two sheets, all combinations f r and s are possible. If r is positive when the curvature is cyclonic, then s is positive vhen the geostrophic flow is divergent and negative when convergent. Secondly, since he field of pressure is in constant flux, the position of the point relative to the confocal ystem of parabolae varies with time. Thus, when the equations of motion are formulated vith respect to the coordinate network of that instant, the velocities u_* and v_* are the components of flow parallel to fixed axes which themselves are parallel to the lines $\xi_1 =$ constant, $\xi_2 =$ constant respectively. The velocities u and v are the geostrophic nd ageostrophic wind components and are thus at all times parallel to the rotating axes.

Now, at that instant, $u_* = u$ and $v_* = v$ of course, but $\partial u_*/\partial t \neq \partial u/\partial t$ and $\partial v_*/\partial t \neq \partial v/\partial t$ since (considering the first of these two inequalities) by the right-hand ide we mean ' the rate of change of the geostrophic component with time at that point,' vhereas by the left-hand side we mean ' the rate of change of the velocity component n the fixed direction which is parallel to the isobar $\xi_1 =$ constant of that instant.' If the ngular velocity of the coordinate system about the point is $\Omega = \Omega(t)$ then,

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho u_*^2) = \frac{\partial}{\partial t}(\tfrac{1}{2}\rho u^2) - \Omega\,\rho uv$$

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v_*^2) = \frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2) + \Omega\,\rho uv$$

Provided there exists such a coordinate network as above, which fits the pressure field in the neighbourhood of the point, the equation of motion in the $\xi_2 =$ constant direction is,

$$\frac{\partial v}{\partial t} + \Omega u - \frac{v}{2|\xi|}\frac{\partial v}{\partial \xi_1} + \frac{w}{2|\xi|}\frac{\partial}{\partial z}(2|\xi| v) + \frac{u}{4|\xi|^2}\frac{\partial}{\partial \xi_2}(2|\xi| v) + \frac{u^2}{4|\xi|^2}\frac{\partial}{\partial \xi_1}(2|\xi|)$$
$$- \frac{1}{2|\xi|}\frac{1}{\rho}\frac{\partial p}{\partial \xi_1} = -2\,\omega\, u \sin\phi + \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right) \quad . \quad . \quad . \quad (1$$

where A is the coefficient of vertical eddy diffusivity. The effect of horizontal shear on $\partial v/\partial t$ is neglected by comparison with the vertical shearing-stress term. Also, the terms including the radius of curvature of the earth's surface are not included as these can easily be shown to be very small compared with the other terms.

The equation of continuity is,

$$\frac{\partial \rho}{\partial t} + \frac{1}{4|\xi|^2}\left[\frac{\partial}{\partial \xi_2}(2|\xi|\rho u) - \frac{\partial}{\partial \xi_1}(2|\xi|\rho v) + \frac{\partial}{\partial z}(4|\xi|^2\rho w)\right] = 0 \quad . \quad (2$$

Now if Eq. (1) is multiplied by ρv and Eq. (2) by $\frac{1}{2}v$ and then the two equations added we obtain

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2) + \Omega\,\rho uv + \frac{1}{2|\xi|}\frac{\partial}{\partial \xi_2}(\tfrac{1}{2}\rho uv^2) - \frac{1}{2|\xi|}\frac{\partial}{\partial \xi_1}(\tfrac{1}{2}\rho v^3) + \frac{\partial}{\partial z}(\tfrac{1}{2}\rho v^2 w) + 2\rho v^2 w\frac{1}{|\xi|}\frac{\partial}{\partial z}|\xi|$$
$$- \rho\frac{v^3}{2r} + \rho\frac{u^2 v}{r} + \tfrac{3}{2}\rho\frac{uv^2}{s} - \frac{v}{2|\xi|}\frac{\partial p}{\partial \xi_1} = -2\,\omega\sin\phi\,\rho uv + v\frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right) \quad . \quad (3$$

which is the equation of energy we require.

When means with respect to time are taken, the first term drops out identically. This equation is integrated with respect to height from $z = 0$ to $z = \infty$. Since the ageostrophic component is much smaller than the geostrophic component we make the hypothesis that the mean transport and dissipation of $\frac{1}{2}\rho v^2$ are negligible compared with the pressure term, the centrifugal and the Coriolis terms which transfer energy from the ageostrophic to the geostrophic motion. In the mean with respect to time and space over the whole globe, the work done on the motion by the down-gradient flow must balance the dissipation terms :

$$-\int_0^\infty \overline{\frac{v}{2|\xi|}\frac{\partial p}{\partial \xi_1}}\,dz = \int_0^\infty \left[\overline{v\frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right)} + \overline{\frac{u}{4|\xi|}\frac{\partial}{\partial \xi_1}\left(\frac{D}{|\xi|}\frac{\partial u}{\partial \xi_1}\right)} + \overline{u\frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)}\right] dz$$

Here the last two terms represent the dissipation of $\overline{\frac{1}{2}\rho u^2}$ due to horizontal and vertical mixing respectively; D is the coefficient of horizontal eddy diffusivity. Generally the former of these two processes is small compared with the latter but in certain circumstances, as for example near the jet stream (see § 5), this might no longer be the case. However, the combined effect of these two terms is responsible for almost all the dissipation and the term occurring in our Eq. (3) is neglected by comparison with the pressure term. Further, if we assume that there is no correlation between s and the wind speed and that $\overline{v^2} \ll \overline{u^2}$ then the dominant centrifugal term is $\int_0^\infty \overline{(\rho u^2 v/r)}\,dz$.

The equation thus reduces to :

$$\int_0^\infty \overline{\frac{v}{2|\xi|}\frac{\partial p}{\partial \xi_1}}\,dz = \int_0^\infty 2\omega \sin\phi \;\; \overline{\rho u v}\,dz + \int_0^\infty \overline{\rho \frac{u^2 v}{r}}\,dz + \int_0^\infty \overline{\Omega \rho u v}\,dz.$$

Finally, the angular velocity Ω is usually much smaller than ω and, in fact, it dies away to zero as $p \to 0$. Therefore, by comparison with the Coriolis term, this term is not included. The velocities are given at fixed pressure-levels and so the remaining terms on the right-hand side are expressed in the alternative form

$$\int_0^\infty \overline{\frac{v}{2|\xi|}\frac{\partial p}{\partial \xi_1}}\,dz = \frac{1}{g}\int_0^{10^6} \overline{\frac{u^2 v}{r}}\,dp + 2\frac{\omega}{g}\sin\phi \int_0^{10^6} \overline{uv}\,dp \quad . \quad . \quad . \quad (4)$$

where 10^6 represents ground-level atmospheric pressure in dyne cm^{-2}. Thus practically all of the kinetic energy which is fed into the ageostrophic motion is transferred by the Coriolis and centrifugal terms to the geostrophic motion where it is eventually dissipated. As pointed out in the introduction, the assumptions made above with regard to the integrated mean-energy equation, although not so restrictive as assuming that the corresponding gradient wind and the real wind are identical, make the energy equation in the form of (4) the same as that which we would have obtained had this assumption been made. The equation for $\frac{1}{2}\rho u^2$ would not be similarly reduced by our assumptions.

3. Dissipation of energy

The mean kinetic-energy dissipation by viscous forces must balance the mean generation of kinetic energy by the pressure forces. Brunt (1926) has estimated the energy dissipation in the atmosphere using in part experimental data and partially the Ekman distribution of velocity. In the lowest kilometre he assumes,

$$\mathbf{V} = \mathbf{G} + 2^{\frac{1}{2}}\,\mathbf{G}\sin\alpha \;\; \exp\{-Bz + i(\tfrac{3}{4}\pi + \alpha - Bz)\}, \text{ (Ekman distribution)}$$

where $\mathbf{V} = u + iv$, $\mathbf{G}$ is the geostrophic wind, $B = (\rho\,\omega\sin\phi/A)^{1/2}$ and α is the angle between the surface wind and the isobars. Then the energy dissipated is equal to the energy generated between $z = 0$ and $z = \{\frac{3}{4}\pi + \alpha\}/B$, where the wind first becomes parallel to the isobars; if the transport terms are neglected this dissipation is given by

$$-v\frac{\partial p}{\partial y} = 2\,\omega\sin\phi\sin\alpha \;\; \rho\frac{G^2}{B}\{\cos\alpha - 2^{-\frac{1}{2}}\exp[-(\tfrac{3}{4}\pi + \alpha)]\}$$

The second term in the bracket is small compared with the first. Brunt takes $\sin\phi = 0{\cdot}6$, $\alpha = 22°30'$, $G = 15$ m sec^{-1}, $K = A/\rho = 5 \times 10^4$, $\rho = 1{\cdot}2 \times 10^{-3}$ g cm^{-3}. Then $(\omega\rho\sin\phi \;\; G^2\sin 2\alpha)/B = 3 \times 10^3$ erg cm^{-2} sec^{-1}. Between the 1 km and 10 km levels he estimates $u\,\partial^2 u/\partial z^2 + v\,\partial^2 v/\partial z^2$ from sample cases and finds a mean value of the order of 2×10^{-4}. He assumes that at the ground $A \simeq 65$ and at 5 km $A \simeq 3{\cdot}5$.

Hence $\int_{1\text{ km}}^{10\text{ km}} A\left(u\frac{\partial^2 u}{\partial z^2} + v\frac{\partial^2 v}{\partial z^2}\right)dz = O(2 \times 10^3)$ erg cm^{-2} sec^{-1}. Above 10 km he claimed that $A(u\,\partial^2 u/\partial z^2 + v\,\partial^2 v/\partial z^2)$ is small except at the base of the stratosphere. Since in 1926 no data were available, this contribution was neglected.

Hence the total estimated mean dissipation is

$$5 \times 10^3 \text{ erg cm}^{-2}\text{ sec}^{-1}$$

4. The data

The geostrophic and ageostrophic components of wind velocity and the radius c curvature of the isobars were deduced from the results of radiosonde ascents over th British Isles for the year 1953, as given in the *Daily Aerological Record* and the surfac values in the *Daily Weather Report* of the Meteorological Office, London.

Charts were analysed at nine pressure levels for every third day. The stations fro which the data were drawn include Lerwick, Stornoway, Leuchars, Aldergrove, Liverpoo Hemsby, Crawley, Larkhill, Camborne and Valentia.

The errors involved in the result arise from :

(i) the data; especially above the 300-mb level where the inaccuracy of th recording instruments is comparable with variations of the data.

(ii) the possibility of wrong interpretation of the pressure system and measure ment of the quantities required.

(iii) the limitation of the data to a single year.

The errors involved in (ii) are not cumulative because both the centrifugal term and the Coriolis term are odd functions of v, the component which is most affected by small errors in the direction of the contours. Again, for those days on which there exists a large pressure gradient and, therefore, strong winds, and hence whose contributions to the totals are above the average, the contour plotting, owing to the simplicity of the system, will be satisfactorily accurate.

TABLE 1. Data derived from the analysis

All velocities are in knots (1kt = 0·515 m/sec); the last row gives the area under the curves (kt mb).

Pressure level (mb)	$\overline{V^2}$	Number of readings	$\bar{u}$	$\bar{v}$	$\bar{u}\,\bar{v}$	$\overline{uv}$	$\overline{uv}/\overline{V^2}$	Number of readings	$\overline{u^2v/V}$
1,000	186	269	9·0	3·55	31·7	46·0	0·25	264	0·61
900	544	366	13·5	1·37	18·5	34·5	0·063	311	1·10
800	665	369	22·1	0·0	0·0	− 2·4	− 0·0036	301	− 0·95
700	831	368	24·8	0·29	7·2	5·0	0·006	352	0·61
500	1,507	368	32·8	0·35	11·3	14·7	0·00975	360	5·74
400	1,850	346	35·4	2·13	75·5	84·2	0·0455	—	—
300	2,898	368	45·3	− 1·09	− 49·5	− 41·4	− 0·0143	363	7·83
200	2,607	171	44·2	− 0·64	− 28·3	− 17·0	− 0·0065	145	4·39
100	487	58	19·4	0·26	5·0	− 8·5	− 0·0174	31	− 1·13
$\int_0^{1,000}(\ )\,dp$		—	27,080	476	6,480	10,200	—	—	2,785

5. The results of the analysis

The results of the analysis are given in Table 1.

The graph of $\overline{V^2}$ against pressure is plotted in Fig. 1 and that of $\bar{u}$ against p in Fig. 3; it will be noticed that the latter curve is remarkably linear within the troposphere. The graph of $\bar{v}$ against p, Fig. 2, is not as smooth as the above two curves and the significance of this is discussed below.

Although the two curves in Fig. 4, showing the variation of $\overline{uv}$ and $\bar{u}\,\bar{v}$ with p, are very similar, as is to be expected, the area under $\bar{u}\,\bar{v}$ is significantly smaller than that for $\overline{uv}$. This implies that there exists a mean positive correlation between u and v, although this is small.

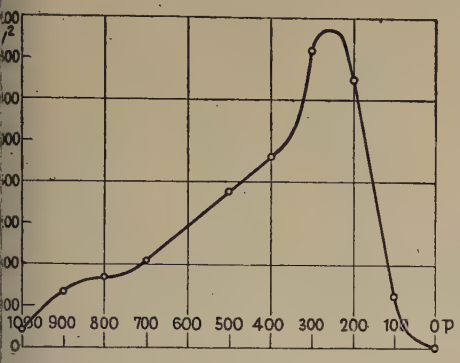

Figure 1. Variation of $\overline{V^2}$ with pressure.

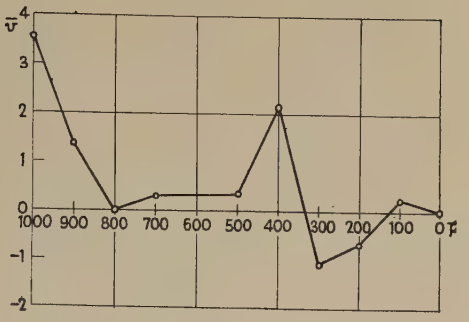

Figure 2. Variation of the ageostrophic component with pressure.

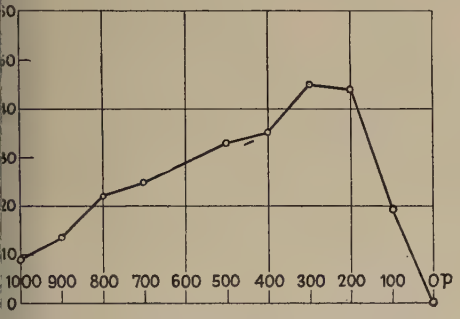

Figure 3. Variation of the geostrophic component with pressure.

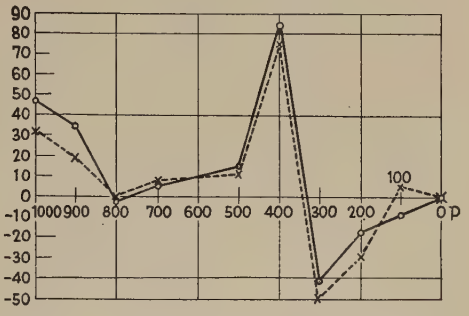

Figure 4. ——— Variation of $\overline{uv}$ with pressure.
- - - - Variation of $\bar{u}\bar{v}$ with pressure.

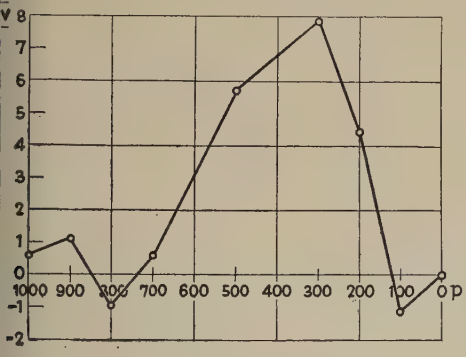

Figure 5. Variation of $\overline{u^2 v/r}$ with pressure.

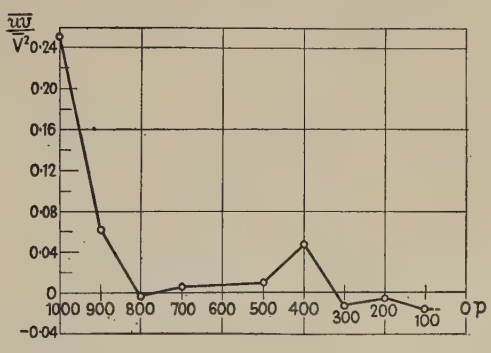

Figure 6. Variation of $\overline{uv}/\overline{V^2}$ with pressure.

We require the magnitude of the Coriolis and centrifugal terms in Eq. (4). Con-rting to cgs units the Coriolis term is :

$$2\frac{\omega}{g}\sin\phi\int_0^{10^6}\overline{uv}\;dp = 4{\cdot}95\times10^3\ \text{erg cm}^{-2}\ \text{sec}^{-1},$$

d the centrifugal term, which is obtained from the graph of $\overline{u^2 v/r}$ against p (Fig. 5) is,

$$\frac{1}{g}\int_0^{10^6}\frac{\overline{u^2 v}}{r}\,dp = 3{\cdot}85\times10^3\ \text{erg cm}^{-2}\ \text{sec}^{-1}$$

here the following values have been taken : $\omega = 7{\cdot}29\times10^{-5}\ \text{sec}^{-1}$, $\phi = 55°$.

Hence the estimate for the mean kinetic-energy generation by pressure forces $8{\cdot}8 \times 10^3$ erg cm^{-2} sec^{-1}.

Although the magnitude of the centrifugal term is nowhere large by comparis with the Coriolis term it is fairly consistently positive. It is of interest to note that t centrifugal term is negative only at those levels where the correlation coefficient negative also.

In contrast the Coriolis term is more variable, so that the total area is more sensiti to small changes in the ordinates. Some consideration is required of this graph or th of $\bar{v}$ against p. If the wind is in a state of acceleration, due to contour divergence or the effect of shearing stresses, then, in general, there will be a component of flow acr the isobars. That is, equilibrium of the forces in steady motion, in which the wi speed is greater, say, than the gradient wind, demands that v should be negative; air w flow from low to high pressure. Hence the positive area between the surface and 800 m is due to the ground shearing stress which reduces the wind below its gradient value.

However, when considering the vertical shear term $\partial (A\, \partial u/\partial z)/\partial z$, it should remembered that A is a function of height and depends upon the stability of the a for example, in the region of the tropopause (400-200 mb), A rapidly decreases wi height, owing to the increase of stability there. At 400 mb there is a well-marked negati correlation between the signs of v and $\partial^2 u/\partial z^2$ (where the latter is approximated by t sign of $u_{300} + u_{500} - 2u_{400}$), and thus, although the mean of $\partial^2 u/\partial z^2$ appears to be actua positive (Fig. 3), when the decrease of A is taken into account it seems likely that the a being more sensitive to the velocity profile beneath it than to that above it, will suff a mean positive resistance and therefore v will be positive.

Within the stratosphere (base height near the 300-mb level) A becomes very sm so that the effect of vertical shear is no longer a dominating factor. Whereas no correlati could be found between the signs of $\partial^2 u/\partial z^2$ and v, for wind speeds greater than 60 k v was found to be negative approximately two times out of every three. The jet strea whose height is roughly 300 mb would frequently be within 200 mi of a point whos wind speed is of this magnitude. The effect of lateral shearing stress acting on such profile produces supergradient winds at the edges of the stream. Murray and Danie (1953) have investigated the transverse flow across jet streams and have shown tha although in conditions of zero contour confluence the mean ageostrophic wind is negativ statistically it is not significantly so. However, owing to the greater frequency of occurrenc of jet exits (regions of contour diffluence) compared with jet entrances (regions of conto confluence) near the British Isles, conditions of diffluence and subsequent supergradie winds and negative v predominate. In the stratosphere, then, $\overline{uv}$ is negative whi implies a net upward flux of energy from below.

These results indicate that, at jet exits, the magnitude of v falls very rapidly, belo the jet core, so that, at 400 mb, $|\bar{v}|$ is less than one-third of that at 300 mb. This suppor our hypothesis that, at 400 mb, v is most influenced by the nature of the vertical velocit profile.

Although both the centrifugal and Coriolis terms remain large between 500 m and 100 mb it should be remembered that $\overline{V}^2$ is also very large there. The non-dimensio ratio $\overline{uv}/\overline{V}^2$ is shown in Fig. 6. This ratio, which behaves somewhat like the mean ang between the flow and the isobars, is, as is to be expected, greatest near the ground.

We found in § 1 that the total mean energy for the British Isles is $1{\cdot}71 \times 10^9$ erg cm^{-2} At the rate determined above this is equivalent to 2·33 days' total production over un surface area. This is about twice the time given by Brunt (1939a) in which all the kinet energy in the atmosphere would be dissipated if no further kinetic energy were produce and assuming a uniform rate of dissipation of 5×10^3 erg cm^{-2} sec^{-1}.

6. Life of depressions

The mean rate of mass flow f across unit length of an isobar is :

$$f = \int_0^\infty \overline{\rho v}\, dz = \frac{1}{g}\int_0^{10^6} \bar{v}\, dp = 25 \text{ kg cm}^{-1} \text{ sec}^{-1}$$

comparison, if the Ekman distribution is used, a value of 20 kg cm^{-1} sec^{-1} is obtained runt 1939b).

A further check on the order of magnitude of $\bar{v}$ is to consider how long it would ke for a symmetric depression, with concentric isobars at all heights, to ' fill ' so that e flux across its perimeter is reduced to $1/e$ of its initial value. If we assume that the gle between the wind direction and the isobars is a function of height only, and that e vertical distribution of velocity differs at any time only by a constant scale factor om the mean distribution (Figs. 2 and 3), then knowledge of the velocity along any dius from the axis of the depression will determine the velocity field throughout the ole depression.

If f = the mass flux across the isobar

m = mass above unit surface area

a = typical radius of depression

r = radius vector from the centre of the depression, then the equation of continuity of mass is

$$\frac{1}{r}\frac{\partial}{\partial r}(rf) = \frac{\partial m}{\partial t} \qquad . \quad . \quad . \quad . \quad (5)$$

ow in the model we have u linearly related to the geostrophic wind G at the ground. n estimate of this mean constant of proportionality is $\frac{2}{3}$ (Petterssen 1940b), that is :

$$\bar{u} \simeq \tfrac{2}{3}\overline{G}, \quad \text{say.}$$

Since $p = mg$ we have

$$\frac{\partial m}{\partial r} = \frac{1}{g}\frac{\partial p}{\partial r} = 2\frac{\omega}{g}\sin\phi \quad \rho G = B\int_0^\infty \rho v\, dz,$$

$$\text{i.e.,} \quad \frac{\partial m}{\partial r} = Bf \qquad . \quad . \quad . \quad . \quad (6)$$

here B is some constant depending on the linear relations between ρG, ρu, ρv and f, etermined by comparison of the graphs.

Eliminating m from Eqs. (5) and (6) we get

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rf)\right) = \frac{\partial}{\partial t}(Bf)$$

Ve try a solution of the form $f = R(r)\,T(t)$. Then, if K is some constant to be deter- ined, the solution is

$$f = Q\, e^{-\frac{K}{A}t} J_1(K^{\frac{1}{2}} r). \qquad . \quad . \quad . \quad (7)$$

Now since $\partial p/\partial r \to 0$ as $f \to 0$ the depression will effectively cease to exist when becomes very small. The order of magnitude of the life of a depression will therefore e given by the time in which f is reduced to $1/e$ of its original value; that is, the time- ale of the depression is B/K.

The constant K can be found as follows; at the edge of the depression we assume o mass accumulation : $\partial m/\partial t = 0$.

Therefore from Eq. (5) we have

$$\left[\frac{\partial}{\partial r}(rf)\right]_{r=a} = 0,$$

that is

$$\left[\frac{\partial}{\partial r}(rJ_1(K^{\frac{1}{2}}r))\right]_{r=a} = 0.$$

Hence

$$J_0(K^{\frac{1}{2}}a) = 0.$$

The first root of this equation is $(K^{\frac{1}{2}}a) = 2{\cdot}405$. If we now choose a typical depressio of radius $a = 8 \times 10^7$ cm say, then $K = 9 \times 10^{-16}$.

The constant A can be obtained as described above by comparison of the graph

$$2\frac{\omega}{g}\sin\phi \;\; \rho G = \tfrac{3}{2}\cdot 2\frac{\omega}{g}\sin\phi \cdot \frac{1}{5{\cdot}1\times 10^4}\int_0^{10^6} u\,dp = \tfrac{3}{2}\cdot\frac{7{\cdot}23\times 10^{-6}}{5{\cdot}1\times 10^4} f$$

Therefore $B = 2{\cdot}17 \times 10^{-10}$, and the time scale $B/K = 2{\cdot}4 \times 10^5$ sec or, in term of days, approximately 2·8 days.

As can be seen from Fig. 7 this is the right order of magnitude for a depression this size. In this figure the life-span of a depression is defined as the time between i arrival over the British Isles and when the weather charts no longer show it as a separa entity. Depressions which coalesced with other low-pressure systems during this tim were not included. One point of this theory can easily be checked. If we assume, a we have above, that B is an absolute constant, independent of a, then since $K \propto a^{-2}$ w have the time scale $l\,(= B/K)$ proportional to a^2. Fig. 7 plots l against a on a log scal The theory predicts a straight line with a gradient of 2. The experimental line has gradient of 1·25. Whether this disagreement is due entirely to the theory, or due i part to the different conditions which the depressions are subjected to after they hav passed over the British Isles, is hard to say, but it seems likely that the latter will at leas have some influence.

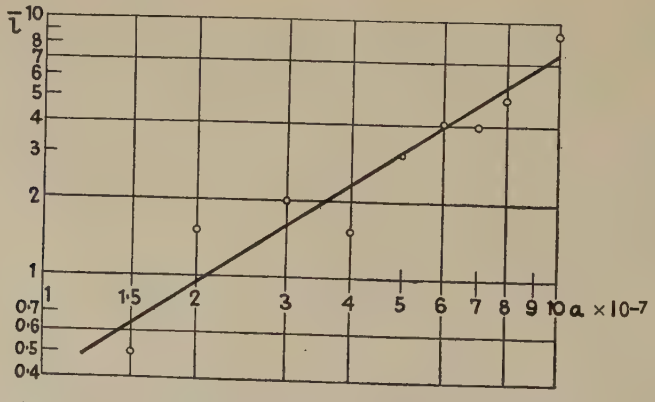

Figure 7. Mean life-span distribution of 30 depressions during Jan.-Sept. 1953.

Acknowledgments

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A potential-theory formulation of radiant-heat transfer

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Summary

The radiant-heat flux in a homogeneous atmosphere constitutes an irrotational vector field. As a consequence, the flux can be represented as the gradient of a suitably-defined radiant-heat potential. The potential is shown to be a functional of the product of the radiant-source function at all points in the atmosphere and an attenuation factor to allow for the absorption of the intervening medium.

The formalism is independent of the choice of coordinate system. The familiar equations of radiant transfer, such as the Milne-Schwarzschild equilibrium equation, are deducible as special cases for the particular geometry of the plane-parallel atmosphere. Most importantly, the radiant heating at an arbitrary point is expressed as a functional of the second derivative of the Planck intensity. This radiant-heat flow equation will form the basis for the construction of a new atmospheric radiation chart.

1. Introduction

The heat flux due to radiant transfer in a homogeneous atmosphere (one in which the absorption coefficient is independent of position) forms an irrotational vector field. This zero-curl property of the flux follows from the second law of thermodynamics which states the impossibility of continuously extracting energy by traversing a closed circuit in the radiation field. This implies that the flux can be expressed as the gradient of a suitably chosen radiant potential. Our choice of potential is justified first on intuitive grounds and proved later by the correctness of deductions made from its assumption.

2. Radiant-transfer theory

We begin by assuming that the radiant-heat potential for an infinite homogeneous atmosphere is given by

$$\psi(r) = \int_0^\infty d\nu \int \frac{d\omega}{4\pi} \int_{-\infty}^{r} \mathcal{J}_\nu(r')\, e^{-\kappa_\nu (r-r')}\, dr', \qquad (1$$

where $\mathcal{J}_\nu = j_\nu/\kappa_\nu$ is the source function, the ratio of the emission and absorption coefficients. The radiant potential is a functional of the source function at all points anterior to r and of a transmission factor depending on the opacity and separation distance of r and r'. This quantity is then averaged over all directions about r and integrated over the spectrum.

For an atmosphere in local thermodynamic equilibrium the source function is given by Kirchhoff's law, as the Planck intensity. The monochromatic radiant potential for such an atmosphere becomes

$$\psi_\nu(r) = \int \frac{d\omega}{4\pi} \int_{-\infty}^{r} B_\nu(r')\, e^{-\kappa_\nu (r-r')}\, dr'. \qquad (2$$

We take the gradient of this scalar point-function to obtain the radiant flux per unit area and solid angle

$$\tfrac{1}{4}\mathbf{F}_\nu(r) = -\nabla\psi_\nu = -\frac{d}{dr}\int_\sigma \psi_\nu\, \epsilon\, d\sigma = -\epsilon\frac{d\psi_\nu}{dr}, \qquad (3$$

$$= -\int \frac{d\omega}{4\pi}\,\epsilon \int_{-\infty}^{r} B_\nu(r')\left[\frac{d}{dr}\, e^{-\kappa_\nu (r-r')}\right] dr' - B_\nu(r)\int \frac{d\omega}{4\pi}\,\epsilon, \qquad (4$$

$$= \int \frac{d\omega}{4\pi} \epsilon \int_{-\infty}^{\tau} \kappa_\nu B_\nu(\tau') e^{-\kappa_\nu(\tau-\tau')} d\tau'. \quad . \quad . \quad . \quad (5)$$

The second term of (4) vanishes in the integration of the unit vector over all directions. The factor of 1/4 in the radiant flux is necessary if we follow the notation of Chandrasekhar (see Eq. (8) *infra*). The quantity

$$I_\nu(\tau, \omega) = \int_{-\infty}^{\tau} \kappa_\nu B_\nu(\tau') e^{-\kappa_\nu(\tau-\tau')} d\tau', \quad . \quad . \quad . \quad (6)$$

is the specific intensity, the integral form of Schwarzschild's equation. Using (6) and (5) we can write the flux and average intensity as

$$\pi \mathbf{F}_\nu = \int d\omega \epsilon I_\nu(\tau, \omega); \quad J_\nu = \int \frac{d\omega}{4\pi} I_\nu(\tau, \omega) = \tfrac{1}{4} \mathbf{F}_\nu \cdot \epsilon. \quad . \quad . \quad (7)$$

The radiant flux is seen to be the sum of the *vector* polygon of the intensities impinging on all directions about τ, while J_ν is the *scalar* average of these intensities. For a radiation field which has symmetry about the vertical axis, the flux becomes

$$\pi \mathbf{F}_\nu = \int d\omega \mathbf{k} \cdot \epsilon I_\nu = 2\pi \int_0^{\pi} I_\nu(\theta) \cos\theta \sin\theta \, d\theta. \quad . \quad . \quad . \quad (8)$$

The heating rate is proportional to the negative divergence of the radiant flux. We use the divergence theorem for an infinitesimal volume enclosing τ, and (3), (7) to obtain

$$\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \pi \mathbf{F}_\nu = -\frac{d}{d\tau} \int_\sigma \pi \mathbf{F}_\nu \cdot \epsilon \, d\sigma = -4\pi \frac{dJ_\nu}{d\tau} = 4\pi \frac{d^2 \psi_\nu}{d\tau^2}, \quad . \quad (9)$$

$$= \int d\omega \int_{-\infty}^{\tau} \kappa_\nu^2 B_\nu(\tau') e^{-\kappa_\nu(\tau-\tau')} d\tau' - 4\pi \kappa_\nu B_\nu(\tau). \quad . \quad . \quad (10)$$

The heating is therefore a functional of the Planck intensity and, implicitly, of the temperature field of the atmosphere. For the special case of radiative equilibrium ($\partial T/\partial t = 0$) and a plane-parallel atmosphere ($d\tau = dz \sec\theta$), (10) becomes the Milne-Schwarzschild integral equation

$$B_\nu = \tfrac{1}{2} \int_0^{\pi} \sin\theta \, d\theta \int_{-\infty}^{\tau} \kappa_\nu B_\nu(\tau') e^{-\kappa_\nu(\tau-\tau')} d\tau', \quad . \quad . \quad (11)$$

$$= \tfrac{1}{2} \int_{-\infty}^{\infty} \kappa_\nu B_\nu(z') E_1(\kappa_\nu |z - z'|) \, dz', \quad . \quad . \quad . \quad (12)$$

where E_n is the exponential integral defined as usual by

$$E_n(x) = \int_0^{\pi/2} e^{-x \sec\theta} \cos^{n-2}\theta \sin\theta \, d\theta. \quad . \quad . \quad . \quad (13)$$

Eq. (10) can be further simplified by noting that for an isothermal atmosphere B_ν is constant, i.e. the heating is zero. Thus the heating must be a functional of the derivatives of the Planck intensity. After a double integration by parts of the radial integral in (10), we finally obtain the remarkably concise *radiant-heat flow equation*

$$\rho c_p \frac{\partial T}{\partial t} = \int d\omega \int_{-\infty}^{\tau} \frac{d^2 B_\nu}{d\tau'^2} e^{-\kappa_\nu(\tau-\tau')} d\tau'. \quad . \quad . \quad . \quad (14)$$

For the plane-parallel atmosphere this becomes simply

$$\rho c_p \frac{\partial T}{\partial t} = \pi \int_{-\infty}^{\infty} \frac{d^2 B_\nu}{dz'^2} 2E_3(\kappa_\nu |z - z'|) \, dz'. \quad . \quad . \quad . \quad (15)$$

The heating at a particular level is therefore the integral transform of the second derivative of the Planck intensity by the kernel $2E_3(\kappa_\nu |z - z'|)$. Physically, this relates the heating at a point to the curvature of the Planck intensity field at every other level multiplied by the probability that the flux penetrates the intervening distance. Eq. (15) shows further that in an infinite atmosphere, for radiative equilibrium to prevail, $d^2 B_\nu/dz^2$ must equal zero. Thus, departures from linearity of B_ν are related to the perturbing effect of the radiant boundaries, which act as heat sources and sinks.

3. The transfer equations in operator form

We may express the radiant potential as a linear operator acting on the source function

$$\psi_\nu(\tau) = \int \frac{d\omega}{4\pi} \int_{-\infty}^{\tau} \mathcal{J}_\nu(\tau') \mathcal{F}_\nu(\kappa_\nu(\tau - \tau'))\, d\tau' = \mathcal{R}(\mathcal{J}_\nu) \quad . \quad . \quad (16)$$

Here we have generalized the transmissivity to include cases other than the Lambert kernel of (1). Thus (16) would apply to narrow bands of intense lines, where the rapid variation of κ_ν causes a breakdown of Lambert's law.

Differentiation with respect to τ yields

$$\frac{d\psi_\nu}{d\tau} = \int \frac{d\omega}{4\pi} \int_{-\infty}^{\tau} \mathcal{J}_\nu(\tau') \frac{\partial \mathcal{F}_\nu(\kappa_\nu(\tau - \tau'))}{\partial \tau} d\tau' + \int \frac{d\omega}{4\pi} \mathcal{J}_\nu(\tau). \quad . \quad (17)$$

By making use of the optical reciprocity of the transmissivity, $\partial \mathcal{F}_\nu/\partial\tau = -\partial \mathcal{F}_\nu/\partial\tau'$, we can integrate (17) by parts, obtaining

$$\frac{d\psi_\nu}{d\tau} = \int \frac{d\omega}{4\pi} \int_{-\infty}^{\tau} \frac{d\mathcal{J}_\nu(\tau')}{d\tau'} \mathcal{F}_\nu(\kappa_\nu(\tau - \tau'))\, d\tau' = \mathcal{R}\left(\frac{d\mathcal{J}_\nu}{d\tau'}\right). \quad . \quad . \quad (18)$$

It follows immediately from (17) and (18) that

$$\frac{d^n \psi_\nu}{d\tau^n} = \mathcal{R}\left(\frac{d^n \mathcal{J}_\nu}{d\tau'^n}\right) \quad . \quad . \quad . \quad . \quad (19)$$

This, then, is the fundamental operator equation of radiant transfer. All the equations of radiant transfer are given by (19) for specified values of n.

TABLE 1. Radiant-transfer equations as particular cases of the fundamental operator equation

n	$d^n \psi_\nu/d\tau^n$	Radiant parameter	Radiant equation
0	ψ_ν	Potential	Potential
1	$-\epsilon\, d\psi_\nu/d\tau = \mathbf{F}_\nu/4$	Flux	Schwarzschild
2	$\frac{4\pi}{\rho c_p} d^2\psi_\nu/d\tau^2 = \partial T/\partial t$	Heating	Heat flow
2	$d^2\psi_\nu/d\tau^2 = 0 = \partial T/\partial t$	Equilibrium	Milne-Schwarzschild

4. Non-homogeneous atmosphere

We can extend the formalism to include the case wherein the absorption coefficient is a function of position in the atmosphere. The flux then becomes

$$\tfrac{1}{4}\mathbf{F}_\nu = \int \frac{d\omega}{4\pi} \epsilon \int_{-\infty}^{\tau} \kappa_\nu(\tau') B_\nu(\tau') e^{-\int_{\tau'}^{\tau} \kappa_\nu d\tau} d\tau'. \quad . \quad . \quad (20)$$

The heating rate is again obtained by taking the negative flux divergence

$$c_p \frac{\partial T}{\partial t} = -\nabla \cdot \pi \mathbf{F}_\nu = \int d\omega \int_{-\infty}^{\tau} \kappa_\nu(\tau')\,\kappa_\nu(\tau)\,B_\nu(\tau')\,e^{-\int_{\tau'}^{\tau}\kappa_\nu d\tau}\,d\tau' - 4\pi\,\kappa_\nu(\tau)\,B_\nu(\tau), \quad . \quad (21)$$

$$= \int d\omega \int_{-\infty}^{\tau} \frac{d}{d\tau'}\left[\frac{\kappa_\nu(\tau)}{\kappa_\nu(\tau')}\frac{dB_\nu}{d\tau'}\right] e^{-\int_{\tau'}^{\tau}\kappa_\nu d\tau}\,d\tau', \quad . \quad . \quad . \quad (22)$$

fter a double integration by parts.

5. Boundary conditions

We have limited our discussion to atmospheres of infinite extent. We now remove his restriction to include finite atmospheres and boundary effects.

Consider an atmosphere bounded by an arbitrary surface having the co-ordinates $r - R|$, ω as viewed from r. We can write (10) as

$$\rho\,c_p \frac{\partial T}{\partial t} = \int d\omega \int_{-\infty}^{R} \kappa_\nu^2\,B_\nu(\tau')\,e^{-\kappa_\nu(\tau-\tau')}\,d\tau' +$$

$$+ \int d\omega \int_{R}^{\tau} \kappa_\nu^2\,B_\nu(\tau')\,e^{-\kappa_\nu(\tau-\tau')}\,d\tau' - 4\pi\,\kappa_\nu\,B_\nu(\tau). \quad . \quad . \quad (23)$$

The boundary effects are included in the first term. Now Kirchhoff's law implies that n isothermal slab of infinite thickness radiates as a black body at that temperature. Hence, ve can write

$$\rho\,c_p \frac{\partial T}{\partial t} = \int d\omega\,\kappa_\nu\,I_\nu(R)\,e^{-\kappa_\nu(\tau-R')} +$$

$$+ \int d\omega \int_{R}^{\tau} \kappa_\nu^2\,B_\nu(\tau')\,e^{-\kappa_\nu(\tau-\tau')}\,d\tau' - 4\pi\,\kappa_\nu\,B_\nu(\tau), \quad . \quad . \quad (24)$$

where $I_\nu(R)$ is the black-body intensity of the boundary. Upon integrating the second erm twice by parts we obtain

$$\rho\,c_p \frac{\partial T}{\partial t} = \int d\omega \int_{R}^{\tau} \frac{d^2 B_\nu}{d\tau'^2}\,e^{-\kappa_\nu(\tau-\tau')}\,d\tau' +$$

$$+ \int d\omega \frac{dB_\nu(R)}{d\tau}\,e^{-\kappa_\nu(\tau-R)} + \int d\omega\,[I_\nu(R) - B_\nu(R)]\,\kappa_\nu e^{-\kappa_\nu(\tau-R)}. \quad (25)$$

The physical interpretation of (25) is similar to that of Green's solution of Poisson's quation. The first term is the heating due to the finite atmosphere, the second repre- ents the contribution of the boundary discontinuity of slope, while the third term is he heating of the double layer $I_\nu(R) - B_\nu(R)$ at the boundary.

Explanatory note on conventions. For radiant-heat quantities the notation of Chandrasekhar, *Radiative Transfer* (Oxford University Press, 1950) is used, except that ve denote by j_ν and κ_ν the *volume* rather than the *mass* emission and absorption oefficients.

551.578.11

The salinity of rainfall as a function of drop size

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SUMMARY

Measurements of the salinity of raindrops as a function of drop size show that there are characteristic variations according to the type of rain. For Bergeron-type rain near Sydney, New South Wales, the dependence may be expressed in the form salinity $\propto$ (radius)$^{-n}$, where n is a positive constant which is small when the cloudbase is low and as high as 2·6 for collection well below the cloud. The mean salinities over all drop sizes range from 1 to 41 parts NaCl per million. The main features of the experimental results are explained by a theoretical treatment which takes into account evaporation and the collection of salt below the cloud.

For rain from warm clouds in the Hawaiian Islands, the salinity has a pronounced minimum at a certain drop size near the median. The daily mean salinities decrease with increasing distance inland, along the direction of trade-wind flow, from 2·0-20 parts per million sodium chloride near sea level to 0·34-8·5 pt/mn near mean cloud base (2,100 ft) and 0·26-3·8 pt/mn at 4,500 ft. The salinity minimum occurs in samples at all three positions and secondary processes below the cloud cannot account for it; nor can it be a result of averaging the considerable variations of salinity which are found to occur from shower to shower.

It is suggested as a possible explanation that the largest cloud drops might form on giant salt nuclei and grow to raindrop sizes first by coalescence with one another. This receives support from measurements on single showers, which show that the first raindrops to fall are large and have a high salinity. A detailed interpretation would seem to require a knowledge of the salt concentration in individual cloud drops.

1. INTRODUCTION

Many investigations have been carried out during the past hundred years to determine the concentration of the soluble impurities in rain. The presence of variable quantities of sea salts has been of special interest, and the way in which their concentration changes with various geographical and meteorological factors has been discussed at length. The work to date has recently been summarized by Eriksson (1952) and will therefore be referred to only briefly. The concentration of chlorides has been taken generally as the best available measure of the total amount of sea salt in rain.

The chloride content varies with distance from the sea, there being a sharp drop in chlorinity quite close to the coast and a slower falling-off inland. Wind strength and direction in relation to the sea also have an influence; strong winds from the sea mean a high chlorinity of associated rain. The rate of rainfall and the total rainfall over a given period have been shown to bear a roughly inverse relation to chlorinity, heavy falls of rain tending to have a low value.

Recent work by Woodcock (1952) lends further support to these results. He has also, for the first time, tried to relate the observed chlorinites to the amount of salt in the air and to the mechanism of rain formation. The suggestion had earlier been made (Simpson 1941) that the initial large drops necessary to start growth by coalescence in warm clouds might form on giant sea-salt nuclei. If this is so, the sea-salt content of rain will not be an incidental property, but should give some fundamental information about the processes occurring in the clouds.

Theories have been developed to predict sizes of raindrops arising from coalescence processes, starting from various assumed populations of cloud droplets and larger drops, but little attention has been paid to the resultant salinities of the individual drops. All

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measurements have been carried out on the gross rainfall, ignoring possible variations with size. It is the purpose of the present investigation to measure salinity as a function of drop size in order to see if such a variation exists, and to compare the cases of ' freezing ' and ' non-freezing ' rain in an attempt to obtain information about the mechanism of growth of raindrops by coalescence.

Simple qualitative arguments can be used to indicate how we should expect such measurements to enable us to distinguish between various mechanisms which have been suggested. For example, the coalescence of equally or randomly saline cloud drops, after initiation of the precipitation by the Bergeron or freezing process, would produce raindrops having a uniform salinity at the base of the cloud. If, however, rain forms on a number of salt particles of the same mass falling through a cloud of relatively pure water droplets, or droplets formed on nuclei other than sea salt, the salinity of the raindrops would be inversely proportional to the final mass of water collected, or inversely proportional to the cube of the radius a.

When an upward air current in the cloud is considered, and nuclei of different sizes are introduced at the base, Bowen (1950) has shown that the smaller particles will rise further through the cloud and will finally become the larger raindrops. A calculation for typical conditions shows that an additional factor of about $1/a^2$ is thus introduced, and the salinity will now be more nearly proportional to $1/a^5$. It is clear that the salinity-size relationship could depend very critically on the mechanism of raindrop growth.

Other processes which can change the salinity will have to be kept in mind when we come to interpret the results. Rain may collect salt on the way to the ground, and evaporation will always take place in the drier air below the cloud; both these procesess will increase the relative salinity of the smaller drops.

2. Experimental procedure

(a) *Position of stations*

Rainwater samples were collected first at two stations near Sydney, Australia; at the Radiophysics Laboratory about 5 mi from the coast, and at Wentworth Falls, 45 mi inland and 2,850 ft above sea level. The second position was chosen once it became apparent that it would be desirable to obtain samples from near cloudbase to compare with those collected at sea level and near the coast.

It was also hoped that falls from warm clouds would occur, and allow a comparison with the more usual Bergeron type of rain. Over a period of nine months, June 1953 to February 1954, there were no cases of non-freezing rain which could be identified with certainty, and it was decided to continue the project in a place where such falls would be frequent and readily identifiable. Three stations were therefore set up on the island of Hawaii, near the town of Hilo, and operated during the months of April to June 1954. These were placed along the Saddle Road, on the windward slopes of Mauna Kea, on a line running close to the usual direction of the trade-wind flow. They were respectively a quarter of a mile inland at 200 ft above sea level; 7 mi inland, at 2,100 ft, which is near the usual level of the cloudbase on the mountain; and 15 mi inland at 4,500 ft, well into the orographic cloud. For a fuller description of the topographical and meteorological conditions in this area, reference can be made to a recent paper by Blanchard (1953).

(b) *Collection of the raindrop samples*

The separation of raindrops into six or seven size ranges was carried out using a simple modification of the raindrop spectrograph described by Bowen and Davidson

(1951). This consists essentially of a horizontal wind tunnel with a slit in the top where rain can enter. For the present purpose, pyrex funnels were set into the bottom of the tunnels in the place of the sensitized paper which was used previously to record the drop-size spectrum. The funnel sizes were chosen so that nearly equal volumes were collected in the several drop-size ranges.

Evaporation from the funnels was eliminated by collecting all samples under kerosene and drawing them off at the bottom of the funnels into well-stoppered vessels. Checks were made from the beginning of the experiments to ensure that the kerosene did not affect the measurements by liberating or absorbing salt, and it was concluded that any such effect was smaller than the measured concentrations and the differences between them over a single period of rain. Care had to be taken to avoid accidental contamination of the most dilute samples, which were sometimes more dilute than the best distilled water available.

Calibration was carried out directly by spraying drops into the wind tunnels from the roof of the Laboratory. A large number of drops was caught on slides covered with oil, and their diameters measured. The mean diameters agreed with those determined from a smooth curve drawn through the values obtained for all the funnels to within $\pm$ 0·02 mm; this gives an estimate of the probable error of the determination. This smoothed mean diameter was used as the measure of drop size at the centre of the funnel under operating conditions, but the actual range of diameters collected on a particular occasion would of course vary according to the distribution of drop sizes in the rain.

(c) *Analysis of samples*

The preliminary analyses were carried out by microchemical methods, chlorine being the element detected. This was inaccurate for the small volumes available. The only method found satisfactory for low concentrations in small samples was to employ a flame photometer to estimate sodium, and the concentration of this element rather than of chloride was henceforth taken as the measure of sea salt.

The samples were preserved and analysed in the same pyrex containers to reduce the risk of contamination. Measurements were made against freshly prepared standards, and a maximum estimate of the probable error would be $\pm$ 5 per cent. The technique is capable of greater accuracy than this, but the facts that samples often varied widely in concentration and that only one reading was possible in most cases have been taken into account.

3. Results for freezing rain

(a) *Relation between drop size and salinity at sea level*

Most of the collections of Bergeron-type rain were made in Sydney at the Radiophysics Laboratory of C.S.I.R.O. From September 1953, samples were obtained also at Wentworth Falls, but on only about ten occasions was the rain general enough to make a comparative study worth while.

The mean salinities over the period showed a wide variation; they ranged from 1·0 to 41 parts per million sodium chloride, the extreme salinities of individual samples ranging from less than 1 to 110 pt/mn. Logarithmic scales were used when plotting salinity against the drop diameters appropriate to the various funnels. Since the largest drops were unresolved (all drops greater than a certain size fell into the vessel directly under the opening of the spectrograph) a median size for these had to be estimated. This was done by using the drop-size distribution curve computed from the volumes of the samples, which were recorded as part of the routine measurements.

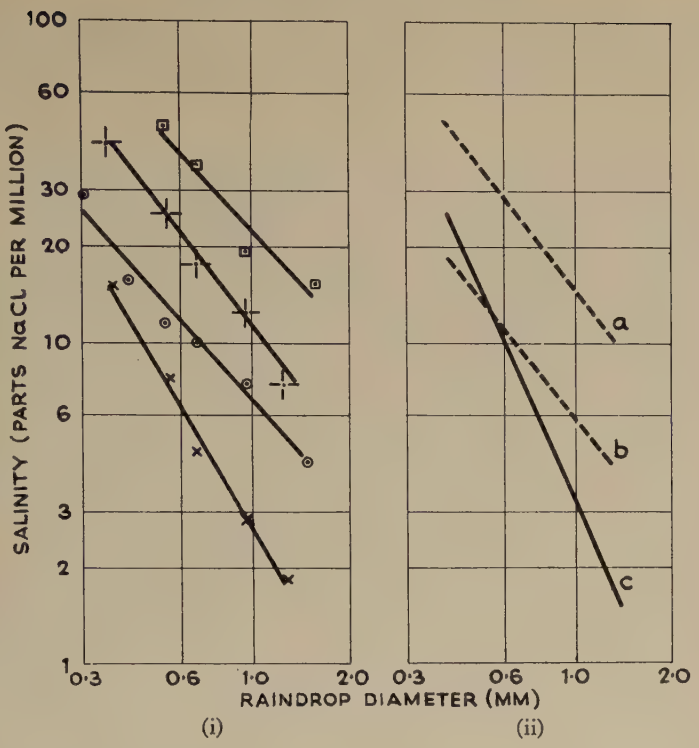


Figure 1. (i) Typical salinity results obtained at the Radiophysics Laboratory. (ii) Comparison of salinity measurements under different conditions. (a) Wind from sea, more than 20 kt. (b) Wind from sea, less than 20 kt. (c) Wind from inland.

Results plotted out in this way in most cases gave points lying close to straight lines, and the line of best fit was drawn in. In Fig. 1 (i) several typical results are given to show the experimental points and the lines drawn through them. The directly-measured sodium concentrations have in all cases been converted to equivalent NaCl values. The scatter of points was greater when the volume of samples was small and, since in these cases only two or three samples were large enough for analysis, such samples were ignored when computing average curves.

The results have for convenience been grouped according to the direction and strength of the wind in the period immediately preceding the rain. In plotting Fig. 1 (ii), the division has been made between periods when the trajectory of the air mass during the previous 24 hr had been from the land or over the sea, with a further arbitrary division in the latter case for strong and lighter winds to give seven or eight cases in each group. Although there is considerable overlap between the groups, the average curves are distinctly different. The relative magnitudes of the salt concentrations are consistent with the results obtained by earlier workers, which were mentioned in Section 1.

The important result brought out by all the curves is, however, that the salinity of raindrops at the ground is not necessarily uniform. The experimental curves represent a dependence of salinity on drop size of the form

$$\text{salinity} \propto (\text{radius})^{-n}$$

where n is a positive constant which was as high as 2·6 for the results obtained at sea level.

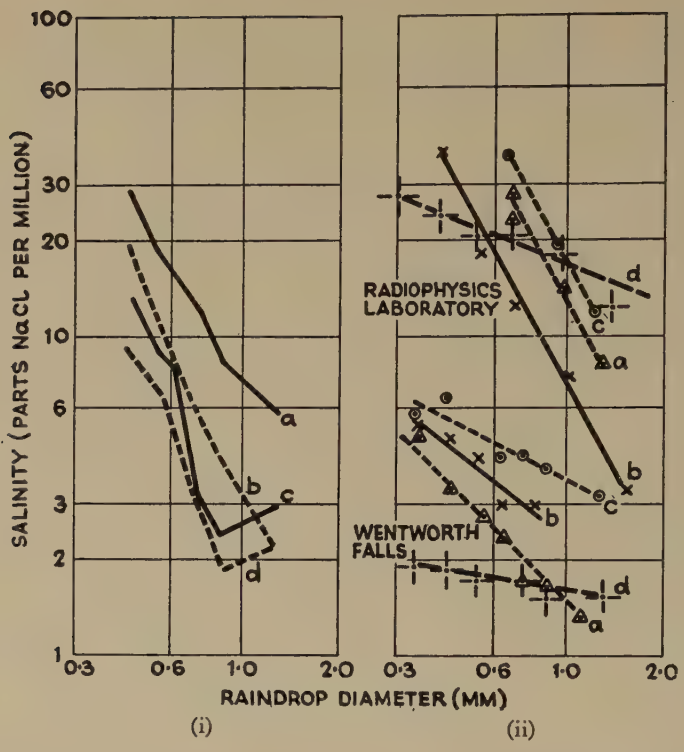


Figure 2. (i) Comparison of salinity measurements at the Radiophysics Laboratory and Wentworth Falls. (a) Radiophysics, wind from sea. (b) Radiophysics, wind from inland. (c) Wentworth Falls, wind from sea. (d) Wentworth Falls, wind from inland. (ii) Salinity of rain from low cloud. The letters refer to corresponding results.

(*b*) *Relation between drop size and salinity at Wentworth Falls*

To compare the results from Wentworth Falls and Sydney, they were again grouped according to the history of the air masses on the days when general rain fell over the areas. The average curves, for three cases in each group when clouds were high, are given in Fig. 2 (i). They show, as we might expect, that close to the sea the salinities are greater than inland, as well as being higher when the wind is from the direction of the sea. The points are more scattered for the Wentworth Falls collections, so that there has been no attempt to fit straight lines to the data in this case. The variation with drop size is of the same general character as before, with the added feature that there is a tendency for the salinity of large drops to increase slightly for diameters exceeding about 1 mm. At the time these measurements were made it appeared probable that this was not a real effect but due to contamination by further falls of rain which were unrelated to the main collection. It should be kept in mind however when we come to discuss the later Hawaiian results.

As mentioned earlier, one of the reasons for setting up a spectrograph some thousands of feet above sea level was the hope of obtaining samples of rain soon after they had left the cloud. On four occasions rain did fall from a cloud layer the lower levels of which constituted a mist on the mountains at Wentworth Falls. These individual results are shown in Fig. 2 (ii), together with those for the same days in Sydney. It appears that, near cloudbase, the salinity tends to be more nearly equal over all drop sizes. Of particular interest is the case marked (d). This was for a period on 12 Feb. 1954 when clouds were

w right to the coast, and it may be seen that the salinity was practically uniform at the diophysics Laboratory as well as on the mountains on this occasion. These observations cloudbase will be important when the comparison is made with results for warm rain.

4. Discussion of results for freezing rain

An examination of the above results leads to an immediate interpretation. The fact at the samples collected when clouds are low on the ground have a salinity relatively dependent of drop size suggests that rain of the Bergeron type might always have this operty soon after it has formed. If this is so, the explanation for the dependence of inity on size in the majority of cases should be sought in processes occurring below e cloud as the rain is falling towards the ground. We must look more closely at evaporation and the collection of salt to see to what extent these could account for the results.

The magnitude of both the effects may easily be estimated. Consider a raindrop ling through air which has salt particles distributed through it. The mass of salt llected by the drop in a given height of fall will be nearly proportional to the volume air swept out, or to the square of the radius a, except for a factor which will depend the collection efficiencies of the raindrop for salt particles of various sizes. This lt will dissolve in a mass of water which is proportional to a^3, giving a resultant factor $1/a$ for concentration. This is still very nearly true when an exact calculation is ade, taking into account the numbers and sizes of salt particles in different air masses different humidities.

Evaporation will also produce a general increase in salinity, the effect being greater e smaller the raindrops. A quantitative investigation of this factor has been carried it, based on the work of Best (1952), and the results will be summarized briefly. Best rived at the result, valid for drops of 0·015 cm radius and above,

$$a_1^2 - a_2^2 = k^2 \qquad (1)$$

here a_1 and a_2 are the initial and final radii and k^2 is a constant which depends on the stance of fall, and on the temperature, pressure and humidity of the air but not on Typical values of k^2 for a ground temperature of 15°C and a lapse rate of 6·5°C/km the ICAN atmosphere are :

(i) Fall to ground through 5,000 ft at 90 per cent RH, $k^2 = 3{\cdot}6 \times 10^{-4}$ cm^2,
(ii) Fall to ground through 10,000 ft at 80 per cent RH, $k^2 = 13{\cdot}0 \times 10^{-4}$ cm^2.

The ratio of the final to the initial concentration of any salt present will be just $^3/a_2^3$, and using Eq. (1) this gives the result

$$\frac{c_2}{c_1} = \left\{1 + \frac{k^2}{a_2^2}\right\}^{3/2} \qquad (3)$$

terms of the radius at the ground, which is the quantity measured.

A similar calculation may be carried out to predict the combined effect of evaporation nd collection of salt. In this case the unintegrated form of Best's evaporation equation ust be used, since collection will depend continuously on a. In Figs. 3 (i) and 3 (ii) e shown calculated relations for the two sets of conditions considered above. In deriving ese it has been assumed as before that the mass of salt collected is proportional to the stantaneous cross-sectional area of a drop, and the constant of proportionality has been timated using typical salt-nuclei counts for eastern Australia. The further assumption as been made that all raindrops are equally saline when they leave the cloud, and that arious proportions of the salt present in the drops at ground level have come from the loud and from the air below.

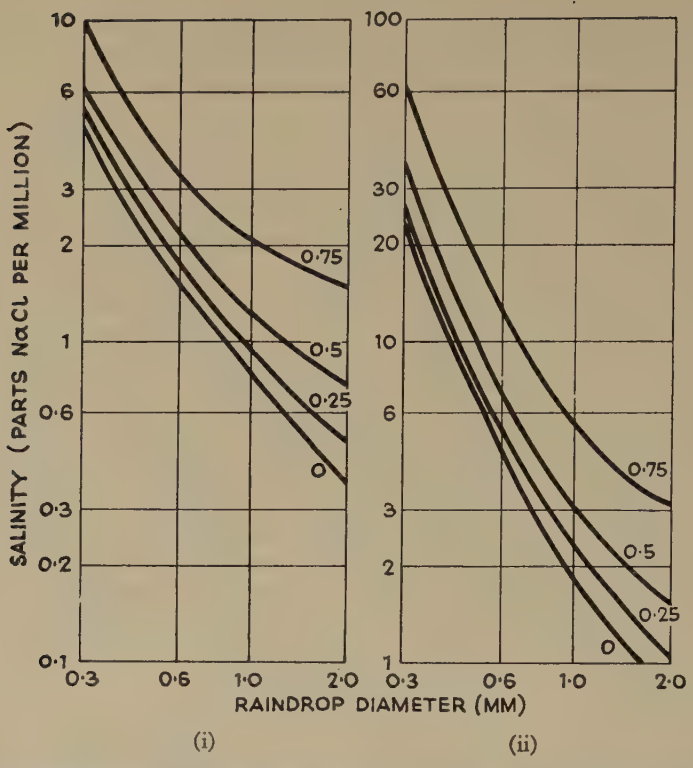


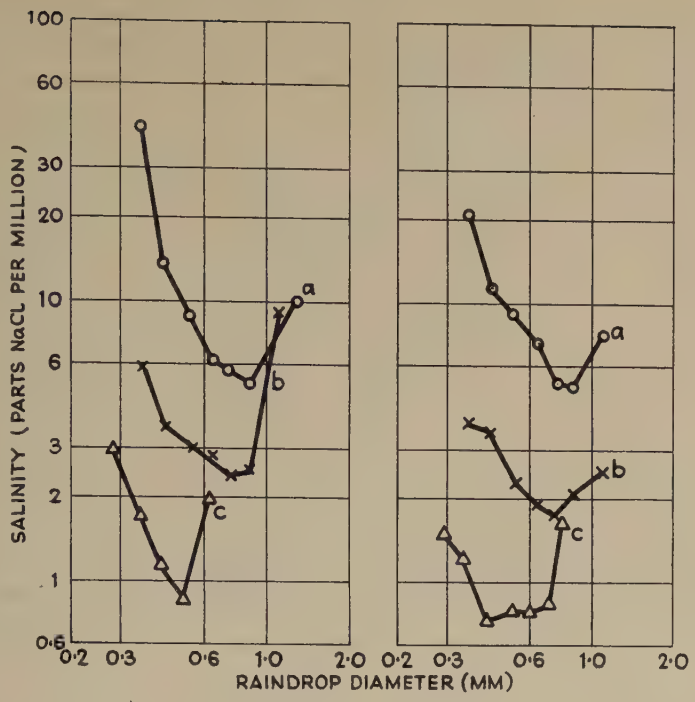
Figure 3. Salinity of raindrops due to combined effects of evaporation and collection of salt in ICAN atmo phere; modified maritime air. (i) Fall from 5,000 ft at 90 per cent RH. (ii) Fall from 10,000 ft at 80 p cent RH. The figures against the curves represent the fraction of salt in 2-mm drops already present at t cloud.

It should be noticed that the curves become steeper for dry air and over long distances of fall. Comparison with Fig. 1 (ii) shows that the experimental results are i agreement with the theoretical predictions. It is obvious that both the total salt conte of our samples and the variation of salinity with size could be explained in terms of th processes occurring below the clouds.

5. Results for non-freezing rain

In addition to the uncertainties of interpretation of results due to the height cloudbase above the ground in New South Wales, there was the added disadvantage tha rain from warm clouds occurs only with a random distribution in space and time, an somewhat infrequently at any given place. Over the months during which measuremen were being made, there were in fact no periods when it was known for certain that no freezing rain had fallen at the two stations. It is this type of rain which is of specia interest, however, since it is only when the salt is supposed to act as the nucleus of th raindrops, or when it enters into the growth process in some consistent fashion, tha salinity measurements are likely to give positive information about the mechanism c formation of the drops.

With these two requirements in mind, the second stage of the experimental wor was carried out in Hawaii. On that island there is frequent rain from warm clouds, an locations could be chosen where it was a simple matter to collect drop samples at cloudbas and in the cloud, as well as at a known distance below.



gure 4. (i) Salinity measurements for the night of 25-26 May 1954, near Hilo, Hawaii. (a) At sea level,) at 2,100 ft, cloudbase, (c) at 4,500 ft, in the cloud. (ii) Mean salinity results at the three Hawaiian stations. (a) At sea level, (b) at 2,100 ft, cloudbase, (c) at 4,500 ft.

(*a*) *Relation between drop size and salinity*

The majority of falls of rain encountered in Hawaii were from warm clouds, with ne cloud tops limited by the inversion to well below the freezing level. Radiosonde ata were available twice daily from the Weather Bureau in Hilo, close to the collecting oints, so that the few exceptional days could easily be picked out. On these particular ays the inversion was absent, moisture was present at high levels, and snow was reported n the tops of mountains in the centre of the island. It seemed likely from a study of ne drop-size spectra (Section 5 (b)) that a combination of the freezing and non-freezing nechanisms was at work, and these unusual results were disregarded when computing verages.

During the three-month period studied, the daily mean salinities ranged between ·0 and 20 parts per million sodium chloride at the coast; 0·34 to 8·5 parts per million t cloudbase; and 0·26 to 3·8 parts per million in the cloud at 4,500 ft. The extreme ndividual readings were 0·20 and 50 parts per million sodium chloride. On occasions vhen samples were collected at all three stations during the same time there was usually decrease in salinity with increasing distance up the mountain; but the relative importance f distance from the sea and elevation is difficult to determine from our results.

In Fig. 4 (i) are shown the drop size – salinity curves for an occasion when consi- erable collections were obtained at the three stations. In Fig. 4 (ii) are the mean curves omputed from all the data available for non-freezing rain during the period studied, sing a weighting factor proportional to the volume of sample. Included are 28 results t sea level, 39 at cloudbase and 16 in the cloud.

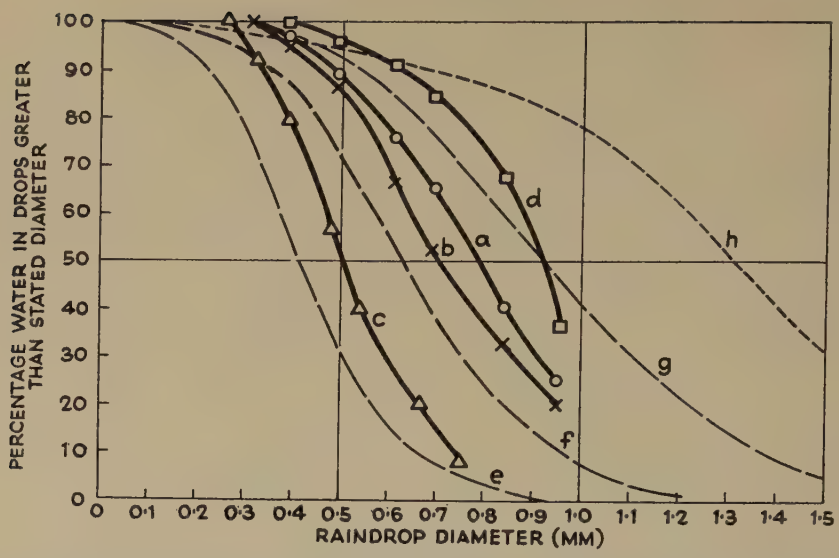


Figure 5. Drop-size distributions in rain collected at ground. (a) Mean at sea level. (b) Mean at 2,100 f (c) Mean at 4,500 ft. (d) Rain from clouds extending above freezing level. (e), (f), (g), (h), Blanchard distributions, for rainfall rates of 1·1, 3·3 and 8·5 mm/hr at cloudbase in warm rain and 0·84 mm/hr freezing-type rain respectively.

The mean results have roughly the same characteristics as the individual curve except that the superposition of sharp minima at different drop sizes has smoothed th feature out of the mean for the 4,500-ft level. It is probably more useful to examin separately the single results; indeed, even these will have lost some information b averaging over several hours or usually all night. In a later section we will discuss brief the way in which a variation with time might affect the shape of the curves.

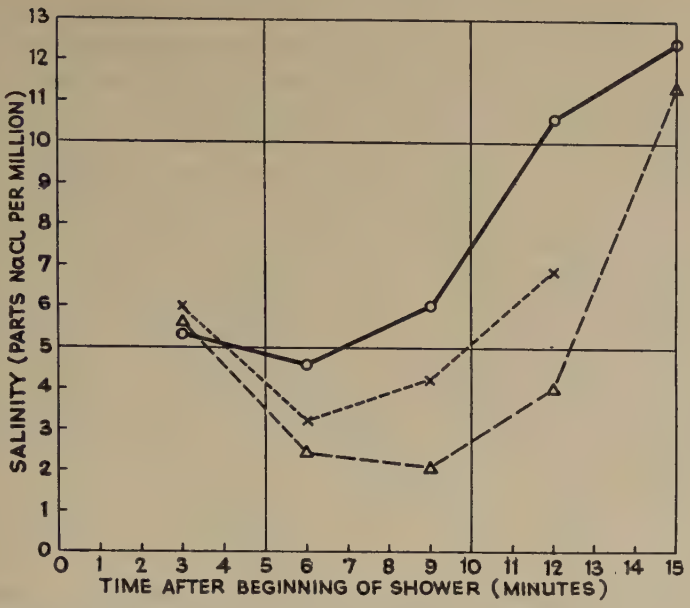
There appeared to be no clear correlation with the direction or strength of th wind; all air masses would have travelled a considerable distance over the sea, and thes detailed history was not known. There was a roughly inverse relation between amoun of rainfall and salinity; but the discussion of these variations is outside the scope of ou present analysis.

The striking feature which is of immediate interest is the minimum in the salinit curves; both the smallest and largest drops measured have higher salinities than thos in the middle of the size range. This appears in all results at sea level and in the cloud and at more than two-thirds of those at cloudbase. No factor common to the days o which the exceptions occurred has been found. There was a minimum in salinity on th several occasions when part of the rainfall was known to have formed above the freezin level.

This consistent variation of salinity with size, which was still present at cloudbas and in the cloud, must be explained in terms of processes occurring in the clouds, no below them.

(*b*) *Drop-size spectra*

Volumes of all the spectrograph samples were recorded, and from these the drop-siz spectra were deduced in order to obtain further information about the type of rain collecte In Fig. 5 are shown average distributions at the three stations for all the samples include in Fig. 4 (ii); cumulative percentages are plotted against drop size over the range of size falling in the spectrograph. We of course lose all information about smaller drops, whic are swept right through the fan.


SALINITY (PARTS NaCL PER MILLION)
0 1 2 3 4 5 6 7 8 9 10 11 12 13
TIME AFTER BEGINNING OF SHOWER (MINUTES)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

gure 6. Variation of salinity with time in bulk samples collected at cloudbase from showers falling from cumulus clouds.

For comparison, some of the results obtained by Blanchard (1953) in the same area Hawaii have been plotted in the same form; instead of numbers of drops per unit olume, which he recorded, we require the number falling per unit area in a given time. here seems to be good agreement in the shape of the curves.

Note that the positions of the salinity minima in Fig. 4 (ii) are at smaller drop sizes e higher we proceed up the mountain. They lie in fact close to the median drop sizes r the corresponding size distribution.

Also shown in Fig. 5 is the mean distribution for the days when some of the rain as known to have originated above the freezing level. It is characteristic rather of rain om warm clouds than of purely ' freezing ' rain, and suggests that in these cases both rocesses played some part in the formation of the rain.

(*c*) *Variation of salinity with time*

It seemed possible that the variation of salinity with drop size might in part be due a variation in time; some measurements were made with this in mind. The ideal rocedure would have been to take samples from the spectrograph at frequent intervals nd to see directly how the salinity-drop size relation changed with time. The possibilities ere limited, however, by the size of sample necessary for analysis; it was only rarely at a complete range of samples was obtained in less than a few hours. The shortest me intervals thus investigated still showed a rise in salinity for the largest drops.

As an alternative, bulk samples were collected, and where possible an estimate was ade at the same time of the drop-size spectrum. The salinity of these samples often aried by a factor of 10 or 20 in a few hours, rising and falling as many as five times in night. If the salinity of drops in an individual shower always increased towards the naller drop sizes, the observed mean dependence of salinity on drop size could be btained provided there was a general increase in salinity during the heaviest showers.

In that case, since the majority of the largest drops collected would fall in the hea showers, their resultant mean salinity would be high, whereas contributions to the sampl of small drops would come from all showers. Results of earlier work are in contradictio to the above hypothesis, however; it is generally agreed that lower salinities are characterist of heavy rainfall.

An explanation along these lines was seen to be inadequate when it was found th drops from individual showers still had an increased salinity for the largest sizes. Fig. 6 the salinities of samples collected over 3-min periods just below the base of th clouds are plotted against time. The three showers for which results are given came fro isolated cumulus clouds which had bases below the general level of the orographic clou These drifted in from the sea and rained as they reached the mountain. The first drop to reach the ground were large, and it may be seen that they had a high salinity. Ther was a decrease in salinity after a few minutes, without any apparent change in the intensit of the rain. As the intensity fell off and the median drop size was obviously decreasing the salinity increased until the end of the shower.

6. Discussion of results for non-freezing rain

Consider first the results for the smaller drop sizes. In Fig. 4, the slopes of th parts of the curves to the left of the salinity minimum lie between the slopes predicte in Section 1 by making two extreme assumptions about the mechanism of growth c raindrops. These were, on the one hand, the coalescence of equally saline drops in th cloud to give equally saline raindrops and, on the other, the growth of saline nuclei b collection of pure water. The results now obtained suggest that, if the initial proces in the formation of raindrops in this size range is condensation on saline nuclei, th subsequent growth must take place by collection of smaller droplets which are not pur but whose mean salinity is near that of the resulting rain. This seems more reasonabl than the arbitrary assumption that there are some pure droplets in the cloud.

The rise in salinity at larger sizes does not fit in with any of the models which wer discussed qualitatively. It implies that there is a tendency either for single giant nuclei or for more than one of the most saline cloud drops to combine, to form a large raindrop since it has been shown that processes which do not involve such a selection increase the relative salinity of small drops. One mechanism could be as follows. If a cloud formed exclusively by condensation on sea-salt nuclei, the largest cloud droplets would form by condensation on the largest nuclei. When growth by coalescence became important these larger drops would increase to raindrop sizes rapidly, and as they grew they could collide with one another as well as with the smaller cloud drops.

Thus the salinity of large raindrops could be high merely because the coalescence of the largest cloud drops became effective at an early stage in the history of the cloud. This is supported by the results of the previous section which indicate that the large saline drops do in fact fall out of the cloud first.

6. Conclusions

The results presented have shown that the salinity of raindrops at the ground is not always uniform, but can depend strongly on their size. It has been demonstrated, however, that the salinity is greatly influenced by the conditions below cloudbase, and that care must be taken to eliminate the effects of collection of salt and evaporation before attempting to interpret the results in terms of the mechanism of growth of drops in the cloud.

Samples of rain collected at cloudbase, which should be unaffected by such secondary ocesses, show characteristic variations of salinity with drop size according to the type rain. In freezing rain the salinity is fairly uniform, varying as a small negative power the radius, whereas in warm rain it shows a pronounced minimum at a certain drop e. It is suggested that these results imply that saline nuclei play a basic role in the mation of warm rain, but are only added incidentally to rain which originates above freezing level as the ice crystals collect randomly-saline cloud drops. An explanation terms of purely geographical or orographic effects seems unlikely, but it would doubtedly have been more satisfactory if both types of rain could have been examined the same locality.

The details of the mechanism are not at all clear, although a tentative explanation s been given. A full interpretation of the results would seem to require further related asurements.

The quantity which has been least studied by direct experiment, and which seems be a crucial one, is the salinity of the individual cloud droplets. Attempts have pre-usly been made to relate salt-nuclei sizes to the salinity of rainwater, but the calculations ve always had to make assumptions about the distribution of salt among the different es of cloud drops. The results of the present investigation emphasize the fact that this ermediate step cannot be ignored if we are to understand how salt enters into the chanism of the growth of raindrops from cloud.

Acknowledgments

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Secular changes of east-coast rainfall regimes

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SUMMARY

The mean rainfall along the east coasts of North America and Australia is shown to have decreas abruptly at the end of the 19th century, in conformity with results for the tropics obtained in an earl paper. A simultaneous decrease of the rainfall in the semi-arid western parts of New South Wales a conforms to conditions in other dry regions such as Arabia. In terms of seasons, the change appears have been largest in autumn and summer for North America and in autumn and spring for Austral Reasons for this are discussed.

A sudden decrease of rainfall at the end of the last century may have occurred also in East Asia a on the east coast of South Africa though it was much less pronounced than in Australia and North Ameri In South America the amplitude of the change was large, but did not extend far south of the Equator.

Dynamic causes and possible implications for research into long-range forecasting are discussed in concluding section.

1. INTRODUCTION

An investigation of secular rainfall variations agrees to a considerable extent wit results obtained by Brooks (1921) from a much shorter series of records. The mo important difference which seems to emerge from the present series of investigation is the almost discontinuous character of the change and the fact that its amplitude seem to be very much larger on temperate east coasts than on the western side of the continent

Two preceding studies (Kraus 1954, 1955) will be referred to as Papers I and I Fluctuations in SE. Australia were described in Paper I. The most salient feature these fluctuations was the abrupt decline of the spring rainfall after 1893. It was als shown that drought seasons tended to be associated with strong upper westerlies an with an absence of east-coast cyclones. Paper II dealt with secular changes in the tropic These, too, were characterized by a rather abrupt change at the end of the 19th centur associated apparently with a narrowing of the equatorial rain belt and with a shortenin of the wet seasons. The change was explained physically as being partly due to a inherent instability of the evaporation-precipitation cycle.

The present paper extends these investigations to the eastern sub-tropical an temperate parts of the continents – particularly North America and Australia. Change will be demonstrated as in Paper II by residual mass curves : graphs of cumulativ percentual deviations from the mean. The quantity plotted is given by :

$$y_n = 100 \sum_{1}^{n} \left(\frac{r_l}{\bar{r}} - 1 \right) - 100 \sum_{1}^{n^*} \left(\frac{r_l}{\bar{r}} - 1 \right)$$

where r_l is the rainfall for the lth year of the record. The first summation adds u percentual deviations from the mean, from the first to the nth year of the record. Th second summation, representing a constant, extends from the beginning of the recor to the end of the year $n^* = 1880$. The mean rainfall, $\bar{r}$, has been computed for th standard 60-yr period 1881-1940. The graph of y_n is zero at the beginning of 1881 an 1941. It is rising during a period with average rainfall greater than $\bar{r}$ and vice versa showing in this way the onset of changes at a glance. The percentual deviation from $\bar{r}$ of the mean for any other arbitrary period, such as the years n to $n + m$, is given simply by $(y_{n+m} - y_n)/m$.

* Now with UN/WMO Technical Assistance Mission for Desert Locust Control, Nairobi, Kenya.

2. Secular changes in eastern North America

Paper II illustrated the abrupt change which affected the rainfall regime of Cuba nd of the West Indies at the end of the last century. The northward extension of this uctuation along the east coast of North America is shown below in Fig. 1. From the eep ascent of the curves it is evident that 19th-century rainfall here tended to be con-stently above the 1881-1940 mean. The same is shown by the following Table :

ABLE 1. Percentual deviations from the 1881-1940 normal of annual rainfall averages in the eastern United States

Station	Normal (in.)	Percentual deviation for period 1821-1860	1861-1900	1901-1940
Charleston	44·02	+ 3·5*	+ 21·4	− 7·8
Washington	40·90	—	+ 5·8	+ 1·0
New York	42·79	− 6·9**	+ 4·3	− 2·9
Albany	32·69	+ 25·0**	+ 17·1	− 2·9
Boston	39·65	+ 10·0	+ 12·3	− 2·3

* 1832-1860. ** 1826-1860.

The 5 stations in Table 1 are those for which sufficiently long unbroken records re given in World Weather Records. The only one to show a lower rainfall prior to 860 is New York. This is contrary to the conditions along the rest of the American ast coast and in the tropics. It is therefore believed to be a spurious effect, due perhaps o the vertical growth of buildings in New York City. Since 1875, the height of the New York raingauge has been raised from 139 ft to 407 ft and it is presumed that for he earlier records, the station was on a very much lower level together with the rest f the New York skyline.

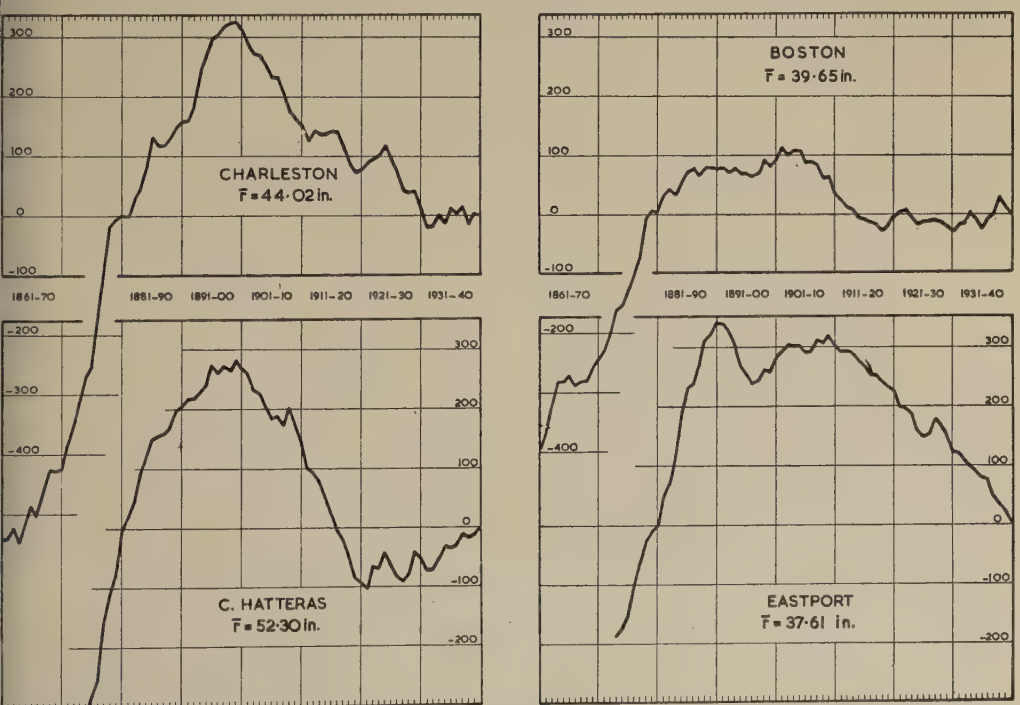


Figure 1. Cumulative percentual deviations of annual rainfall from normals (1881-1940) for stations on the ast coast of North America : Charleston (33°N 80°W), Hatteras (35°N 76°W), Boston (42°N 71°W), Eastport (45°N 67°W).

The higher rainfall regime extended apparently backwards well into the 18th century as is evidenced by Table 2.

TABLE 2. PERCENTUAL DEVIATIONS FROM THE 1881-1940 NORMAL OF ANNUAL RAINFALL AVERAGES AT CHARLESTON

Period	1738-1765	1785-1791	1807-1811
Percentual deviation	+ 8·4	+ 12·8	+ 9·0

The three periods given in Table 2 are the only ones for which records from Charleston are available. They might be considered as independent samples, if it is assumed that there was no significant change in observational technique. With this reservation it may be concluded that rainfall in most of the 18th century was probably about 10 per cent higher than in 1881-1940.

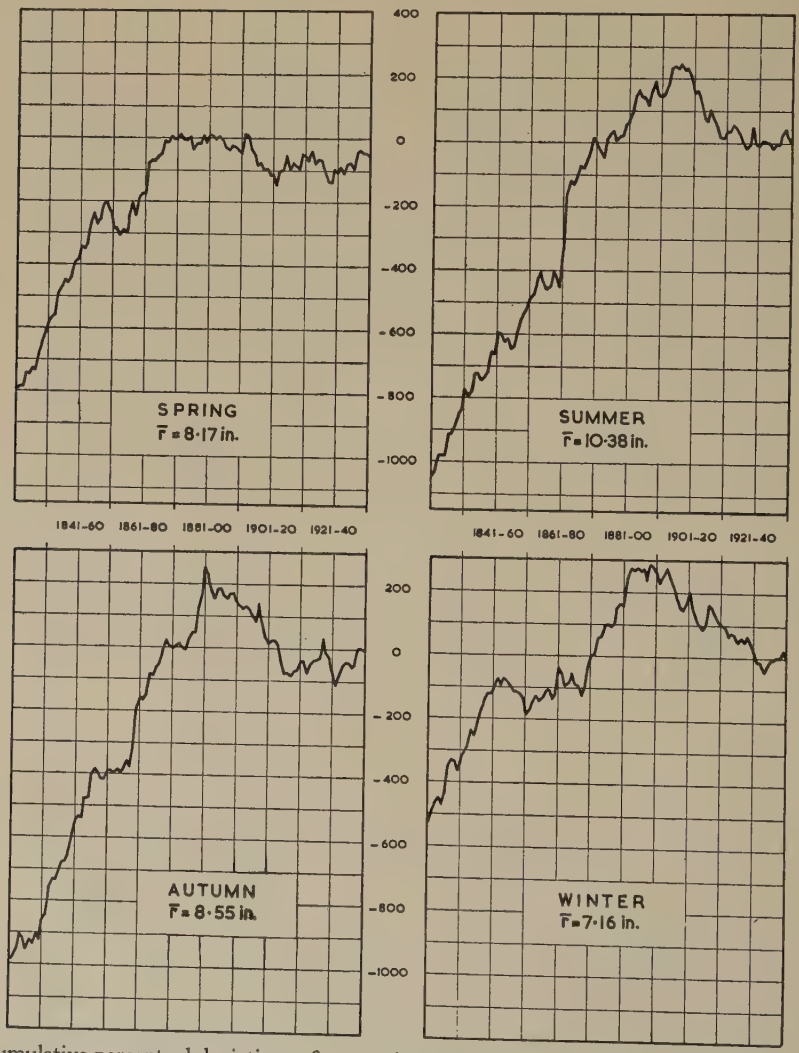


Figure 2. Cumulative percentual deviations of seasonal rainfalls from normal: Albany (43°N 74°W) – spring (March-May), summer (June-August), autumn (September-November), winter (December-February).

The diagrams in Fig. 1 suggest that the climatic change at higher latitudes was less arp and clear-cut than further south, at Cape Hatteras or Charleston, where the ange is more of the equatorial type. There is also some indication that the decline ay have started earlier in higher latitudes. The available data are insufficient to state is with any certainty, but the assumption of an earlier change in the rainfall regime higher latitudes seems to be supported by similar observations on the east coast of ustralia.

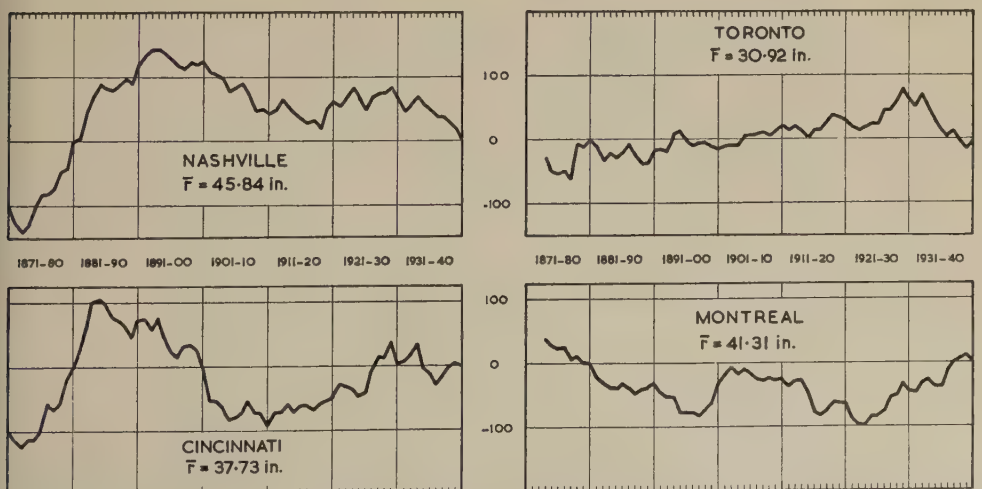
From Fig. 2 it may be inferred that the change in the rainfall regime was more onounced in summer and autumn than in winter and spring. The latter seasons should characterized mainly by a westerly frontal weather regime whilst the former may well of a more sub-tropical character. This assumption is confirmed by the work of Stidd 954) which seems to indicate that, in summer, precipitation in the eastern states is rrelated with the incidence of south-easterly or southerly winds.

It was inferred in Paper II that any decline of equatorial precipitation must be ssociated with a decrease of the tradewind circulation and a corresponding decrease evaporation. This effect would tend to amplify the action of extra-terrestrial or xtra-tropical causes. It seems possible that a reduction in the amount of moisture, btained by the atmosphere from the ocean in the tradewind region, may have been ssociated with the sudden decline of rainfall in the West Indies and along the American ast coast.

The limitation of the change to the east coast is indicated clearly by the contrast etween Figs. 1 and 3. The 4 stations in Fig. 3 are located on the eastern side of the ppalachian Mountains at approximately the same latitude as the 4 stations of Fig. 1. nly the southernmost station, Nashville, shows anything like the same type of climatic hange and, even there, the amplitude is very much smaller than that of Charleston or ape Hatteras. The other stations show no clear trend and – in the case of Montreal – here is even some indication of a reversal.

3. Secular changes in eastern Australia

Fig. 4 is comparable with Fig. 1. The change in Brisbane, for example, occurred t the same time as in Charleston, which has approximately the same latitude. The



igure 3. Cumulative percentual deviations of annual rainfalls from normals: Nashville (36°N 87°W) Cincinnati (39°N 85°W), Toronto (44°N 79°W), Montreal (46°N 74°W).

second peak in the residual mass curve in the Townsville and Gabo Island curves ha been found in Paper II to occur in many stations of SE. Asia and Australia with pro nounced rainfalls during the November-April season. It is observable as far afield a N. Japan, Hawaii and W. Europe, and was presumed to be associated with a pronounce frontal activity in the northern westerlies.

Fig. 4 shows again a decrease of amplitude in the climatic fluctuation towards highe latitudes. The flat character of the Gabo Island curve is, however, somewhat deceptive In Paper I it was shown that an abrupt change in the spring and autumn rainfalls occurre about 1894 all through SE. Australia. Gabo Island is no exception as may be seen i Fig. 5. The decrease in spring rainfall is submerged to some extent in the annual figures

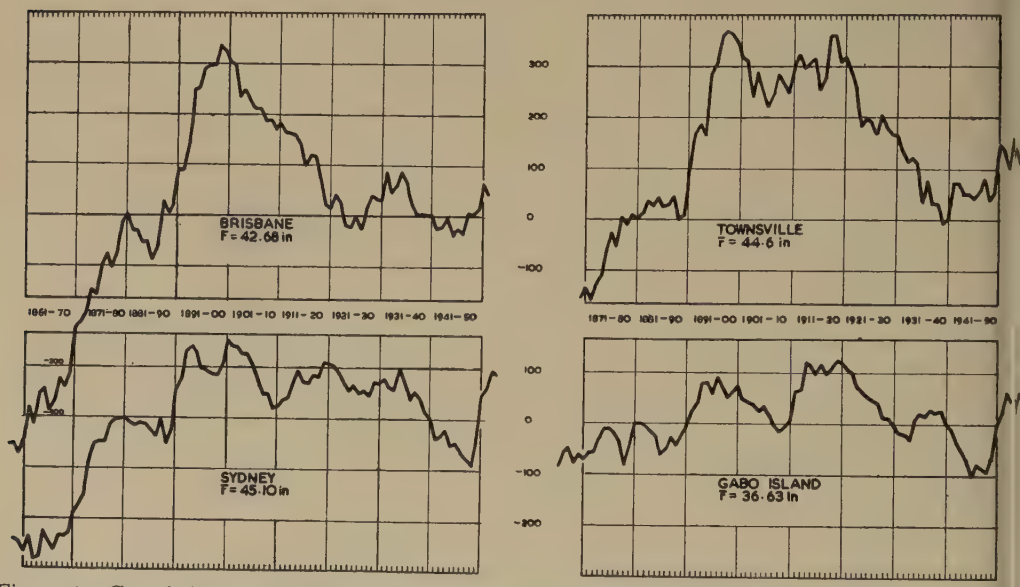


Figure 4. Cumulative percentual deviations of annual rainfalls from normals for stations on the eas coast of Australia : Townsville (19°S 147°E), Brisbane (28°S 153°E), Sydney (34°S 151°E), Gabo Island (38°S 150°E).

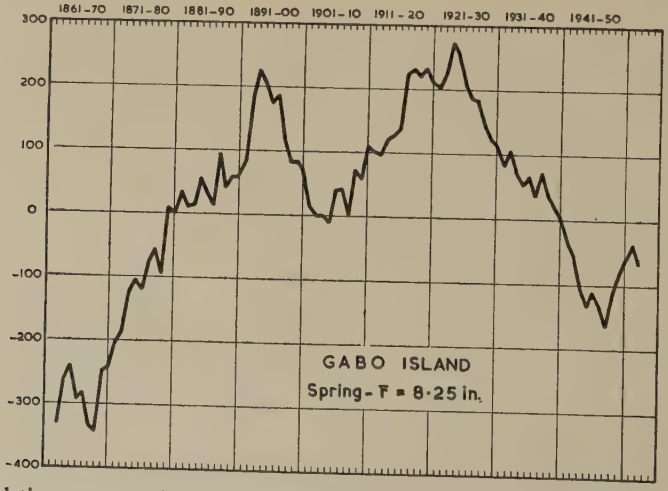


Figure 5. Cumulative percentual deviations of spring (September-November) rainfalls from normal : Gabo Island.

affected southern New South Wales and also southern Victoria, where the amplitude the change decreases in a westerly direction. In a careful statistical analysis, it was und by Cornish (1954) to have occurred as far west as Adelaide.

The seasonal pattern of eastern Australia is unlike that of eastern North America ving to differences in latitude and in the general character of the two hemispheres. Heavy ring and autumn rainfalls in eastern Australia are often associated with east-coast clones – disturbances which receive some of their energy from the westerlies but also aw in moist tropical or equatorial air from the northern Tasman and Coral Seas. This pe of weather situation is rare in summer when the westerlies are weak and far to the uth. It also is comparatively rare in mid-winter when the supply of equatorial air is nited and when the weather in the southern half of Australia is mainly of a rapidly oving frontal type. East-coast cyclones appear to occur most readily in autumn, early inter and spring. The change in the SE. Australian rainfall regime after 1894 was robably associated with a change in the occurrence and intensity of these disturbances the spring and autumn seasons. The greater magnitude of the change during these

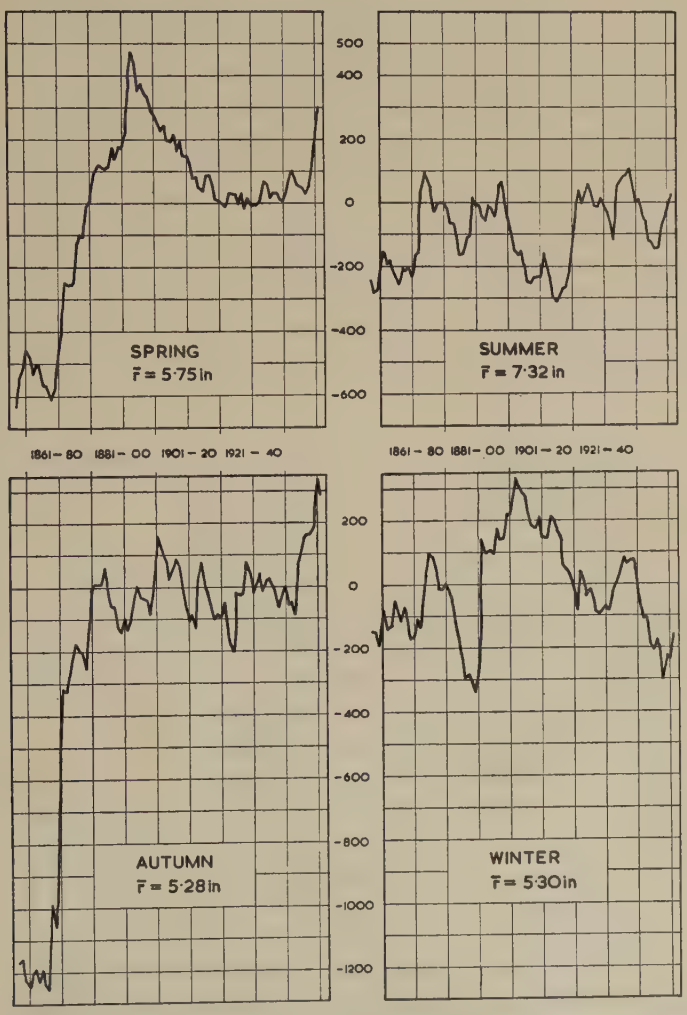


Figure 6. Cumulative percentual deviations of seasonal rainfalls from normal : Bukalong (37°S 149°E).

seasons as compared with summer, in particular, is demonstrated in Fig. 6. Bukalor is situated about 60 mi from Gabo Island at a height of 2,563 ft in the inter-mounta plateau of the Monaro.

The change of the rainfall regime at the end of the 19th century was due not on to a decline in the mean rainfall intensity but also to a reduction in the number of da with rain and hence presumably in the storminess. This is shown in Fig. 7 whic represents cumulative percentual deviation from the mean number of spring rain-day $\bar{d}$, in Sydney.

As in N. America, the amplitude of the Australian secular rainfall variation tend to be smaller on the western side of the mountains. Toowoomba and Bathurst are on a short distance inland from Brisbane and Sydney on the western slope of the Dividin Range; but the curves in Fig. 8 are more irregular and of smaller amplitude than tho for the two capitals on the coast. Deeper inland, however, the decline of the amplitud in the climatic fluctuation with distance from the east coast is over-compensated b another fact. It was shown in Paper II that the fluctuation assumed very great amplitud

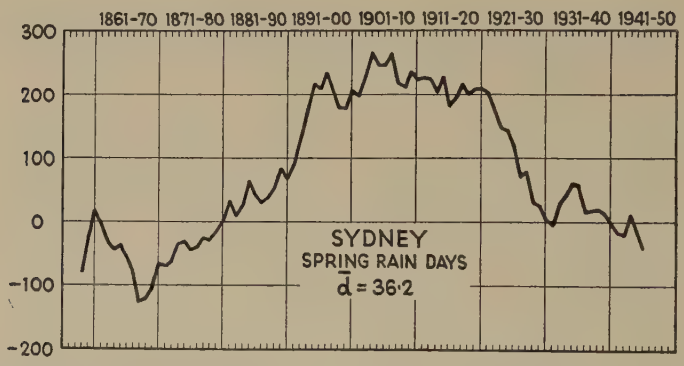


Figure 7. Cumulative percentual deviations in the number of spring rain-days from normal : Sydne

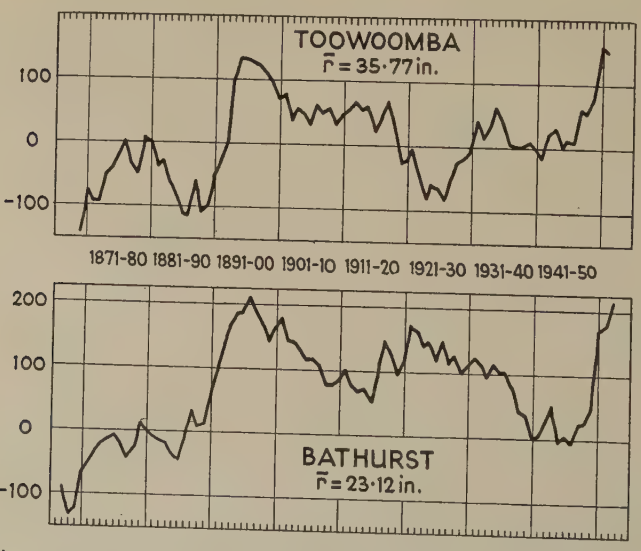


Figure 8. Cumulative percentual deviations of annual rainfall from normal : Toowoomba (28°S 152°E) Bathurst (33°S 150°E).

long the margin of the desert and in some arid regions. The same applies to Australia, s may be seen from Fig. 9 which represents conditions in the semi-arid parts of western New South Wales.

It should be noted that, with the exception of Sydney and Brisbane, none of the tations discussed in this section experienced any major urban development.

4. Secular changes along other sub-tropical east coasts

(*a*) *East Asia*

The changes which are so pronounced along the east coast of Australia and North America are very much less accentuated in Japan and along the Chinese coast. They are not entirely absent. The residual mass curves for both Nagasaki and Zi-ka-Wei show a peak in 1891. However, the amplitude is comparatively small and in the case of Zi-ka-Wei there is a second larger peak culminating in 1921, in conformance with conditions in the westerlies and in the monsoon region as mentioned in Section 3 above.

(*b*) *South America*

In Paper II it was shown that the typical tropical pattern of secular rainfall variation was present on or near the S. American continent in places such as Bogota, Trinidad or Recife. On the east coast, south of the Brazilian bulge, the prevailing wind is not onshore but travels partly overland. The coastline is more open to air-streams from the

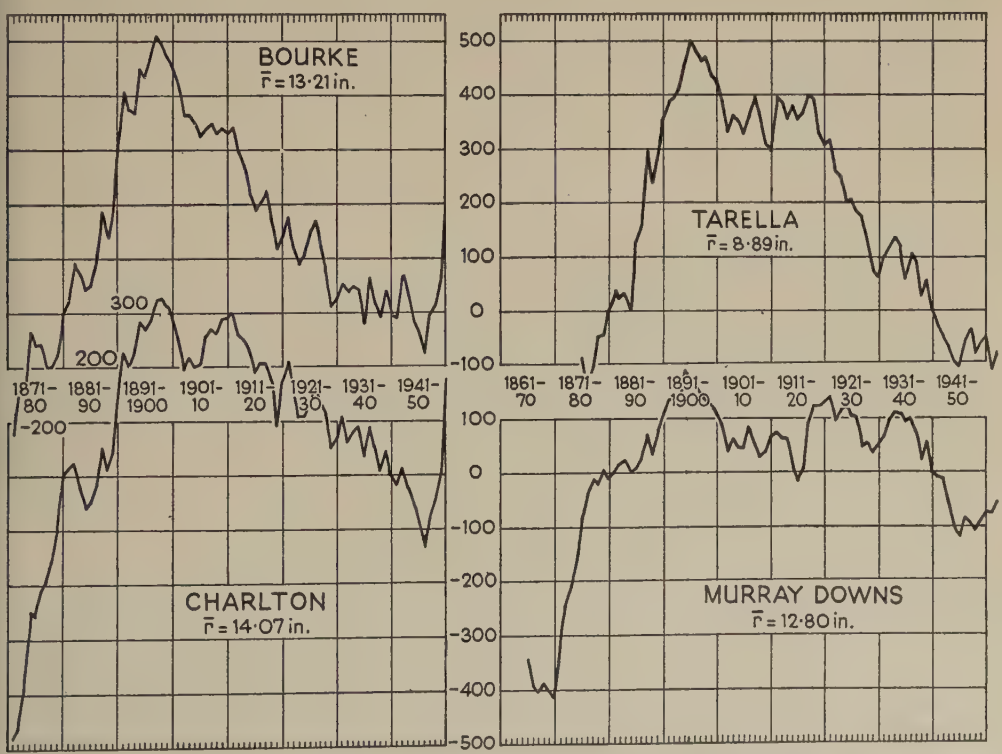


Figure 9. Cumulative percentual deviations of annual rainfall from normal : Bourke (30°S 146°E), Charlton (30°S 147°E), Tarella (31°S 143°E), Murray Downs (35°S 144°E).

pole than from the equator and, in the Argentine, rainfall is probably affected to a considerable extent by cyclonic disturbances in the westerlies and by waves generated on the lee side of the Andes. This must cause conditions very different from those of Australia, and it is not surprising that nothing like the Australian fluctuations pattern can be observed in any of the Argentinian records. Further north, at Rio de Janeiro the normal pattern of change can be discerned though its amplitude is small and partly obscured by other fluctuations. It is of interest that the pattern is somewhat more pronounced at Alto da Serra at an altitude of 2,500 ft.

(*c*) *South Africa*

The east coast of S. Africa, like S. America and the Australian east coast south of the Queensland border, has a polar rather than an equatorial exposure. The only available station for analysis is Durban. Unfortunately, the exposure and location of this station was changed repeatedly and the record is not internally consistent, which makes it useless for analysis of the present type.

The location of the Durban raingauge did not change from 1873 to 1912 and, for that period correlations were computed with Maryborough, Queensland (25°33′S), and Brisbane (27°30′S) which have approximately the same latitude as Durban (29°51′S). The correlation coefficient between annual rainfall at Maryborough and annual rainfall at Durban is 0·50 which a '*t*' test would show to be significant on the 1 : 1,000 level. The correlation coefficient between Brisbane and Durban is 0·31 which is just significant on the 1 : 20 level.

These correlations are interesting in themselves, suggesting, as they do, similarities of weather type in a large part of the southern hemisphere. From this it is deduced that some climatic fluctuations of the same type as in eastern Australia, but with smaller amplitude, did probably occur in eastern S. Africa. This deduction is supported by the shape of the residual mass curve for Capetown, a station comparable to Adelaide, S. Australia in its rainfall regime.

5. Discussion

It was argued in Paper I that the abrupt decline of the rainfall in eastern Australia at the end of the last century was probably associated with a strong development and equatorial displacement of the mean upper westerlies at 300 mb and above. Strong upper westerlies over the tropical and sub-tropical regions – particularly when they attain the character of a jet stream over the east coast – would tend to inhibit both the local growth of big monsoon cumuli and that of depressions on their poleward side.

In Paper II it was suggested that the decline was also associated with a reduction of the direct meridional circulation between the equator and temperate latitudes. This reduction, once started, must have been amplified by the inherent instability of the evaporation-precipitation cycle. The two arguments are mutually consistent as a strong development of the upper westerlies will tend to balance meridional temperature contrasts.

It was shown also in Paper II that the change of the climatic pattern at the end of the last century was characterized not only by a weakening of the inter-tropical convergence, and hence with dry conditions in the tropics, but also with an expansion of the sub-tropical high pressure belts. This caused relatively storm-free conditions in the lower middle-latitudes, and a wider separation of the tropical and westerly rainbelts which often tend to overlap along the east coasts of the continents, particularly in autumn and spring. Together with the warming in high latitudes reported by Willett (1950), Lysgaard (1950) and other authors, these changes are analogous to those which differentiate a glacial from an interglacial period as described by Craig and Willett (1951).

The differences between the recent fluctuation and the larger glacial to inter-glacial cle are perhaps less in the magnitude of the change than in its duration, though the o cannot be separated. It takes time before a small change in atmospheric circulation ects the volume of tropical rainfall, the temperature of the sea and the extent of ciation. However, once these other factors have been affected in a significant manner, ey will tend to amplify and perpetuate the change which has already occurred.

There are some indications that the period of low rainfall which began at the end the last century is drawing – at least temporarily – to a close. A higher rainfall regime the tropics since about 1931 is indicated by the figures in Paper II. The same is ggested for America by some of the diagrams in Fig. 1 of the present paper. Figs. 4 d 9 show a strong increase of the Australian rainfall since about 1945.

It is of interest that the drought period which affected eastern Australia from about 96 to 1915 was of world-wide extent in tropical and east-coast regions. The more cent SE. Australian drought which lasted from about 1939 to 1944 was not only shorter it also far more limited geographically. A similar relation between the duration and e geographical extent of deviations from the mean seems to characterize series of wet ars – not only in low latitudes but also in the westerly belt. Dealing with paleoclimato-gical evidence, Faegri (1950) proposed the principle that the shorter the duration of e climatic fluctuation, the smaller is the area similarly affected; the larger the cycle, e greater the area within which it is felt in the same sense. The present investigation ggests that the same applies to the fluctuations which are discernible in modern meteoro-gical records. If that can be confirmed in a statistically satisfactory manner – and it ay not be possible to do so with the available material – it would provide a means to recast the duration of droughts or series of wet years, at least in a qualitative way.

Acknowledgments

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On the space and time distribution of showers in a tropical regio

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Summary

Charts of daily rainfall in Southern Rhodesia show that the distribution of rain in space and time cann be associated in any detail with features of the synoptic map.

Displays of precipitation echoes on the screen of a 10-cm radar situated at Salisbury have been phot graphed by time-lapse camera giving great detail and continuity of record.

It is found that echo displays associated with cumuliform cloud can be resolved into a few areas s echo activity which move continuously by a process of new echo formation. These moving areas have be termed ' progressions ' and progression behaviour is described in terms of individual echo behaviour a in relation to other meteorological observations. The discovery of progressions has introduced order in a type of tropical shower activity which has previously been considered incoherent.

1. Introduction

The rainy season in Southern Rhodesia is of short duration. There are irregul outbreaks of showers in October and April, but most of the rain falls between la November and early March. During this period, spells of fine weather occur but, the main, showery conditions prevail.

Since December 1951, radar displays of precipitation echoes over a 100-mi radi have been recorded from time to time by photographing the PPI* screen by means of time-lapse camera. For one typical day on which filming had extended over nine hou an estimate was made of the area covered by precipitation echo. The maximum ech coverage at any one time was less than 3 per cent of the whole observation area; the tot area covered by echo at some time during the nine hours amounted to about 32 per ce of the area. Thus, at any one time, the shower activity affects a very small portion the whole area and, over a period, the activity is displaced so that the area which h received rain continually increases.

It has been found that precipitation echoes may be broadly classified into two type The one consists of relatively large areas of weak echo, having diffuse boundaries an associated with stratiform cloud; the other type consists of relatively small areas of stron echo with sharp boundaries and associated with cumuliform cloud. The first type, whe it occurs close to the radar, shows a ' bright band ' of very strong echo situated at a lev where the temperature is about 0°C to 2°C. The second type, which does not show bright band, is the one most commonly found here and all further discussion of precipita tion echoes in this paper refers to the second type.

When the PPI films are projected as ' movies,' having a contraction in the time scal of the order of 200, echoes are seen to be nearly always short-lived and to move ove small distances. Changes in their distribution come about mainly through the appearanc of new echoes and the disappearance of existing ones. Distinct line formations of echoe are frequently seen but two or more lines occurring simultaneously or at different time on the same day may have widely different orientations. Study of the films as ' movies did not reveal any basic orderliness of the echo distribution. These radar observation have confirmed the impressions previously obtained from daily rainfall maps that (1) th

* ' Plan position indicator ' – showing the position, as on a map, of the precipitating clouds giving radar echoes.

ower activity may miss out large areas of the country, (2) the time of rain at stations the same area on the same day may be very different, and (3) the distribution of shower tivity does not appear to be connected or consistent with travelling pressure systems fronts.

It frequently happens that neither the rainfall map nor the echo pattern can be sociated in any detail with the synoptic situation. However, by detailed examination the photographs of the PPI screen, two kinds of order have appeared in the behaviour the showers. Firstly, it has been found that it is a rare event for an echo to be quite ainly isolated in space and time. Secondly, it has been found that the echo activity any one time can be resolved into a few groups or areas of activity which move across e country with time and which we have called ' progressions.' This analysis of the ho pattern into progressions has introduced order into an otherwise incoherent rainfall ttern. The properties of these progressions have been examined in relation to the haviour of the individual echoes, to upper winds, the synoptic chart and the topography the observation area. The following three sections describe the actual observations ade, the method of analysing the pattern into progressions and the results of this alysis.

2. Observations

The radar is situated at Salisbury Observatory (17°50′S, 31°01′E) at an altitude of ,850 ft ASL. Over the 100-mi radius of the observation area, the ground level varies etween about 3,000 and 5,000 ft. The main topographical features of the area are hown in Fig. 1. The radar has a wavelength of 10 cm, peak power of 500 kw, pulse ength of 1·9 μsec and a pulse repetition frequency of 500/sec. The beam is 3° wide to alf power both in azimuth and in elevation.

Figure 1. Map of radar-observation area. Range rings on all figures are shown at 50-mi and 100-mi radius.

In some early observations it was found that the maximum intensity of echo fro a distant shower, as judged visually on a range-amplitude display, was usually obtaine by adjusting the beam in elevation so that the echo was returned by a volume of rai just below the freezing level. At higher elevation angles the echo intensity decrease rather gradually and it is presumed that this decrease indicates the presence of ice i the upper levels of the shower cloud. The height of the freezing level is about 16,000 ASL and varies by no more than about 3,000 ft through the rainy season. Hence, i order to obtain optimum presentation of echoes over 100-mi radius, the beam was se at 1° elevation at which angle it reaches freezing-level height at about the maximu range.

During the 1951-2 rainy season, photographs of the PPI screen were taken ever 8 sec over periods of $\frac{1}{2}$ to 4 hr. In the following season, photographs were taken ever 20 sec and the period of recording varied from 6 to 10 hr. On each occasion the camer was set in operation at a time of the day when a number of precipitation echoes wer already visible on the screen; on almost every occasion there were still echoes in evidenc at the end of the period of filming.

Upper winds are measured by pilot balloons and theodolite at Salisbury. Thes ascents are normally made three times a day but, during the rainy season, the measure ments are sometimes reduced by cloud to one ascent up to 10,000 ft or less.

3. Progression analysis

For our analysis of precipitation echoes into progressions on any one occasion, w make tracings of the echo outlines at hourly intervals. By inspection of the first one c two tracings, we divide the echo pattern into groups or areas of activity. On succeedin tracings the areas of activity are found to be displaced to positions adjacent to or over lapping the preceding positions. Over a period of time, the displacement of a particula group proceeds continuously in the same general direction, forming a sequence of activit in time and space which we have called a ' progression.' The areas of activity usuall consist of rather elongated groups of scattered echoes and the areas are subject t considerable changes of size and shape from hour to hour. By separating the echo activit into these groups, it has been found possible to resolve the whole echo pattern for each day into a few progressions.

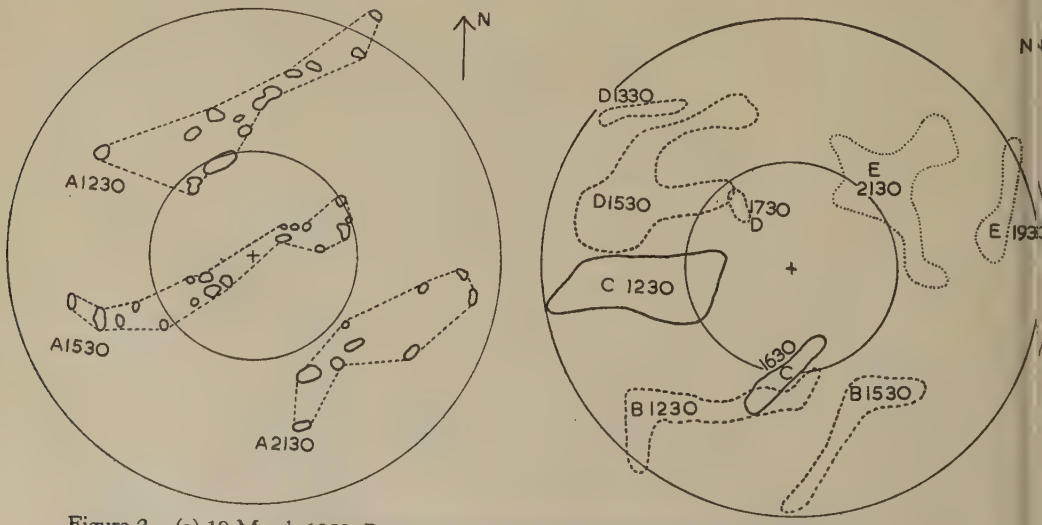

Figure 2. (a) 19 March 1953, Progression A; (b) 19 March 1953, Progressions B, C, D and E.

Examples of progressions on two different days are shown in Figs. 2 and 3. The ogressions shown in these figures include all the echoes seen on those days. Figs. 2 (a) d (b) together comprise the whole progression picture for 19 March. In Fig. 2 (a) the tline of each echo has been drawn and a dotted line drawn around the area of activity each particular time. In Fig. 2 (b) only the areas of activity have been outlined and ch progression is marked with an identifying letter. In order to make the diagrams arer, the areas of activity are not shown at every hour, but only at the times indicated. r our detailed study of progressions, we have drawn diagrams of the progression sition at each hour. We find that the clearest method of representing the progression by marking only the leading edges of the echoes, using different colours for the edges echoes at different times. Most of our observations on progression behaviour are sed on these more detailed diagrams which we have not illustrated here because the tail cannot be shown clearly in black and white.

4. Results of analysis

For the 1952-3 season we have 11 film records. Of these, one showed almost entirely e diffuse type of echo which is not susceptible to analysis into progressions. The cords for the 1951-2 season are of too short duration for the analysis to be usefully plied. Thus we have records for 10 days which are suitable for the progression analysis. he results are summarized in Fig. 4.

The maximum number of progressions on any one day was 5, not all present multaneously. On 3 days (3 March, 29 Dec. and 16 Feb.) there were two progressions aving opposite directions of movement, approaching each other. On one day, 20 Feb.

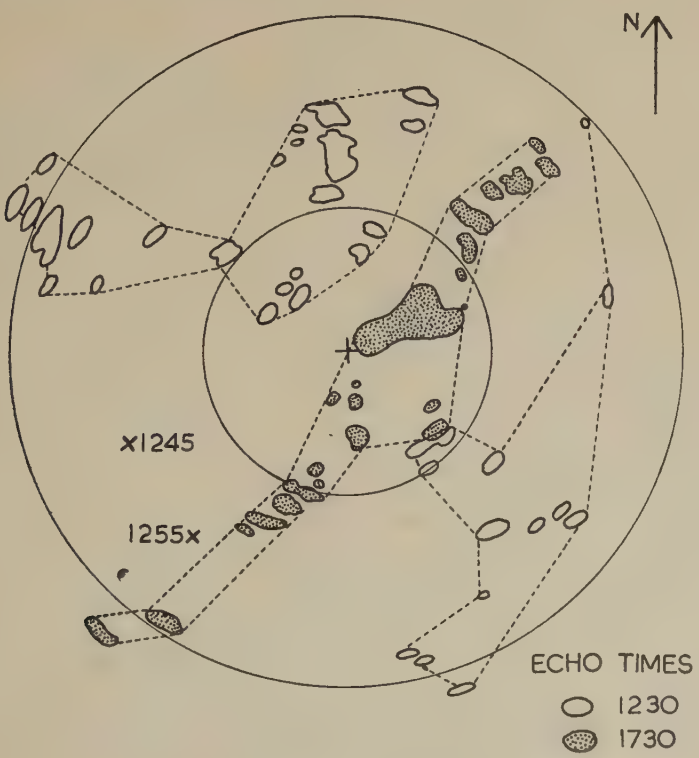

Figure 3. 29 December 1952.

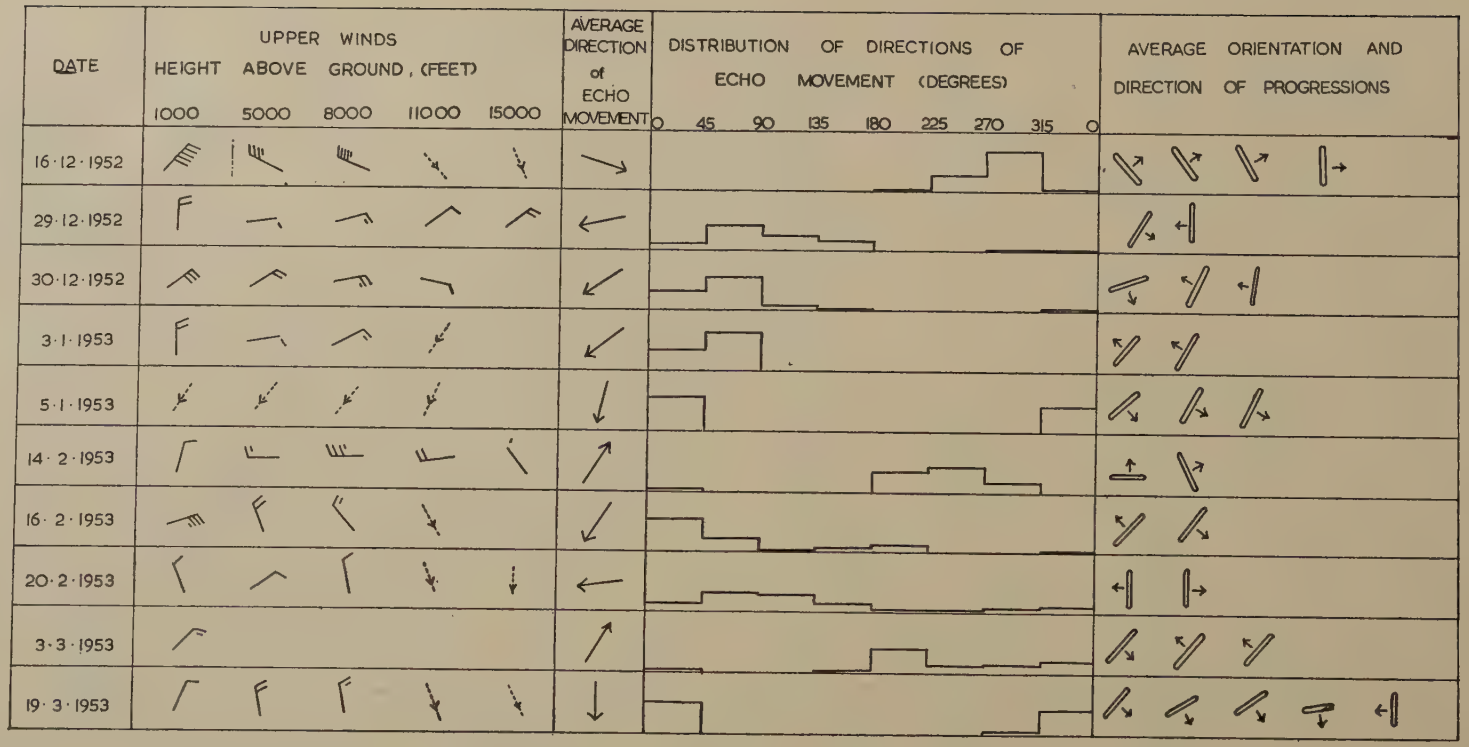

Figure 4. The relation between upper winds, individual echo movement and the orientation and direction of progressions. Upper winds shown by arrows are measured at Salisbury; those shown by dotted lines are deduced from the upper-wind map. One full feather represents 5 kt.

two progressions were directed away from each other. One day, 19 Mar., showed rogressions, all fairly uniform in direction, together with a later one having a completely erent direction.

The area of activity at any one time has a maximum width up to about 40 mi, the al width being up to about 20 mi. Nearly all progressions have at least one end ex- ding to the edge of the PPI screen. On every occasion at least one progression lasted oughout the period of the record; on most days there were two such progressions l on one day, three. Every progression either was first seen at the start of the record advanced into the observation area from outside at a later time. Three progressions ayed and disappeared within the observation area. The remainder either moved out the area or were still in existence at the end of the record.

There is sometimes a relatively small number of echoes which cannot be described progressions because the echo activity is too short-lived and too irregularly arranged. ese echoes do, however, occur in groups and there is nothing apparent in their behaviour suggest that there is any fundamental difference between these groups and progressions. ually the areas of shower activity representing the progressions at one time on any asion are readily determined since they are separated in space. When two progressions erlap, through meeting from opposite directions or from overtaking each other, there no justification for attempting an analysis of the echoes into the two progressions at t time. This happens on three occasions during the 10 days' records.

As mentioned in the Introduction, lines of echoes are frequently seen on the radar een at particular times. During the 10 days' records, there were 9 lines more than mi long and 62 more than 20 mi long. It is found that the longer lines lie along the ociated progression, i.e., have the same orientation, but that short lines may be at a large gle to the orientation of the progression.

5. Individual echoes and progressions

In the two preceding Sections we have described the analysis of the echo pattern to progressions and the apparent behaviour of those progressions. In this Section the haviour of individual echoes is discussed and related to the behaviour of the progressions ich they form. We apply the term ' individual echo ' to discrete areas of echo such those shown in outline in Fig. 2 (a). When the film is viewed as a 'movie' these areas e seen to change in shape rather spasmodically while still retaining a continuous undary. These changes in shape may be interpreted as being due to new cells forming jacent to the existing echoes, or old cells decaying. As long as the new cells cannot resolved on the radar photographs from the original echo, the whole continuous echo ea is termed an ' individual echo.' The length of life of these individual echoes varies different days. An estimate was made of the number of these echoes lasting for 1 hr more, expressed as a percentage of the total number of new individual echoes appearing. his figure varied between 1 and 20 per cent on different days so it is clear that the majority echoes have a lifetime of less than 1 hr.

It is rare for an echo to be clearly isolated in space and time. We have taken the bitrary criterion that an echo is isolated if, within a radius of 7 mi of the place of the echo's pearance and during the preceding $1\frac{1}{2}$ hr, there has been no other echo. On this basis we nd that the number of isolated echoes varies between 5 and 10 per cent of the total mber of new echoes on different days. If the criterion of isolation is extended to a 20-mi dius with the same interval of $1\frac{1}{2}$ hr, the number of isolated echoes varies between 0 and 2 r cent on different days. Such isolated echoes, although small in number, may have very large effect upon the shape and orientation of the active area of a progression. For

example, in Fig. 3, the area of activity originally situated to the north-west of the stati progressed in a south-easterly direction. Between 1230 and 1730 hr, there was a lar change in the orientation of this progression. The change resulted largely from t appearance of two isolated echoes, whose positions are marked by crosses, at 1245 a 1255 hr. Following on these appearances, activity grew in this area, i.e., additional echo appeared, while the original progression advanced and overtook the new development.

In addition to the effects of new isolated echoes, there are considerable hour-to-ho changes in the area of activity due to the succession of non-isolated echoes. Fig. 5 sho the detailed behaviour of a progression over a period of 4 hr (Progression A, 19 Marc as in Fig. 2 (a)). Echo outlines are shown at every hour. The dots indicate where ne echoes appeared or existing echoes disappeared at intermediate times. The arrows sh the direction of movement of the echoes. Echoes which have no direction arrows last for such a short time that their direction could not be determined. It will be seen th the progressive displacement is largely due to the growth of new echoes ahead of existi ones. All this portion of the progression had disappeared by 1830 hr.

The direction of movement of a progression cannot be specified very exactly, part because of the large effects of new echo development and partly because, as mentione in Section 4, almost invariably the progression probably extends beyond the edge of t screen. In practice therefore, the direction of a progression can only be taken as t perpendicular to the average orientation of the progression, which can be specified mo easily and accurately. It appears that the orientation may be at almost any angle to t average direction of echo movement. We find that the direction of a progression, defined above, is not opposed to, i.e., is not more than 90° away from the average directi of echo movement in that progression. The speed of movement of the progression of the same order of magnitude as the upper-wind speeds.

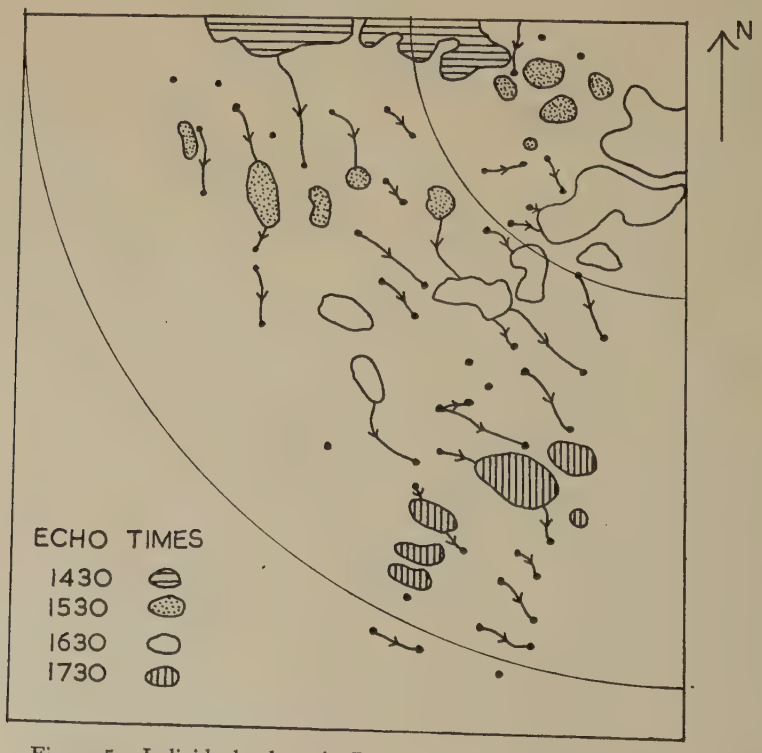

Figure 5. Individual echoes in Progression A, 19 March 1953.

6. Other meteorological observations

In the preceding sections we have described our method of analysis of convective-type precipitation echoes into progressions, the sort of results we obtain from this analysis and the characteristics of progressions in relation to the behaviour of the individual echoes. It remains to consider these progressions in relation to other factors. We have examined progressions in relation to the topography of the observation area, to the upper-wind patterns, the weather maps and the autographic records of surface pressure, temperature and wind at the radar station. The small number of radar films is inadequate for drawing very firm conclusions from these studies.

The distribution of echoes in relation to topography has been examined firstly by superimposing the tracings of echo outlines on to the contour map of Fig. 1, and secondly by projecting the films as 'movies' on to the map. In this we paid particular attention to the two most prominent topographic features – the valley running away to the north-east and the ridges of high ground to the east-south-east and the north-west. As a result of this study we are unable to say more than that there appears to be a tendency for echoes to occur mainly on the lee side of ridges of high ground.

Examination of the upper-wind maps shows that on six of the days of radar-film record, there was a zone of confluence situated over or very close to the observation area. The zone lies roughly east-west, having, for example north-westerly winds to the north and easterly winds to the south. On four days of radar record there was no such zone; the upper winds were either uniform in direction or backed with height over the whole area. There is no obvious connection between the direction of progression and the direction of upper-air flow. For example, on 16 Dec. the upper winds were north-east backing to north-west, while progressions advanced from the south-west and west. However, on 19 Mar., upper winds were again north-east backing to north-west and the earlier progressions all advanced from the north-west. On 19 Mar. there is a later progression from the east which may be connected with the fact that, by the following day, the upper winds were north-east at all levels up to 20,000 ft. There is one case of progressions moving in opposite directions on a day with no confluence zone in the upper winds, so these two phenomena do not appear to be connected. As an indication of the magnitude of the upper winds in this area, the mean wind speeds up to 20,000 ft at Salisbury during the rainy season are all between 8 and 12 kt.

On 6 of the 10 days, a progression actually passed the radar station and surface records have been examined about the time of passing of the progression. This time can only be established to about the nearest half hour owing to the loss of detail at the centre of the PPI when it is switched to 100-mi radius. With this limitation, we find the following :

(1) At the time of arrival of a progression there is a rapid drop of air temperature and a small drop, rise or wave in the micro-barograph record. The magnitude of these pressure variations is between 0·3 and 0·9 mb (the mean day-to-day pressure change at Salisbury is about 1 mb in the rainy season).

(2) Rain is reported in sight from 2 to 3 hr before the arrival of a progression.

(3) In nearly all cases, there is a rapid increase of surface wind speed followed by a rather gradual decrease of speed.

(4) In nearly all cases the surface-wind direction changes so that it blows from the quarter occupied by the progression. This direction change tends to precede the increase of speed by as much as 1 hr. The pressure change mentioned above is sometimes coincident with the direction change and sometimes with the increase in speed.

(5) The surface-wind direction is, in nearly all cases, changed for several hours, during which it may vary considerably and thereafter it tends to return to the original direction prevailing before the progression came near to the station.

7. Discussion

In any attempt to explain the behaviour of progressions, one has to consider two questions which may be quite separate; the one concerns the initial outbreaks of activity in particular areas and the other is that of the mechanism of the progressive displacement of the activity. The initial outbreaks must depend on some external influence, such as ground heating or convergence of air streams, whereas the subsequent displacement may be due either to an external influence or to a mechanism of self-propagation.

As far as the initial outbreaks are concerned, we have very little information, since all the radar films began after the activity had started. Also, the only station within the observation area at which upper-wind measurements are made is that at the radar site so the detection of small-scale convergence is impossible.

As far as the progressive displacement is concerned, we believe that self-propagation may occur to a considerable extent, partly because the great majority of echoes fall within the isolation limits of a 7-mi radius and a 1½ hr interval (see Section 5) and partly because as will be seen from Fig. 5, much of the activity of a progression can be traced through almost continuously. However, there are isolated echoes which are followed by further development, and this development fits very well into a progression. For this reason we suspect that an external influence may be concerned in the progressive displacement.

As to the exact mechanism of displacement, it is necessary to try to distinguish changes which might produce the necessary conditions for shower activity in an area from the changes which are a direct result of shower activity nearby. This distinction, if possible, will require accurate timing of events at the radar station which is not possible with existing long-range PPI pictures.

Our present programme consists of collecting further data on progressions (long-range PPI films) and gathering information for detailed studies of events at the radar observatory (shorter range PPI films).

8. Conclusions

The radar precipitation echoes (convective) for 100 mi around Salisbury have been found to occur practically entirely in groups. Isolated echoes are extremely rare. The groups of echoes are usually elongated in shape and have an apparent movement perpendicular to their length. There is considerable evidence that these groups frequently extend outside the range of the radar photographs and, therefore, that they are often more than 150 mi in length. The continuity of the movement of these groups is largely brought about by the appearance of new echoes, rather than by the translation of existing echoes which usually last for less than an hour.

There appear to be conditions causing both initial grouping of echoes and the continuous new development of echoes in the same general direction. We have not observations yet of the initial formation of these groups. The behaviour of the moving groups has been considered in terms of the theory of the self-propagation of showers and as an effect of a moving external influence which might be causing the showers. We find that either of these explanations is reasonably consistent with the existing observations.

Acknowledgments

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ıvestigation into the accuracy of four types of rainfall recorder

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(Communicated by R. Frith; manuscript received 23 November 1954, in revised form 27 April 1955)

Summary

This paper describes four rainfall recorders and gives the results of laboratory and field investigations rried out at the Road Research Laboratory to determine the accuracy of the instruments.

The instruments used were :

(1) Standard Meteorological Office total-rainfall recorder,
(2) Meteorological Office rate-of-rainfall recorder,
(3) Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder (orifice type).
(4) Road Research Laboratory rate-of-rainfall recorder (channel type).

A laboratory study of the accuracy of the instruments was followed by examination of the records btained of 49 storms occurring between the summer of 1951 and the autumn of 1952.

It was found that mean rates of rainfall between given times could be measured within about 5 per cent om the records from all the instruments, but that the chart speed of the Meteorological Office total-rainfall corder is too slow for peak periods of short duration to be located accurately. The principal advantage the Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder is that it provides clear picture of the shape of the storm curve which is very helpful in the analysis of the records.

1. Introduction

The Road Research Laboratory is investigating the problem of designing surface-vater drainage systems. For this purpose instruments are required for measuring and ecording both the rate of rainfall at any given time and the quantity of rainfall that has allen over a given period of time. This paper describes rainfall recorders developed by he Meteorological Office and the Road Research Laboratory and gives the results of aboratory and field tests carried out by the Road Research Laboratory to determine the ccuracy of the instruments. Although these investigations were carried out to assess he suitability of the recorders for the particular fields of research concerned with surface-vater sewer design, it is thought that the results obtained will be of value for other urposes.

2. Description of instruments

The four instruments for recording rainfall, studied in the present investigation, vere :

(*a*) *Standard Meteorological Office total-rainfall recorder*

In this instrument (Meteorological Office 1952) the chart speed is 0·45 in./hr 11·4 mm/hr) and the vertical scale is 1 in. (25·4 mm) to 0·1 in. of rain.

(*b*) *Meteorological Office rate-of-rainfall recorder*

The record produced by this instrument (Bibby 1944) takes the form of a series f ordinates of the storm curve, drawn as mean values over periods of either one or three ninutes. The range of the instrument depends upon the diameter of the collecting unnel and the duration of each time step; with a collecting funnel 2·887 in. (72·18 mm) n diameter the range is 0-135 mm (0-5·3 in.) per hr at the higher speed and 0-45 mm 0-1·8 in.) per hr at the lower speed.

(c) *Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder (orifice type)*

This instrument (Fig. 1, Plate III) comprises two units, one of which records the rate of rainfall and the other the total rainfall. In the first unit a continuous weighing is made of the quantity of rainfall collected during a small period of time. This period of time is so small (a few seconds) that for all practical purposes an instantaneous record is provided of the weight, and hence the rate of rainfall. The second unit operates in a similar manner to that of the standard Meteorological Office total-rainfall recorder.

Details of the mechanism for measuring and recording are given in Fig. 2. The rain is collected in a funnel 30 in. (762 mm) in diameter and is led into the top of the cylinder (A); this is suspended from one end of a balance arm which has an adjustable counter weight. In the bottom of the cylinder is a circular sharp-edged orifice through which passes a fixed tapered needle. A pen arm (B), which also carries a counterweight, is fixed to the balance arm and carries a pen across a moving chart. As a head of water builds up in the cylinder, the cylinder drops and the size of the orifice increases, and at any given rate of rainfall a position of equilibrium is reached when the inflow to the cylinder equals the outflow from it, and the weight of water is balanced by the weight of the pen arm and attached counterweight. At this position the reading on the chart is a measure of the rate of rainfall.

After leaving the cylinder, the water passes into a float chamber (C). As the float rises, a second pen (D) is moved across the chart providing a record of total rainfall. When the float reaches the top of its travel the float chamber is emptied by a siphon (E).

The recording unit incorporates either a synchronous motor or a clockwork motor which drives a strip chart with a total width of $3\frac{1}{2}$ in. (88·9 mm) between the minimum and maximum readings. The speed of the chart can be set at any of a range of speeds between $\frac{1}{2}$ in. (12·7 mm) and 12 in. (305 mm) per hr and charts up to 100 ft in length can be used.

The range of the instrument is 0-5 in. (0-127 mm) per hr on the rate-of-rainfall scale and on the total-rainfall scale a full-scale movement of $3\frac{1}{2}$ in. (88·9 mm) represents 0·1 in. (2·54 mm) of rainfall. The syphons of the pens dip in opposite sides of the inlet trough and this results in the records produced being separated by 1 in. (Figs. 6 and 7). To prevent freezing during cold weather the instrument is warmed by a thermostatically controlled electric heater.

(d) *Road Research Laboratory rate-of-rainfall recorder (channel type)*

The channel type of rainfall recorder uses the same size of collecting funnel (30 in. dia.) as the orifice-type recorder described above but a different mechanism (Fig. 3, Plate IV) is employed for providing a continuous record of the rate of rainfall.

The rain passes from the collecting funnel into a light V-section channel (A). This channel is pivoted at the end at which the water enters, the weight of the empty channel being balanced by a counterweight (B). As the water runs down the channel the weight of water causes the channel to tilt. This tilting movement is opposed by the balance weight (C) which is fixed to the pivot end of the channel in such a way that the counter-balancing moment increases with increase in the angle of tilt of the channel. Thus, for any given rate of rainfall, an equilibrium position for the channel is reached. A pen arm is attached to the pivot of the channel and as the channel tilts, a pen (D) is moved across the chart of a recorder. The movement of the channel, which is damped by a dash-pot (E), gives a measure of the weight of water in the channel and therefore of the rate of rainfall.

The range of the instrument is 0-5 in. (0-127 mm) per hr and the strip chart recorder is of the same type as that incorporated in the orifice-type instrument described above.

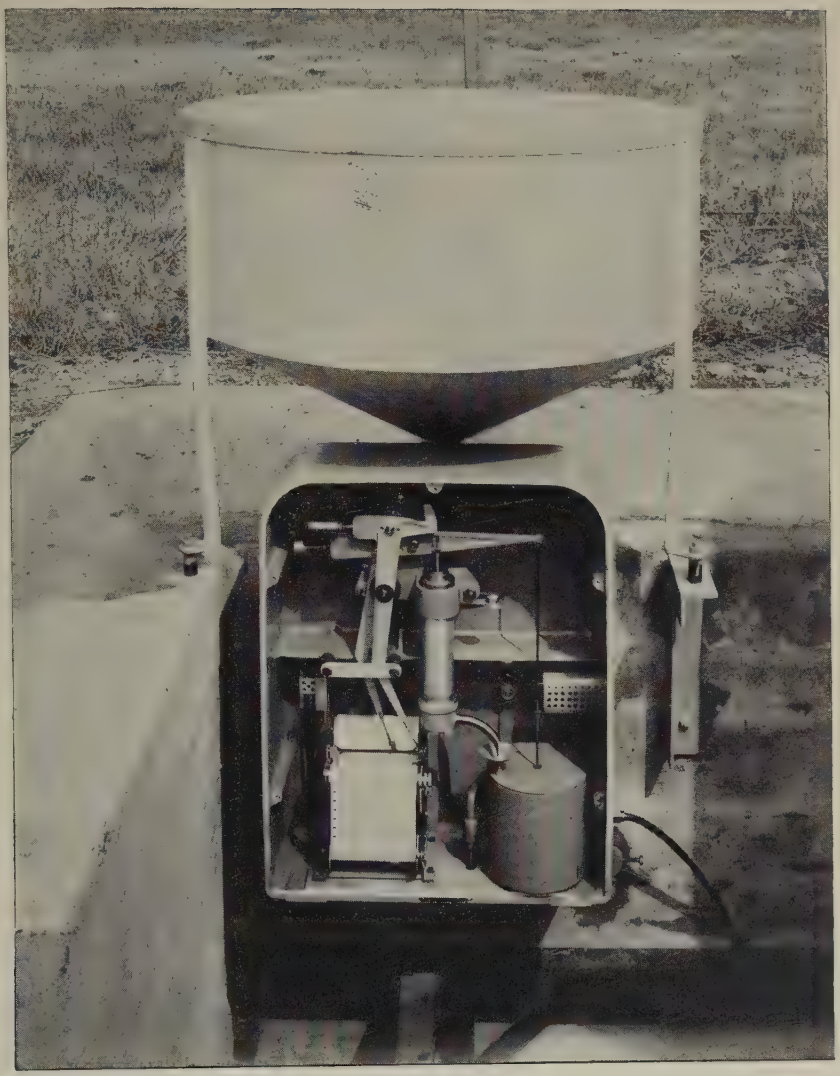

Figure 1. Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder (orifice type) with front cover removed to show mechanism.

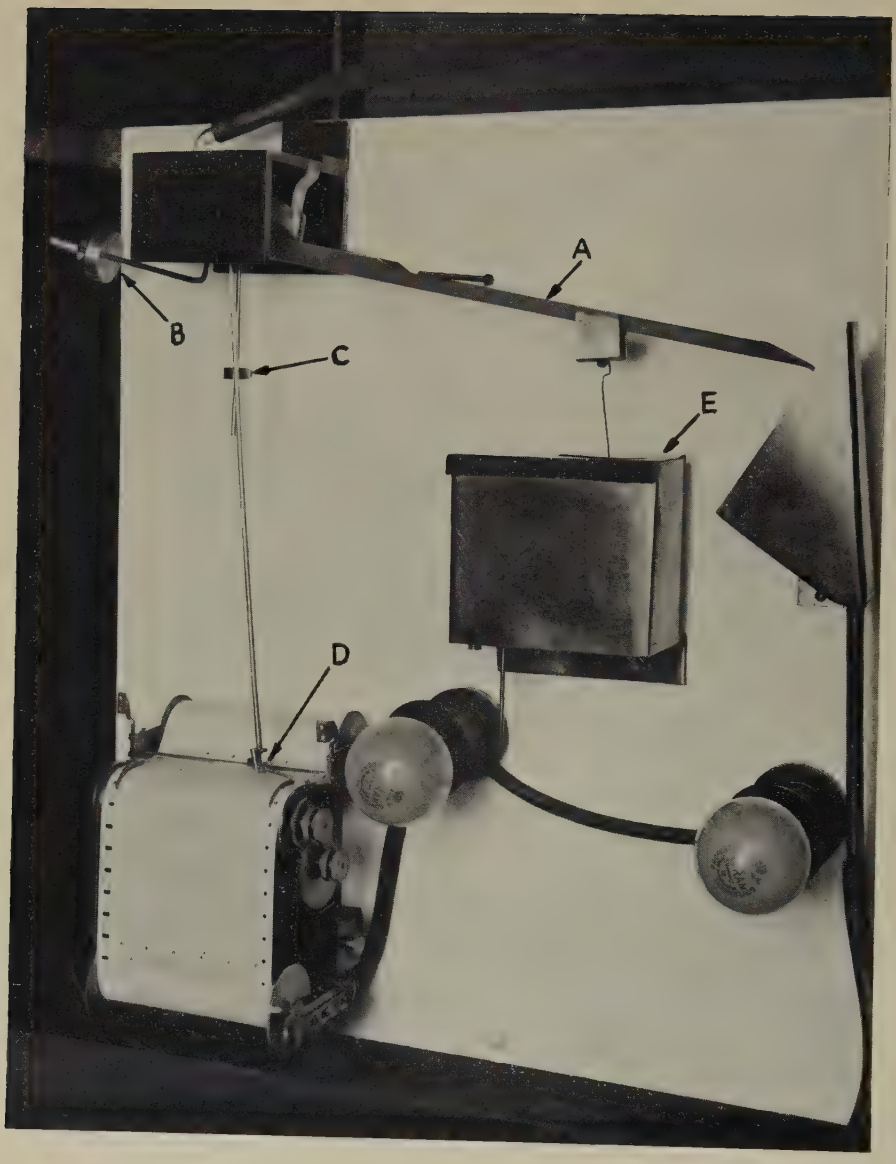

Figure 3. Mechanism of Road Research Laboratory rate-of-rainfall recorder (channel type).

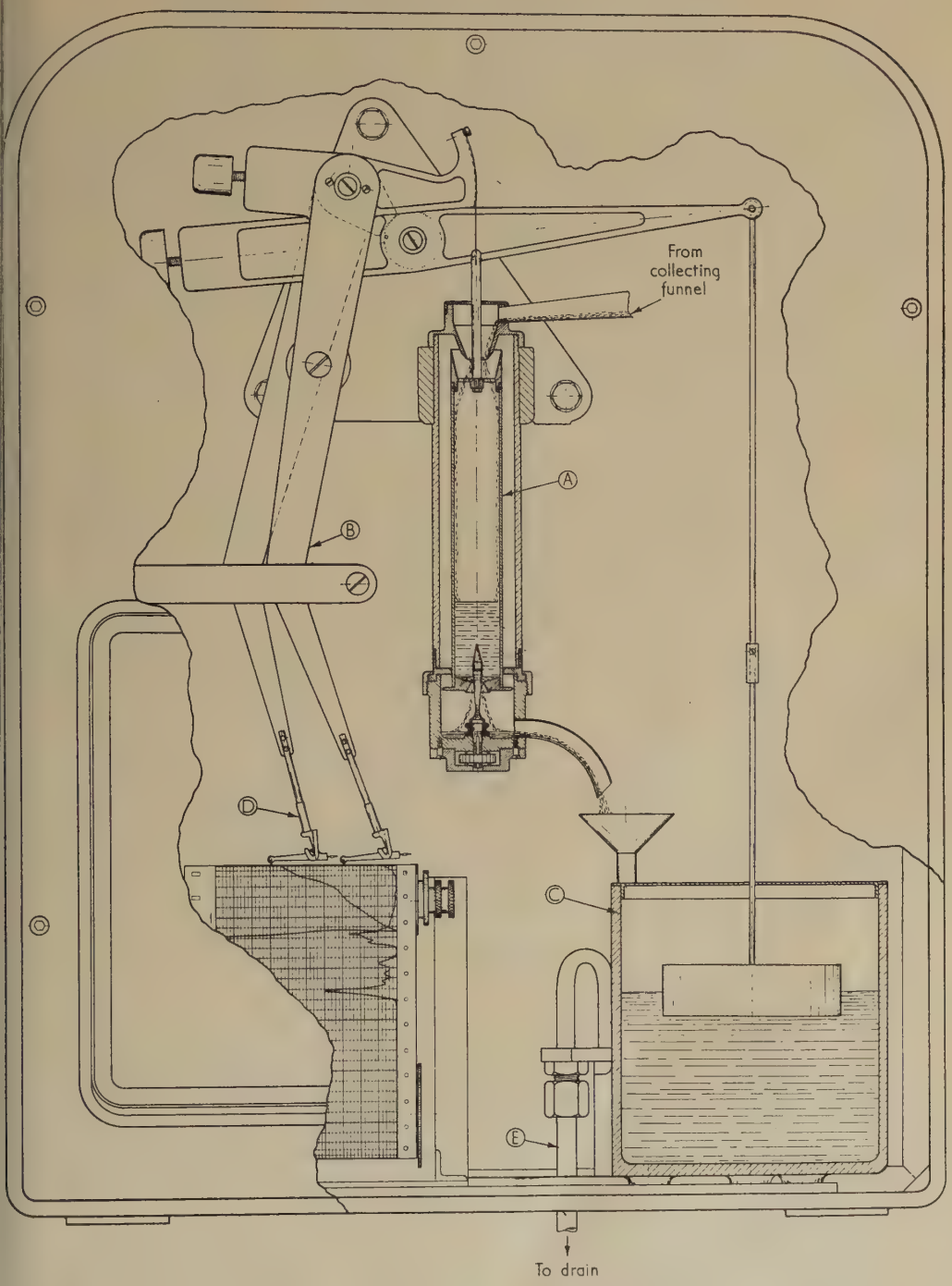

Figure 2. Mechanism of Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder.

3. Laboratory investigations of the accuracy of the raingauges

(a) *Standard Meteorological Office total-rainfall recorder*

Laboratory tests were made simulating rain falling at constant known rates. From the records obtained the total rainfall and the mean rate of rainfall were measured for durations of approximately 6, 12 and 30 min. The flow of water in each test was approximately that corresponding to the rate of rainfall given by published curves (Ministry of Health 1930) for the particular duration. The duration of rainfall was measured on the charts between well-defined points, that is, between the positions of the pen immediately after successive operations of the siphon.

The results (Table 1) indicate that the instrument when working under ideal conditions gives accurate values of both the total rainfall and the mean rate of rainfall during a given period, especially when a correction is made for the quantity of water passing unrecorded during the siphoning period. The accuracy of the results increased with increase in the duration of test, because the longer tests gave larger records which could be measured more accurately.

TABLE 1. Results of laboratory tests with standard Meteorological Office total-rainfall recorder. (The results are an average of 10 repeat tests)

Duration of test (min)	Total rain (actual) (in.)	Total rain (measured) (in.)	Error in measurement of total rain (%)	Error in measurement of total rain (corrected)* (%)	Actual rate of rainfall (in./hr)	Error in measured rate of rainfall (%)	Error in measured rate of rainfall (corrected)* (%)
5·85	0·210	0·20	5	1	2·16	6	4
11·60	0·206	0·20	3	1	1·06	4	3
28·40	0·409	0·40	2	1	0·863	3	1

* Results corrected for loss of water during siphoning period.

(b) *Meteorological Office rate-of-rainfall recorder*

In this instrument the rain collected is caused to fall from an orifice in drops: the impact of each drop on a paddle operates a mercury switch which actuates the recording pen. The calibration of the instrument is stated to be 0·030 mm of rain per drop. To determine the extent to which this calibration was affected by both the rate of rainfall and the temperature of the water, a known quantity of water was run into the collecting funnel at a constant rate and the number of drops produced by the orifice was counted. This was repeated for rates of flow up to 7 in. (177·8 mm) per hr using tap water at a temperature of 0°C and 21°C and distilled water at a temperature of 21°C. Sufficient water was used in each test to produce approximately 100 drops and the work was carried out in constant-temperature rooms to ensure that both the instrument and the water were at the same temperature. The results (Fig. 4) show that there was considerable variation (up to ± 5 per cent) in the quantity of rain per drop for any given rate of rainfall. These variations may in part be due to variations in the surface tension of the water; they cannot be attributed entirely to experimental error as this was unlikely to have been greater than ± 1 per cent.

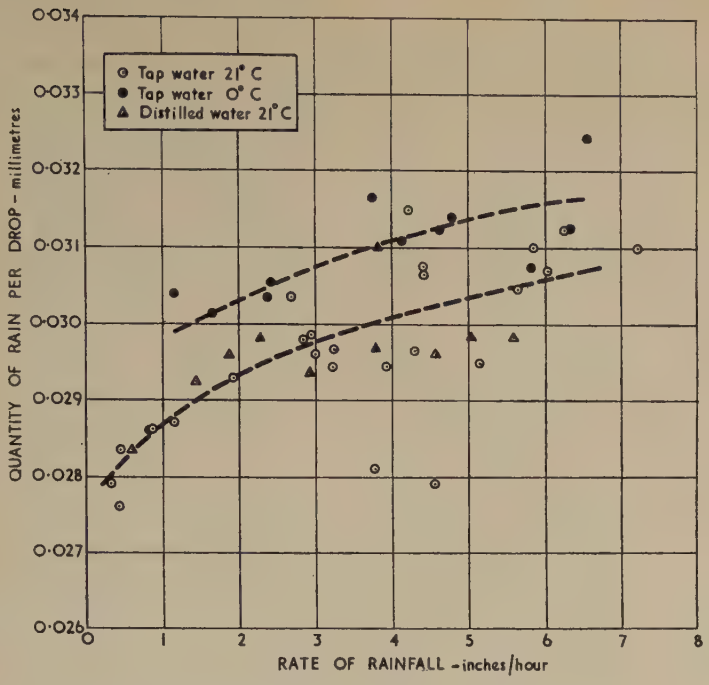

Figure 4. Calibration of Meteorological Office rate-of-rainfall recorder.

The results also show that the calibration of the instrument is affected by both the rate of rainfall and the temperature of the water. There was no significant difference, however, between the results obtained with distilled water and with tap water. Although over the rate-of-rainfall and temperature ranges investigated, the mean value of the calibration of the instrument was 0·030 mm/drop, confirming the value of the calibration given by the Meteorological Office, only 90 per cent of the results lay within ± 6 per cent of this mean value. This indicates that appreciable errors could be introduced into the records of rainfall produced by the instrument.

(*c*) *Road Research Laboratory combined total-rainfall and rate-of-rainfall recorder* (*orifice type*)

The total-rainfall recording unit in this instrument was investigated by tests similar to those carried out with the standard Meteorological Office total-rainfall recorder outlined above. Approximately the same durations of test (6, 12 and 30 min) and corresponding rates of rainfall were employed.

The results (Table 2) show that provided a correction is made for the quantity of water passing through the instrument during siphoning periods very accurate results are obtained. For high rates of rainfall, however, an appreciable error is introduced if no correction is made for the siphoning period. The siphoning period occurs twice as frequently as, and lasts slightly longer (18 sec as compared with 15 sec) than, with the Meteorological Office total-rainfall recorder and therefore has a greater effect (cf. Table 1). A correction will, therefore, always have to be made in the case of the records produced by the Road Research Laboratory instrument if accurate results are to be obtained.

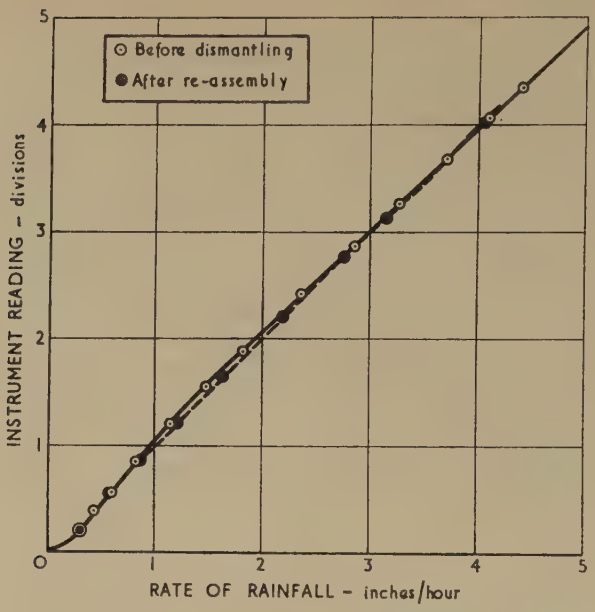

Figure 5. Calibration of Road Research Laboratory rate-of-rainfall recorder (orifice type) before and after dismantling.

TABLE 2. RESULTS OF LABORATORY TESTS WITH THE ROAD RESEARCH LABORATORY TOTAL-RAINFALL RECORDER. (The results are an average of 10 tests)

Duration of test (min)	Total rain (actual) (in.)	Total rain (measured) (in.)	Error in measurement of total rain (%)	Error in measurement of total rain (corrected)* (%)	Actual rate of rainfall (in./hr)	Error in measured rate of rainfall (%)	Error in measurement of total rain (corrected)* (%)
5·90	0·219	0·194	11	2	2·23	12	2
11·21	0·205	0·195	5	1	1·04	4	1
29·70	0·404	0·387	4	1	0·82	4	1

* Results corrected for loss of water during siphoning period.

Investigations were carried out into the effect of the temperature of the water on the calibration curve of the rate-of-rainfall recording unit and into the reproducibility of the curve. One instrument after calibration was dismantled, re-assembled and then re-calibrated. The calibration of an instrument in use in the field was also determined at intervals of approximately a week over a period of several months.

A typical calibration curve of the rate-of-rainfall recording unit in the instrument is given in Fig. 5. After dismantling the unit and re-assembling it, the calibration curve was reproduced to within the limits of experimental error of the tests. Over the range of temperatures likely to be experienced in the field (0°C to 20°C) no significant change occurred in the calibration curve of the unit.

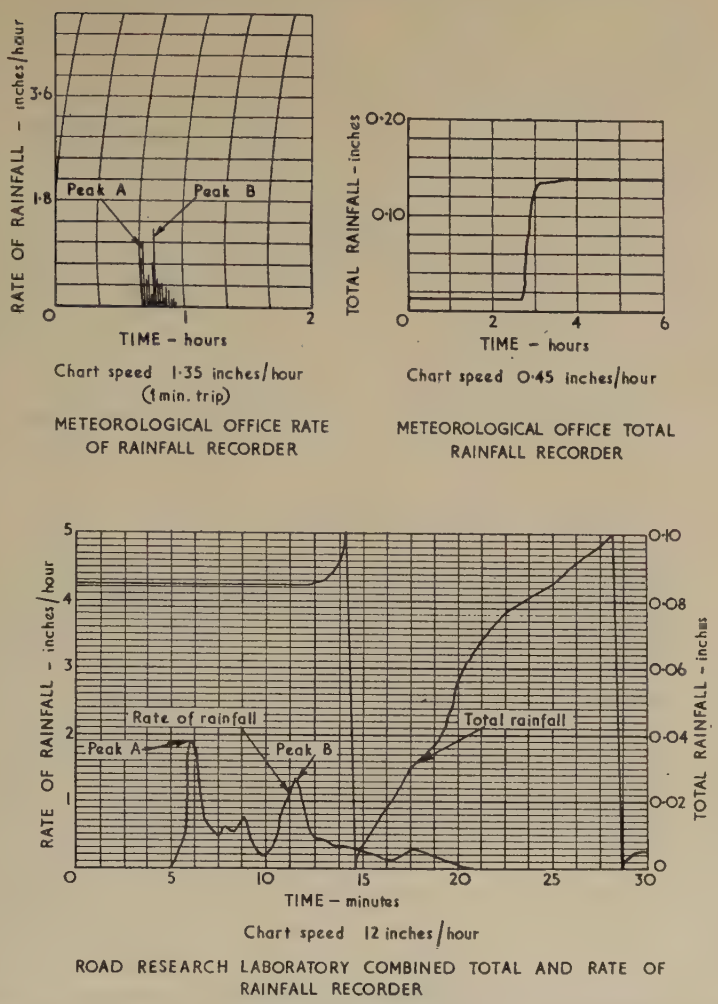

Figure 6. Rainfall of 31 Aug. 1951 recorded at Harmondsworth by three different instruments (times measured from arbitrary zeros).

(*d*) *Road Research Laboratory rate-of-rainfall recorder (channel type)*

The tests with the Road Research Laboratory rate-of-rainfall recorder (channel-type) were similar to those made with the orifice-type instrument. The reproducibility of the calibration curve of the instrument and the effect of the temperature of the water were determined in the laboratory. Two approximately linear calibration-curves are obtained for the channel-type recorder, depending on whether the rate of rainfall is increasing or decreasing. This effect is probably due to the difference between the advancing and receding contact angles of the water-metal interface. Although different types of metal with various surface treatments were tested it was not found possible to eliminate this effect, which, with the best type of surface found, introduced an error of $\pm$ 4 per cent into the records.

The calibration curve is also affected considerably by the temperature of the water, owing possibly to the change in the contact angle with change in temperature. Since the temperature of rainfall could vary over a range of $\pm$ 10°C, an error ranging from $\pm$ 10 per cent at 1 in. (25·4 mm) per hr to $\pm$ 2 per cent at 5 in. (127 mm) per hr could result if no correction for temperature were made.

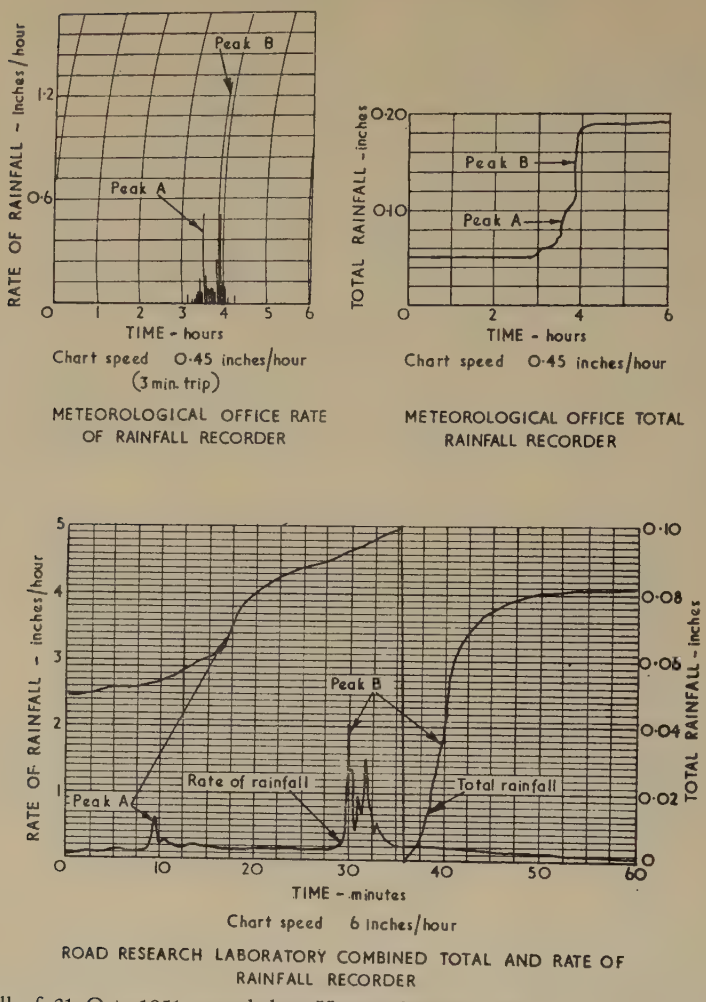

Figure 7. Rainfall of 21 Oct. 1951 recorded at Harmondsworth by three different instruments (time measured from arbitrary zeros).

Although the reproducibility of the calibration curve obtained in the laboratory was good, when operating under field conditions the surface of the metal channel became oxidized and covered with a fine layer of dirt and grease resulting in a steady drift in the calibration curve.

As the channel-type rainfall recorder appeared to be liable to greater errors than the orifice-type instrument, development work was concentrated on the latter instrument.

3. Investigation of records of actual storms obtained with rainfall recorders

All the rainfall recorders described in this paper were set up in the Laboratory grounds within a few feet of each other. Records of the rainfall measured by the individual instruments were obtained for 49 storms during a period of about 15 months from the summer of 1951 to the autumn of 1952. The amount of rain was obtained from the rate-of-rainfall recorders by integrating the records. The records obtained with both the Meteorological Office and Road Research Laboratory total-rainfall recorders were corrected for the water passing through the instruments during siphoning periods.

In general all the instruments recorded substantially the same amounts of rainfall or any given storm. The Meteorological Office rate-of-rainfall recorder was frequently ound to be out of adjustment. The Road Research Laboratory total-rainfall recorder ppeared to give the most reliable records whilst the least reliable records were obtained ith the Road Research Laboratory channel-type rate-of-rainfall recorder. The results re in agreement with those obtained from the laboratory investigations and confirm the ccuracy of the various calibration curves.

Two typical sets of records obtained with the Meteorological Office total-rainfall nd rate-of-rainfall recorders and the Road Research Laboratory total-rainfall and orifice-ype rate-of-rainfall recorders are given in Figs. 6 and 7. In Fig. 6, whilst the general hape of the storm curves recorded by both rate-of-rainfall recorders is similar over he total duration of about one-quarter of an hour, the first peak on the record obtained ith the Meteorological Office instrument is lower than the second, whereas, on the ecord obtained with the continuously recording Road Research Laboratory instruments, he first peak is the higher of the two. This difference occurs because, on the record roduced by the Meteorological Office rate-of-rainfall recorder, each ordinate is a mean alue over a one-minute or three-minute period, thus causing some distortion of the torm curve. Fig. 7 shows a storm curve smoothed by the Meteorological Office rate-f-rainfall recorder when three-minute steps are used; the whole of the peak 'B' is ompressed into only two ordinates.

It is frequently necessary to determine the highest mean rate of rainfall over given durations. To investigate the relative value of the records produced by the various nstruments in this connexion, ten storms of which good records were available were elected for detailed analysis. From the records obtained with each instrument for each storm, two observers acting independently selected periods of 6, 12 and 18 min duration with the highest mean rate of rainfall. For each storm the mean of the readings of the wo observers was obtained and the percentage difference between the readings and the mean value was calculated. The average values so obtained from the records of the 10 storms produced by the four instruments are given in Table 3.

TABLE 3. INDICATION OF THE LIKELY OBSERVER ERROR IN ASSESSING, FROM THE RECORDS PRODUCED BY THE FOUR INSTRUMENTS, THE HIGHEST MEAN RATE OF RAINFALL OVER GIVEN DURATIONS

Type of instrument	Observer error (per cent) Duration (min) 6	12	18
Meteorological Office total-rainfall recorder	4	2	1
Meteorological Office rate-of-rainfall recorder	0	0	0
Road Research Laboratory total-rainfall recorder	1	2	1
Road Research Laboratory rate-of-rainfall recorder	3	2	3

There was no significant difference between the two observers' values from the Meteorological Office rate-of-rainfall recorder as the peaks are always clearly shown, and he measurement simply consists in counting the number of drops recorded on the chart. The biggest variation was obtained with the records from the Meteorological Office total-rainfall recorder for short durations because of the difficulty in selecting the precise period during which the mean rate of rainfall is greatest. This is confirmed by he smaller variation for the short duration obtained from the records of the Road Research Laboratory total-rainfall recorder, which has a considerably greater time scale than the Meteorological Office instrument.

The variations obtained from the records of the Road Research Laboratory rate-o rainfall recorder are considered to be due mainly to errors in planimetering the appropria areas of the storm curves. A subsidiary investigation with the planimeter employe indicated that an error of about ± 2 per cent is to be expected in the measurement of th type of area involved.

The examination of records of storms, described earlier in this paper, indicated th the Road Research Laboratory total-rainfall recorder produced the most reliable record The values obtained with this instrument have thus been used as a basis for comparin the accuracy of the values obtained with the other three instruments (Table 4). In orde to reduce observer error the values given in Table 4 were obtained from the average of the measurements made by the two observers. The results indicate that, apart fro the values obtained with the Meteorological Office total-rainfall recorder for the sho duration (6 min), there was little to choose between the various instruments.

TABLE 4. AVERAGE PERCENTAGE DIFFERENCE BETWEEN THE MEASUREMENTS OF THE HIGHEST MEAN RA OF RAINFALL OVER GIVEN DURATIONS OBTAINED FROM THE ROAD RESEARCH LABORATORY TOTAL-RAINFA RECORDER AND THE OTHER THREE INSTRUMENTS

Type of instrument	Duration (min) 6	12	18
Meteorological Office total-rainfall recorder	11	8	4
Meteorological Office rate-of-rainfall recorder	7	8	7
Road Research Laboratory rate-of-rainfall recorder	9	8	9

4. Conclusions

The conclusions drawn from this investigation may be summarized as follows :

1. Mean rates of rainfall over any selected period of time greater than about 6 mi can be measured with a high order of accuracy from the records of all four instruments.

2. The chart speed of the Meteorological Office total-rainfall recorder is too slo for periods of peak rainfall of short duration to be located accurately.

3. The Road Research Laboratory combined total and rate-of-rainfall recorde provides a clear picture of the shape of the rainfall curve which is very helpful in th analysis of storm records. However, where such analysis is not required this instrumen has little advantage over the Meteorological Office total-rainfall recorder.

Acknowledgments

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551.513.1

On the mean meridional drift of air in the frictional layer of the west-wind belts

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(Manuscript received 10 May 1955)

The existence of a mean poleward drift of air in the lowest layers of the westerlies has generally been considered a necessary result of the frictional drag between the earth (including the oceans) and the atmosphere. Recent investigations (Sheppard, Charnock and Francis 1952), however, have shown that such a conclusion is not necessarily valid for all regions. Their investigations indicate that the surface wind over the North Atlantic often blows almost parallel to the surface isobars. This seems quite regularly to be the case in westerly winds when the surface wind is approximately parallel to the thermal wind in the lowest layers. In such cases the shearing stress remains nearly constant in the frictional layer.

For the following discussion of the shearing stresses in the frictional layer of the westerlies, and of the associated cross-isobaric flow, the numerous surface-wind observations, collected from more than 50 years ship data and published in the *Atlas of climatic charts of the oceans* (McDonald 1938) were used. From these data Mintz and Dean (1952) have computed the zonal and meridional wind components for January and July for each 5° latitude-longitude area of ocean. The mean values of these components for zones of 5° in latitude are presented in Table 1.

TABLE 1. MEAN ZONAL AND MERIDIONAL COMPONENTS OF THE SURFACE WIND, $\bar{u}_0$, $\bar{v}_0$, OVER THE OCEANS IN LATITUDINAL BELTS (VELOCITY IN m sec^{-1})

Latitude (°N)	Northern hemisphere January $\bar{u}_0$	January $\bar{v}_0$	July $\bar{u}_0$	July $\bar{v}_0$	Latitude (°S)	Southern hemisphere January $\bar{u}_0$	January $\bar{v}_0$	July $\bar{u}_0$	July $\bar{v}_0$
50	2·00	0·48	1·00	0·60	50	3·30	− 1·58	3·05	− 1·42
45	1·88	− 0·03	0·90	0·42	45	2·50	− 1·01	2·83	− 1·06
40	1·74	− 0·60	0·55	0·25	40	1·50	− 0·12	2·31	− 0·48
35	1·37	− 0·54	0·24	− 0·05	35	0·25	0·36	1·55	0·01
30	0·70	− 0·42	− 0·83	− 0·50	30	− 1·25	0·60	0·56	0·31
25	− 0·50	− 1·05	− 2·02	− 0·43	25	− 2·63	0·72	− 1·00	1·46
20	− 2·00	− 1·57	− 2·15	− 0·49	20	− 3·00	1·13	− 2·90	2·33
15	− 3·46	− 2·38	− 1·50	− 0·12	15	− 2·55	0·96	− 4·10	2·75
10	− 3·90	− 2·89	− 0·46	0·75	10	− 1·60	0·42	− 3·85	2·59
5	− 2·71	− 1·36	− 0·38	2·15	5	− 1·00	0·00	− 2·80	2·20
0	− 1·40	− 0·30	− 1·59	2·42	0	− 1·40	− 0·30	− 1·59	2·42

The wind components in the Trades have been used by Riehl and Yeh (1950) and by Palmén (1954) for an estimate of the mean meridional mass-circulation in the trade-wind cells and will therefore not be considered here. As characteristic of the west-wind belts, the wind components in the zones 40°-50° in both hemispheres will be selected. The mean values for these belts are presented in Table 2.

TABLE 2. MEAN ZONAL AND MERIDIONAL COMPONENTS OF THE SURFACE WIND IN THE OCEAN REGIONS OF LATITUDES 40°-50° OF BOTH HEMISPHERES (VELOCITY IN m sec^{-1}).

Month	40°-50°N $\bar{u}_0$	40°-50°N v_0	40°-50°S $\bar{u}_0$	40°-50°S $\bar{v}_0$
January	1·87	− 0·05	2·43	− 0·90
July	0·82	0·42	2·73	− 0·99

The most striking feature in the above table is the non-existence of any poleward dri in the northern hemisphere in January. In July, however, a pronounced poleward drift can l observed. In the southern hemisphere no essential difference between summer and winter ca be seen in that respect.

In the southern hemisphere the mean meridional component is almost equivalent to a mea cross-isobaric flow, since the oceans extend practically over the whole region. In the norther hemisphere, however, the continents occupy more than half of the zone 40°-50°N. The mea cross-isobaric component can here be computed by considering the mean pressure-field ov the ocean regions. For this purpose the sea-level pressure data published by the Institute Meteorology and Geophysics, Free University of Berlin (1953) were used. For the geostroph zonal and meridional components over the oceans the following mean values were computed January, $\bar{u}_g = 2{\cdot}63$, $\bar{v}_g = 0{\cdot}16$ m sec^{-1} and July, $\bar{u}_g = 2{\cdot}04$, $\bar{v}_g = 0{\cdot}37$ m sec^{-1}. Consequentl the angle between the mean isobars and the latitude circles is about 3·5° in January and 1 in July. Since, according to Table 2, the angle between the mean wind vectors and the latitud circles is − 2° and 27° respectively, the angle between the mean wind vectors and the mea surface isobars can be estimated as − 5° for January and 17° for July. The corresponding valu for the southern hemisphere are, according to Table 2, about 20° both for January and Jul Positive values here indicate flow from higher to lower pressure.

For the lowest atmospheric layers, where the acceleration and the influence of both smal scale and large-scale lateral stresses can be neglected, the difference between the mean surfa drag $\bar{\tau}_{x_0}$ and the mean zonal stress $\bar{\tau}_{x_H}$ at level H can be expressed by

$$\bar{\tau}_{x_0} - \bar{\tau}_{x_H} = \pm \frac{2\,\Omega \sin\phi}{g} \int_{p_H}^{p_0} \bar{v}\, dp \qquad . \quad . \quad . \quad (1$$

Here Ω denotes the angular velocity of the earth's rotation, ϕ is the latitude, g is the accelera tion of gravity, and p_0, p_H denote the mean pressure at the ocean surface and at level H. Th positive sign is valid for the northern hemisphere, the negative sign for the southern hemisphere Vanishing $\bar{v}$ then means that the mean zonal stress remains constant. From Table 2 we ca therefore conclude that the mean zonal stress in the zone 40°-50°N in January is practicall constant throughout the frictional layer or increases slightly with height, whereas in July th mean stress decreases with height. For the belt 40°-50°S the mean zonal stress both in Januar and July decreases with height.

The mean zonal stress in the belts considered can be written in the form

$$\bar{\tau}_{x_H} = \bar{\mu}\,\overline{\frac{\partial u}{\partial z}} + \overline{\mu' \left(\frac{\partial u}{\partial z}\right)'} \qquad . \quad . \quad . \quad . \quad . \quad (2$$

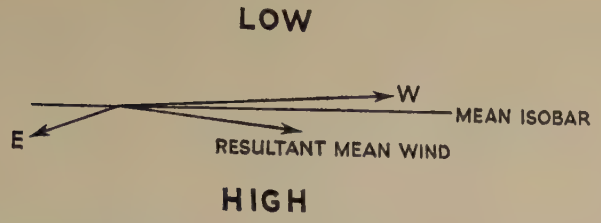
Here μ is the coefficient of eddy viscosity. The bars denote mean values over the whol zones of oceans, and μ' and $\left(\frac{\partial u}{\partial z}\right)'$ denote the deviations from the mean values. Since nothin is known about any systematic correlation between μ and $\frac{\partial u}{\partial z}$ we must assume that $\overline{\mu' \left(\frac{\partial u}{\partial z}\right)'}$ vanishes. The mean zonal stress therefore can be determined from the product of the mean valu of μ and the vertical shear of the mean zonal wind-component.

The surface drag has generally been computed from the empirical formula

$$\bar{\tau}_{x_0} = k\,\rho_0\,\overline{V_0^2 \cos\alpha} \qquad . \quad . \quad . \quad . \quad . \quad (3$$

where k is a ' drag coefficient,' V_0 is the total wind velocity at the surface (' anemometer height ') and α denotes the angle between the wind vector and the west-east direction.

Priestley (1951) has determined $\bar{\tau}_{x_0}$ for the ocean areas in different seasons using formul (3). From his computations we get for the surface stress in January in the zone 40°-50°N a mean value of 0·80 dyne cm^{-2} and for zone 40°-50°S in July a mean value of 0·95 dyne cm^{-2}. The mean vertical shear of the zonal wind has been computed by Mintz (1953). For the level 900 mb corresponding approximately to the upper boundary of the ' frictional layer ' in the west-winc belts, the values of $\frac{\partial \bar{u}}{\partial z}$ are about 3×10^{-3} sec^{-1} and 2×10^{-3} sec^{-1}, respectively. In the northern hemisphere the mean value of $\bar{v}$ in the whole layer is approximately zero, whereas the mean value of $\bar{v}$ in the southern hemisphere in the layer between 1,000 mb and 900 mb could be



gure 1. Resultant mean surface-wind vector in the belt of westerlies and the mean vectors for cases with westerly (W) and easterly (E) winds.

imated as about $-0{\cdot}4$ m sec^{-1}, corresponding to $-1{\cdot}0$ at the surface and 0 at the 900-mb vel. The corresponding value of $\bar{\mu}$ would then for both hemispheres be 270 g cm^{-1} sec^{-1}, mputed from the approximate formula

$$\bar{\mu}\frac{\partial \bar{u}}{\partial z} = k\,\rho_0\,\overline{V_0^2\cos\alpha} \mp \frac{2\,\Omega\sin\phi}{g}\int_{900}^{1,000}\bar{v}\,dp \qquad . \qquad . \qquad . \qquad (4)$$

his mean value of the eddy viscosity in the free atmosphere seems reasonable.

Hence the pronounced difference between the northern and southern hemispheres concerning e poleward drift of air in the frictional layer can be explained when we consider the difference both surface drag and in vertical wind shear. Since the latter, however, has not been determined parately for the ocean areas of the northern hemisphere, our previous conclusions are not unded upon sufficiently exact empirical data. The conclusions made therefore cannot be con-dered as final.

It should also be pointed out that the surface flow can, in fact, have a component across the bars from higher to lower pressure in practically all individual cases, in spite of the mean ow being directed slightly from lower to higher pressure. One needs only to assume that the mponent from higher to lower pressure is considerably stronger in cases with east wind than cases with west wind, in accordance with the ideas presented by Sheppard, Charnock and rancis (1952) and Sheppard and Omar (1952). It is easy to see that, under such conditions, ie resultant mean-wind vector can have a slight component from lower to higher pressure, the schematic diagram (Fig. 1) indicates, if the frequency of cases with east wind is sufficiently gh.

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551.5

Synoptic evidence for a direct circulation about a jet stream

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(Manuscript received 10 December 1954, in revised form 2 April 1955)

Summary

The examination of a uniformly-sloping tropopause surface during the eastward progression of a je stream maximum reveals that the observed perturbation of the tropopause can be interpreted in terms the vertical branch of a direct circulation occurring above a jet stream in the entrance region. Indire evidence is also obtained for the existence of a corresponding direct circulation below the jet stream.

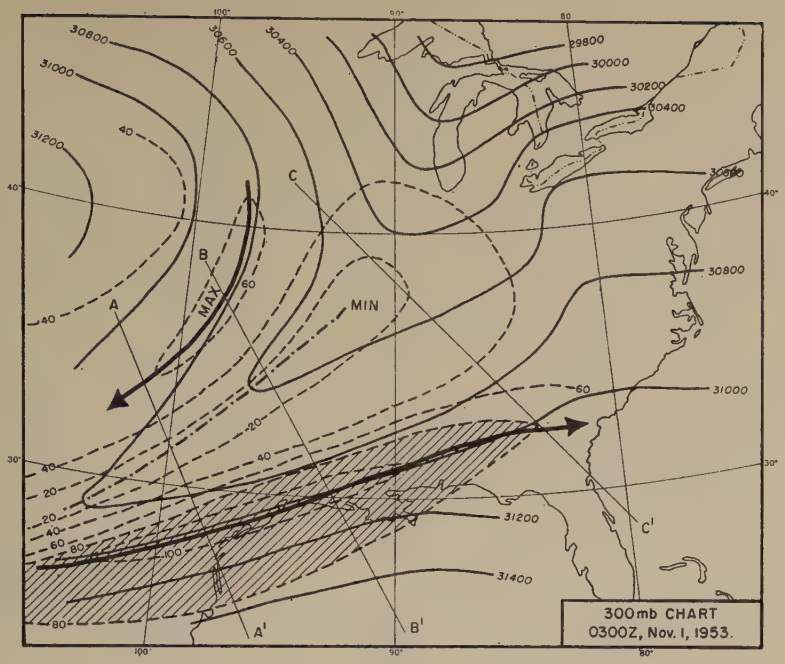
1. Introduction

Two atmospheric models have been presented to account for jet-stream intensificatio That proposed by the Staff Members of the Department of Meteorology of the University Chicago (1947) led to an indirect thermal circulation in planes perpendicular to an intensifyin jet stream. On the other hand, Namias and Clapp (1949) postulated a direct thermal circulatio in their model of jet-stream formation or intensification. The contradiction between the tw models has led to several attempts to resolve it by appeal to observation. The investigation proceeded along two lines : those which set out to establish the sense of the vertical branch of the circulation by vertical-velocity computations, and those which sought to determine th sense of the horizontal branches by examining cross-isobar components near jet streams. Amon those in the former category, Nyberg (1950), Newton and Carson (1953) and Hubert (195 found the jet-stream circulation to be of the direct thermal type in specific instances; in particula Hubert determined it for the diffluence region of a jet stream. Endlich (1953), in a study of vertica velocity patterns over the northern hemisphere for a 10-day period, found limited evidence indicate that the mean circulation about the jet was thermally indirect. In the latter categor Murray and Daniels (1953) examined statistically the cross-isobar flow near jet-stream maxim to determine the sense of the horizontal branch of the circulation cells. They found, in the je stream entrance region, evidence of a transverse flow consistent with the existence of a dire thermal circulation either above or below the jet stream, or both; and in the jet-stream ex region, the transverse flow was consistent with the existence of an indirect circulation abov or below the jet stream, or both.

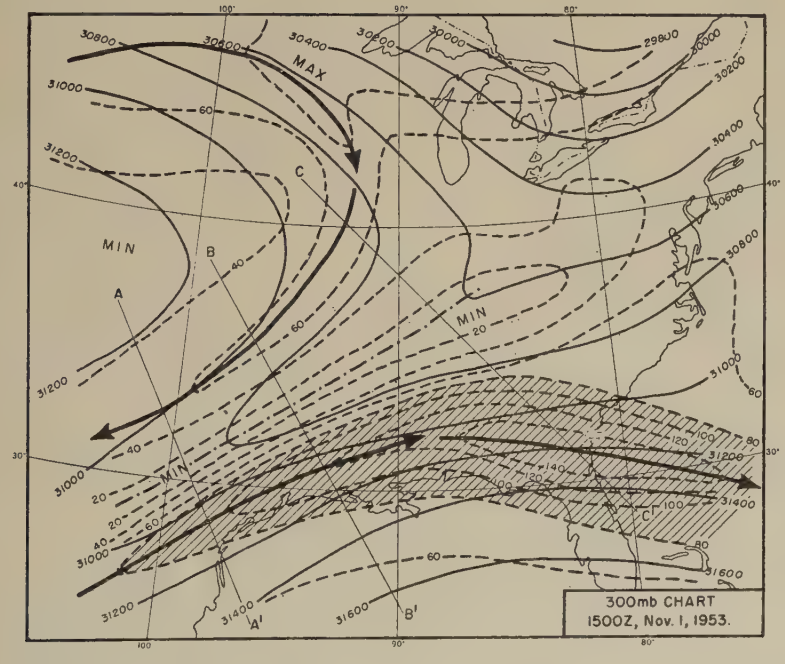
There are inherent difficulties in obtaining reliable vertical-velocity computations in ar given situation. In view of the desirability of obtaining further direct evidence for the existen of the vertical branches of the jet-stream circulation, it appears possible that the examinatio of a non-laminated tropopause (essentially an isentropic surface) near a jet-stream maximu would reveal such evidence. The essence of this paper is a summary of the observational eviden which, it is claimed, demonstrates the existence of a vertical branch of a direct thermal circulatio above a jet stream in the entrance region, in agreement with the horizontal branch establishe by the investigation of Murray and Daniels.

2. Circulation above a jet stream in the entrance region

During the period 31 October-2 November, 1953, a shear line of the type described b Hsieh (1950) and Newton and Carson (1953) formed over the central U.S.A. The maximu intensity of the shear-line trough occurred around 03-15 hr GMT., 1 Nov., for which the corre ponding 300-mb maps are shown in Fig. 1. South of the trough, a single westerly jet strea persisted over Texas and Louisiana for a period of 36 hr, along which a single intensifying je stream maximum progressed eastward. The jet-stream maximum was centred over Texas 03 hr, (Fig. 1(a)) and downstream over Florida at 15 hr (Fig. 1(b)). The fixed vertical plan containing AA′ in Fig. 1 was thus located near a jet-stream centre at 03 hr and a jet-stream entranc region at 15 hr.

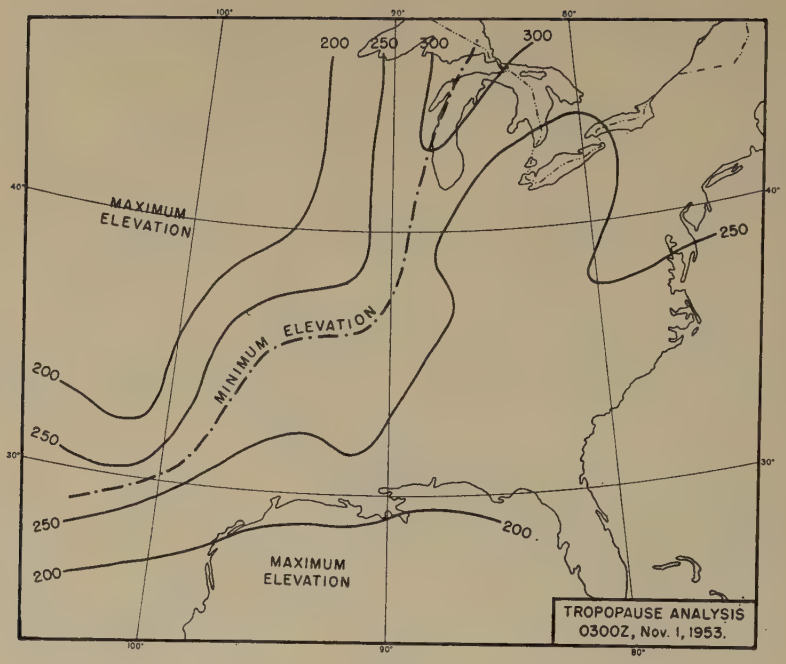


(a)

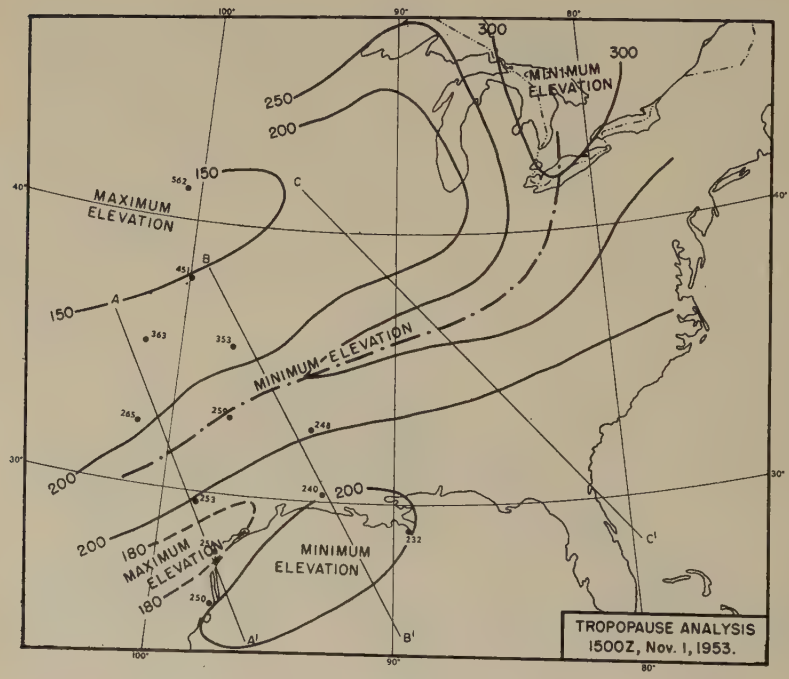


(b)

Figure 1. 300-mb charts 1 Nov. 1953: (a) 03 hr GMT, (b) 15 hr.
Isotachs in broken lines. Hatched areas along southern jet stream indicate wind speeds greater than 80 kt.



(a)



(b)

Figure 2. Tropopause analyses 1 Nov. 1953: (a) 03 hr, (b) 15 hr.

Solid lines are isobars.

Analyses of the tropopause over the eastern part of the U.S.A. for the same periods are shown in Fig. 2. A single tropopause was identifiable on each ascent. For each station, its orientation and slope were determined in the manner described by Godson (1951). The tropopause in Fig. 2(a) has a minimum elevation along the shear line and slopes upwards both to the NW. and SE. of the shear-line trough, this slope extending as far south as the Gulf of Mexico. Fig. 2(b) shows much the same tropopause configuration except for the development of a slight elevation in the tropopause at Corpus Christi (251) such that its slope reversed between Corpus Christi and Brownsville (250) during the 12 hr period.

In order to determine the relation between the local change in tropopause level and other synoptic features, vertical cross-sections along AA′, BB′, and CC′ were constructed for 15 hr. Sufficient data were available for the construction of a cross-section along AA′ at 03 hr also. Those along AA′ at 03 hr and 15 hr are shown in Fig. 3, corresponding to Figs. 1 and 2 respectively for which data were selected from Amarillo (363), Big Spring (265), San Antonio (253), Corpus Christi (251) and Brownsville (250). The positions of these stations are shown in Fig. 2(b). The cross-sections include components of the measured wind speeds exclusively. Above the NE. and SW. jet streams flanking the shear line in Fig. 3(a), the tropopause was found to slope upwards to either side from its minimum elevation of 260 mb at the shear line. In Fig. 3(b) the tropopause at Corpus Christi appears at a level about 30 mb (3×10^3 ft) higher than it was 12 hr earlier, the rise having occurred in the 12-hr period during which a jet-stream maximum in the plane AA′ moved eastward so that its entrance zone appeared at AA′ at 15 hr. This upward displacement is consistent with the presence of a direct thermal circulation in the entrance region above the jet axis.

An estimate of the vertical velocity in the upward branch may be obtained from the average local vertical velocity of the tropopause at Corpus Christi (251), assuming it moved continuously as an isentropic surface : it is of the order of 2 cm/sec. This assumption appears to be reasonably

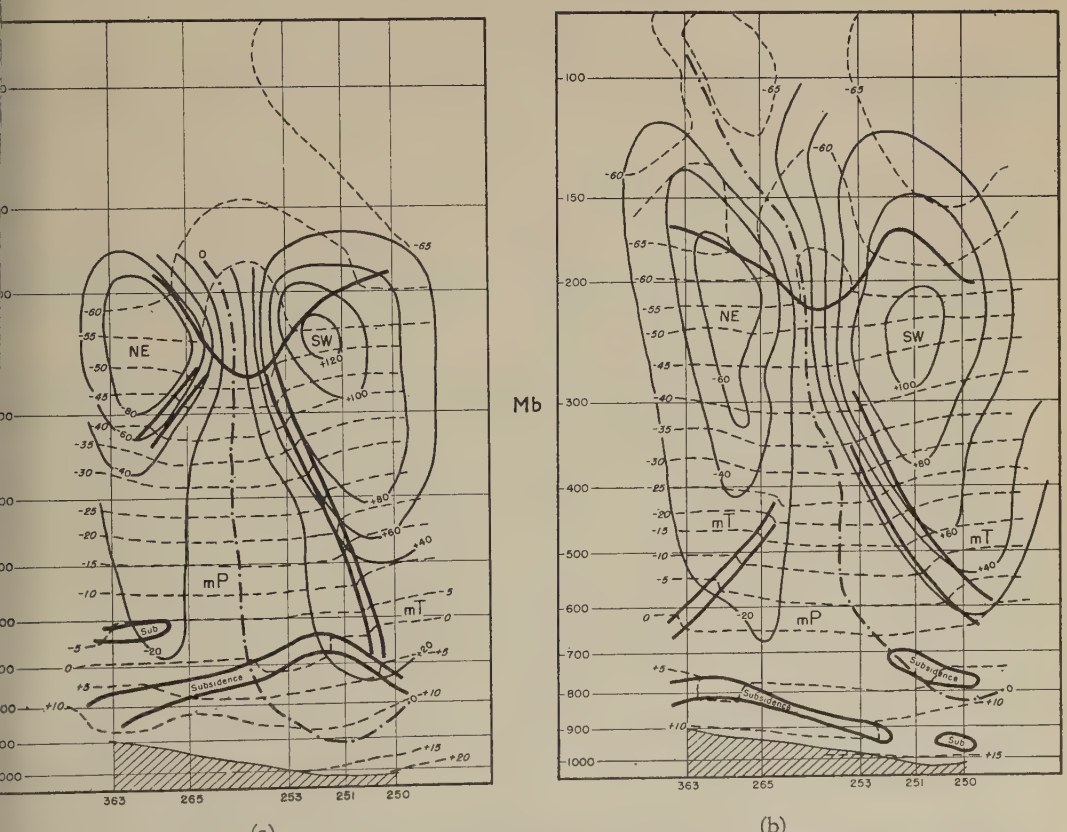


Figure 3. Cross-sections Amarillo – Brownsville, along AA′ in Fig. 1. (a) 03 hr, (b) 15 hr. Fronts, tropopause and subsidence inversions indicated by heavy solid lines; isotherms – broken lines, isotachs – light solid lines.

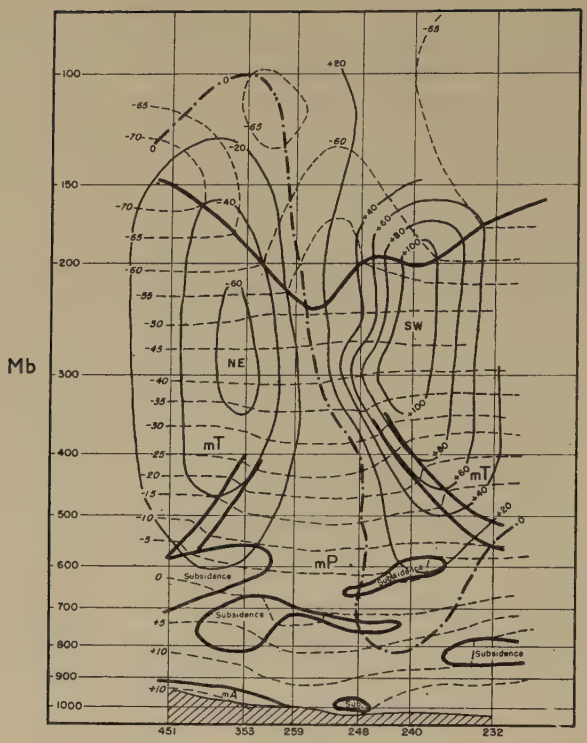


Figure 4. Cross-section Dodge City – Burrwood along BB′ in Fig. 1. 15 hr. Fronts, tropopause and subsidence inversions indicated by heavy solid lines; isotherms – broken lines, isotachs – light solid lines.

valid for the tropopause surface, away from the sloping portion near the shear line. For example, at 13 stations over the southern part of the United States at 15 hr, 1 November, the average potential temperature at the tropopause was 343°K with an average deviation of ± 2°K. The largest individual deviation was 4°K. In comparison, the corresponding potential temperatures at 03 hr at stations similarly located with respect to the sloping portion were 342°K at Big Spring (265) and 343°K at Corpus Christi (251). Thus, on the basis of the foregoing evidence, the assumption is plausible if it is restricted to the tropopause surface south of the shear line where the deformation occurred.

Further corroborative evidence for the reality of the upward displacement of the tropopause may be found in the vertical cross-section in the plane containing BB′, which is nearer the jet centre at 15 hr. (Fig. 4). It likewise showed the local upward perturbation in the tropopause between Barksdale (248) and Lake Charles (240). The smaller amplitude of the vertical displacement is consistent with the shorter time during which the air parcels were under the influence of a direct circulation after the passage of the jet centre. The cross-section along the plane containing CC′ (not shown), near the jet centre, shows the same tropopause configuration as in Fig. 3(a). Unfortunately, insufficient high-level data were available to permit the construction of reliable cross-sections further downstream. In any event, an indirect thermal circulation above the jet-stream exit would only effect a steepening of the tropopause, so that a deformation of the type described would be very unlikely to appear.

3. Circulation below a jet stream in the entrance region

A study of the 12-hr average vertical velocities, in the troposphere beneath the jet stream, was attempted in order to seek evidence for the existence of the vertical branches of a corresponding direct thermal circulation beneath the jet stream. The isobaric method of Graham (1947) was used ; it assumes adiabatic motion and involves the computations of 12-hr trajectories from constant-pressure charts. The procedure was carried out at the standard isobaric levels up to 200 mb and the computed 12-hourly displacements were plotted at the end points of the

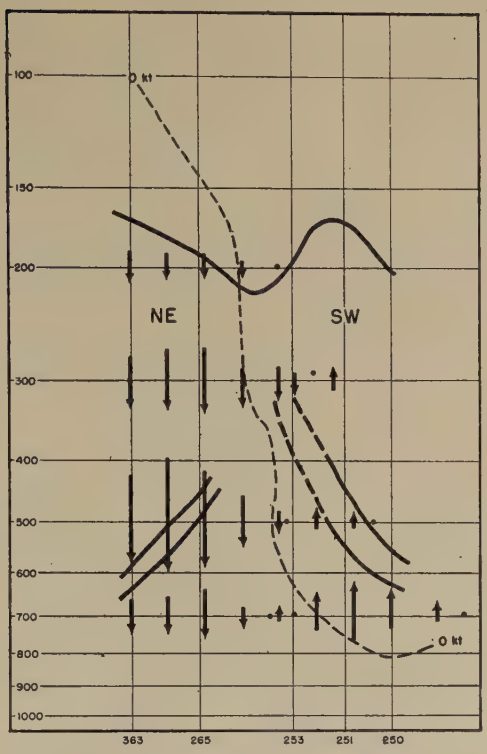


Figure 5. Computed 12-hourly vertical displacements along AA′ 03-15 hr, 1 November 1953. Lengths of arrows are equal to computed displacements in mb. The broken line is the locus of points where the horizontal component of the wind speed is zero.

trajectories. They were then combined on a single cross-section shown in Fig. 5 which corresponds in time to Fig. 3(b). The lengths of the vertical arrows are equal to the computed 12 hr displacements in mb.

The shear-line trough, designated by the zero-kt isotach, roughly separates the regions of ascent and descent in the troposphere. The indicated velocities suggest a direct thermal circulation beneath the SW. jet stream although the evidence is somewhat less conclusive than for the upper circulation. However, some indirect support for the suggested sense of the circulation appeared in the reports of clear skies and steady pressure rises beneath the upper shear line, similar to those observed by Hsieh (1950) and Newton *et al.* (1951), indicating subsidence. Moreover, cloudiness with light precipitation at Galveston, Texas, whose corresponding location on the cross-section is between Brownsville and Corpus Christi, agrees with the few computed positive vertical velocities in that region.

The NE. jet stream is evidently associated with greater negative vertical velocities to the right of the jet, looking downstream ($-$ 6 cm/sec), than to the left of the jet stream ($-$ 2½ cm/sec). The distribution of vertical velocities suggests the presence of an indirect thermal circulation associated with the approaching exit-region of the southward-moving jet-stream maximum which appeared over North Platte (562) at 15 hr, 31 October. It can be observed approaching the plane AA′ 12 hr later in Fig. 1(a). In view of the uncertainties in the computed vertical velocities at 200 mb, no conclusions can be drawn concerning the existence of a corresponding circulation above the NE. jet stream.

ACKNOWLEDGMENT

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551.55

The local wind at Milfield, Northumberland

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Summary

There is a very strong local westerly wind at Milfield, which blows along the foot of the escarpment o the Cheviot Hills. It occurs in any synoptic situation which gives a surface wind between south-west and west and the presence of an inversion is not necessary for its appearance. The wind is not purely katabatic, and its main cause appears to be some feature of the local topography.

1. Introduction

During the war of 1939-45, an airfield was constructed, for the Royal Air Force, just south of the village of Milfield, in the valley of the River Till. The aerodrome is on level ground, but two miles to the south of it is the escarpment of the Cheviot Hills, rising steeply about 1,000 ft (Fig. 1). West of the site, the valley of the River Glen runs two miles back to Kirknewton where several small valleys converge. The airfield site is apparently in a sheltered position; but in fact it is subject to very frequent strong winds blowing down the Glen valley.

2. Methods of measurement

The basis for this study of the local wind was an analysis of the routine observations made at R.A.F. Station Milfield during part of 1945. Hourly observations of the direction and speed of the wind were made with a wind vane and an anemometer, mounted on masts 10 ft and 8 ft high respectively, on the roof of the Control Tower, an isolated building about 25 ft high. A portable anemometer, which agreed closely with the mast anemometer in check readings, was used to measure wind speeds in the Glen valley. Estimates of the geostrophic wind were made by measurements on the routine synoptic charts.

3. Observations

(a) *Daily frequency of strong and gale winds*

The impression gained, after a few weeks at Milfield airfield, that it is an exceedingly windy place, was confirmed by the wind records made during the first six months of 1945. During

Figure 1. Sketch-map of surroundings of airfield site. Ground above 500 ft m.s.l. level is shaded; contours at 500 ft intervals.

this time the wind speed attained Force 6 or more on 93 days out of 181, and there were 27 gales, 23 of them blowing from points between south-west and west. In January, February and March the maximum wind speed was Force 6 or more on nearly two days out of three. In April May and June strong winds were not quite so frequent; even so Force 6, or more, was attained on 40 per cent of the total days.

(*b*) *Direction from which strong and gale winds blew*

To obtain a more detailed picture of the frequency and direction of strong winds at Milfield, the hourly wind observations for the 90 days 17 January to 16 April, 1945 were analysed. The results are summarized in Table 1. An examination of this table shows the following points of interest : During the 90-day period studied, Milfield ranked with the most exposed recording stations, both in the frequent occurrence of strong winds and gales, and in the maximum speed which the wind attained. There were 109 hr of gale-force winds (5 per cent of the total), and 522 hr of winds of Force 6 and over (24 per cent of the total). The wind reached Force 10 at nine, and Force 11 at five, hourly observations. (ii) Three wind directions, SW., WSW. and W., account for nearly all the gales, and a very high proportion of the strong winds. In fact, the stronger the wind is, the more likely it is to blow from this quarter. Only one-third of the light winds (223 observations out of 694), but half the winds of Forces 4 and 5, three-quarters of the Force-6 and Force-7 winds, and nine-tenths of the gales (95 observations out of 109), are from directions between south-west and west.

TABLE 1. FREQUENCY TABLE OF HOURLY WIND OBSERVATIONS, 17 JANUARY TO 16 APRIL

Force	Calms 0	Light 1-3	Moderate 4-5	Strong 6-7	Gales 8	9	10	11	Total All forces
N.	—	25	3	0	0	0	0	0	28
NNE.	—	32	7	0	0	0	0	0	39
NE.	—	26	1	0	1	0	6	0	34
ENE.	—	7	1	0	0	0	0	0	8
E.	—	31	0	0	0	0	0	0	31
ESE.	—	48	16	0	0	0	0	0	64
SE.	—	37	45	0	0	0	0	0	82
SSE.	—	27	23	0	0	0	0	0	50
S.	—	26	9	1	0	0	0	0	36
SSW.	—	30	12	9	3	0	0	0	54
SW.	—	68	65	45	7	2	0	0	187
WSW.	—	56	164	152	34	5	0	2	413
W.	—	99	217	150	33	6	3	3	511
Total of 3 last rows		223	446	347	74	13	3	5	1111
WNW.	—	85	110	40	0	2	0	0	237
NW.	—	49	50	8	0	0	0	0	107
NNW.	—	48	22	8	2	0	0	0	80
Total (all directions)	199	694	745	413	80	15	9	5	2160

(c) *Comparison of Milfield with neighbouring stations*

To test whether the very high frequency of strong winds, during the first six months of 1945, was confined to Milfield, wind observations from Milfield were compared with simultaneous observations from five other recording stations in Berwickshire and Northumberland : St. Abbs Head, Charterhall, Acklington, Eshott, and Ouston. When the wind was blowing from any other point than those between SW. and W., no consistent difference in wind speed or direction could be noticed between Milfield and the other five stations. But winds from the critical directions, SW., WSW. and W. were consistently stronger, and were sometimes more veered, at Milfield than at any of the other places. Some examples of striking differences in wind speed

TABLE 2. CONTRASTS IN WIND SPEED BETWEEN MILFIELD AND NEIGHBOURING STATIONS.

Date	Time (GMT)	Milfield D	Milfield F	St. Abbs D	St. Abbs F	Charterhall D	Charterhall F	Acklington D	Acklington F
7 Feb.	09	WSW.	9	SSW.	5	SW.	6G	SW. by W.	6G
15 Feb.	09	WSW.	7	SW.	4	SW.	3	SW.	2
15 Mar.	06	SW.	7G	SW.	5	WSW.	4	WSW.	4
17 Apr.	07	W.	7	SW.	4	SW. by S.	4	W.	3
16 May	15	W.	7G	WSW.	4	SW.	5G	WSW.	4G
23 Jul.	12	W.	8G	SW.	4	S by W.	5G	WSW.	6G
15 Oct.	13	WSW.	5G	CALM		SW.	4	WSW.	3

D = Wind direction (compass points); F = Force of wind on the Beaufort scale; G = Gusty.

are given in Table 2. In the first six months of 1945 there were no fewer than 30 days when the wind was between SW. and W., when Milfield had stronger winds than any of the other stations. It may fairly be concluded that the frequent strong winds from this quarter are a purely local phenomenon.

TABLE 3. OBSERVATIONS ON THREE ' LOCAL WIND DAYS '

ample 1, 7 February, 1945.

ssage of fronts at Milfield	Cold front between 06 and 07 GMT. Occlusion between 14 and 15 GMT.
eostrophic wind	07 till 15 hr 250/40 (deg/mi hr^{-1}) After 15 hr 290/50
eather type	Unstable polar air, snow and hail showers

Surface wind

me	Milfield		St. Abbs		Charterhall		Acklington	
MT)	D	F	D	F	D	F	D	F
00	WSW.	7	SW.	4	SSW.	6G	SW.	4
00	WSW.	8	SW.	5	SW.	6G	— —	
00	WSW	9	SSW.	5	SW.	6G	SW. by W.	6G
00	WSW.	9	SSW.	5	SW.	7G	SW. by W.	7G
00	WSW.	7G	SW.	6	SW.	6G	SW by W.	7G
00	WSW.	7G	W	6	WSW	6G	WSW	7G
00	WSW.	8G	W.	6	—	—	WSW.	6G
00	WSW.	9G	W.	6	WSW.	6G	W.	7G
00	W.	7G	W.	6	WSW.	6G	W.	5G

xample 2, 15 October, 1945

ronts	None
eostrophic wind	At 12 GMT 300/15-20
eather type	Anticyclonic, stable air, warm fair weather

Surface wind

ime	Milfield		St. Abbs		Charterhall		Acklington	
	D	F	D	F	D	F	D	F
00	W.	4	W.	2	SW. by W.	3	W.	3
00	WSW.	5G	CALM		SW.	3	SW.	3
00	WSW.	4	CALM		SSW.	3	W.	4
00	WSW.	5G	CALM		SW.	4	WSW.	3
00	W.	5G	SW.	1	SW.	4	WSW.	3

xample 3, 13 May, 1945

ssage of front at Milfield	Occlusion between 14 and 15 GMT
eostrophic wind	Ahead of occlusion 220/40 behind occlusion 260/30-40

Surface wind at Milfield

Time	Direction	Force	mi/hr
1200	SW.	5	21
1400	SW.	6	27G
1500	WSW.	7	38G
1600	WSW.	8	41G
1700	WSW.	9	47G

D = Wind direction; F = Beaufort force; G = Gusty.

(*d*) *Occurrence of the local wind in different synoptic situations*

The charts for a number of ' local wind days,' when the wind blew more strongly at Milfield han at neighbouring stations, showed only one feature of the weather in common, namely, the irection of the geostrophic wind. The critical direction seems to be slightly south of west; 4 estimates were made, and varied between 230° and 330°; but 41 of them fell within the range 40° to 270° inclusive. As long as the direction of the geostrophic wind was within the critical ange the local wind would blow, whether the air mass above Milfield was warm or cold, stable r unstable. The wind records on three ' local wind days ' of very different weather types are iven in Table 3; similar records were made on many other occasions. Example 1 is particularly nteresting, as it shows the local wind occurring in deep polar air; it was a day of strong winds verywhere, but the strongest were reported from Milfield (e.g., Force 9 at 09 hr, when no other tation was reporting more than Force 6). It may also be remarked that the wind at Milfield uring the morning was veered from that at the other places, and, indeed, more veered than

might be expected from the geostrophic wind. Example 2 was chosen as a complete contra to 1; it was a day of anticyclonic weather and light airs, except at Milfield, where there we moderate winds all day. Example 3 shows the local wind appearing in response to a chan of direction of the geostrophic wind; the wind-veer at the passage of an occlusion resulted in marked increase in the speed of the surface wind, although the geostrophic wind had slight decreased in speed.

(*e*) *The question of katabatic effect*

The best-known cause of strong winds in hilly country is the katabatic effect, i.e. the flo of cold air down a slope under the influence of gravity. If the Milfield wind could be entire accounted for by the katabatic effect, it should blow most strongly during the early part of t night, and be accentuated when the ground is covered with snow. The wind observations we examined to see if either sign of a katabatic wind could be detected.

(i) Day and night winds showed no striking differences. The strongest wind of the d might occur at any hour of the twenty-four and, in the six months observed, it occurred mo often in daylight (107 occasions) than in darkness (74 occasions). The observations in Tables and 3 were all recorded in daylight, and show great differences in wind speed between Milfie and other places, which in itself makes it seem improbable that the Milfield wind is purely katabati A more detailed analysis did, however, make it seem possible that the local wind was sometim intensified by the katabatic factor. Curves of the mean wind speed at each hour of the da prepared for six successive fortnights between mid-January and mid-April, showed the expect maximum in the middle of the day, and minima at sunrise and sunset. But they were not quite expected, because rises in wind speed between sunset and midnight were sometimes observe It is probable that the katabatic effect comes into play on cold clear nights, and reinforces t main cause of the local wind.

(ii) Snow cover existed at Milfield for the twelve days, 18 to 30 January 1945. If the loc wind had been purely katabatic, this time should have been very windy indeed, especially in th early part of each night. But nothing of the kind was observed. They were days of variab and mostly light winds; the strongest winds observed were : Force 6 from WNW (21st, morning and Force 5 from NW (20th, evening), NNE (27th, evening) and from WSW at 11 hr, and ther fore in daylight, on the 21. 25 January was a day of flat calm throughout, which would ha been impossible had the wind been purely katabatic.

(*f*) *The wind and local topography*

The typical local wind at Milfield, as has already been said, does not blow straight dow from the mountain mass of the Cheviots, but along the bottom of its northern edge; and the is no characteristic cloud formation connected with it, as there is with the Helm wind in the Ede valley (Manley 1945). However, on one occasion only in 1945, something very like the clou bar of the Helm wind was seen at Milfield, but this was not on a day when the typical local wi was blowing; the wind was light southerly, blowing straight down the escarpment. During th morning, a roll cloud was seen about 1,500 ft from the ground, between the escarpment and th airfield. The cloud was evidently continuously rotating, moving away from the hill below, an back towards it at the top. Two pilots reported that there was violent turbulence in and nea the cloud.

Some observations on the extent of the local wind were made in the vicinity of the airfield and it was found to be on a comparatively small scale. In the area of the Till valley affecte by the wind, all the small trees are bent away from it, as they commonly are on sea coasts. Th strip of wind-bent trees is only about two mi wide and three or four long; it includes the airfiel site, and almost reaches the hills on the east side of the Till (see Fig. 1).

A few measurements of wind speed were made in the Glen valley, and up the side of th Cheviot escarpment. For this purpose three ascents were made of Yeavering Bell, a 1,200 hill which forms part of the escarpment, on the afternoons of days when the local wind ha been apparent in the morning observations. The first reading in each case was made right i the bottom of the valley, at Coupland bridge, followed by a series of readings on the open hillside any shelter, such as from bushes or stone walls, being carefully avoided. A check reading wa made at Coupland bridge on return, to make sure that the wind speed had not dropped.

The readings, which are given in Table 4, show that, on all three occasions, the wind spee on the open hillside between 200 and 900 ft above sea level was less than the speed at the botto

TABLE 4. READINGS OF WIND SPEED ON THREE ASCENTS OF YEAVERING BELL

	Height ft. ASL (approx.)	Wind direction and speed		
		23 July Deg/mi hr^{-1}	23 Aug. Deg/mi hr^{-1}	15 Oct. Deg/mi hr^{-1}
efore ascent Geostrophic wind at 12 GMT		300/40	270/25	300/18
Surface wind at airfield	150	300/24	280/23	270/20
osition		mi/hr	mi/hr	mi/hr
1 Coupland bridge	130	15·6	13·1	16·5
2 Terrace in valley	160	17·6	—	16·6
3 Hillside	200	9·9	—	—
4 ,,	300	—	11·2	—
5 ,,	400	9·9	—	11·2
6 ,,	500	—	10·8	—
7 ,,	550	14·1	—	—
8 ,,	600	—	11·5	11·3
9 ,,	700	14·9	12·6	14·3
10 ,,	900	21·6	—	17·5
11 ,,	1,000	20·9	—	—
12 ,,	1,050	—	—	13·5
13 Top of Yeavering Bell	1,182	39·0	—	20·5
14 As 1, on return	130	18·1	18·7	—
fter scent		Deg/mi hr^{-1}	Deg/mi hr^{-1}	
Surface wind at airfield	150	290/28	260/23	

N.B. Each figure for wind speed during the ascent is calculated from the average of three anemometer readings).

f the valley. The airflow which forms the Milfield wind seems to be less than 100 ft deep, and to be analized in the valley bottom. Incidentally, the wind speed at the top of Yeavering Bell, on the wo days when the top was reached, was found to be about equal to the estimated speed of the geostrophic wind.

4. Discussion

It has been pointed out by Brunt (1934) that descriptions of local winds originating in several different ways are included in meteorological writings under the general heading of ' mountain nd valley winds.' Unusually strong winds occur in many places among and near mountains, ut the causes which produce them are not yet fully understood. The katabatic effect has been ound, by Newnham (1918) and Heywood (1933), to produce light winds on cold clear nights n two places in England. It is generally believed that, in order to produce winds of gale force, he katabatic effect must operate over very mountainous, steep, snow-covered country, e.g., the dge of the Greenland ice-cap.

The local wind at Milfield is not purely katabatic, and the main cause of it must be some eature of the local topography. It is suggestive that the wind blows down a valley, and that he wind speed appears to be greater in the valley bottom than on the hillside above it; but the cale of the phenomenon is much smaller than that of the well-known valley winds. The *mistral* ffects some hundreds of square miles of country, and the *Genoa Maestral* described by Hare 1943) blows through a gap in the maritime Alps, which are high enough to act as a barrier to arge-scale movements of air; the Cheviots are scarcely of these dimensions.

The Milfield wind is on an even smaller scale than the Helm wind of Crossfell, described y Manley (1945). It also differs from the Helm wind, in that it blows along the bottom of a slope nd not straight down one, in having no characteristic cloud associated with it, and in not needing he presence of an inversion for its occurrence. This is shown by the fact that it occurs in deep olar air. (Example 1, Table 3). But in spite of its small scale, the local wind has a great effect n the local climate of Milfield, as it blows so often, and as the average speed there of any wind etween SW. and W. is about twice that which might be expected in a reasonably sheltered place.

Acknowledgments

I would like to thank the Director, Meteorological Office, London, for permission to ma use of official observations; the staff of the Meteorological Office at Milfield for friendly c operation, and Professor F. K. Hare for a most illuminating discussion of the problem.

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551.577.35 : 551.5

24-hr tendencies and weather trends at Singapore

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(Manuscript received 29 March 1955, in revised form 2 May 1955)

Summary

Rainfall over Singapore, on each day for a year, was compared with that of the preceding day and the d was classified according to the change of weather which had occurred. A 24-hr tendency parameter, availab on the previous day, was computed. A relation between the parameter and weather trends is demonstrate

1. Introduction

Pressure enters the expression of rate of change of horizontal velocity divergence mo importantly in the form $-\alpha \nabla^2 p$ (Scott 1951) in which ∇^2 is the Laplacian in the horizont surface, α is specific volume and p pressure. Hence, of all the pressure-field parameters, ∇^2 and its time derivatives appear most likely to give indication of weather trends. Over the perio investigated, daily values of $\nabla^2 p$, measured necessarily by finite differences between stations considerable distance apart, were observed to vary about monthly mean values which themselv varied considerably throughout the year. The variation of the monthly means was thought be associated with the seasonal variation of the wind field and its local action of producing l troughs and windward ridges. So that adequate comparison of a pressure parameter could b made throughout the year the 24-hr tendency of $-\nabla^2 p$ was used. This selection also eliminat the effect of any systematic errors of the barometers involved.

2. Day classification and pressure data

Hourly rainfall data from eleven autographic raingauges, well scattered over Singapor island, were available. Because the average time of observation of the pressure-data used w about 16 hr ZT (GMT plus 7 hr), each 24-hr period, 18 to 18 hr, was compared with its predecesso having regard to time and space distribution and intensity of rainfall, and was classified int one of four types. These were :

B a substantial improvement (not necessarily a complete clearance),

D a dry or nearly dry day following a similar day,

S a day of no substantial change following a day which was not dry or nearly dry and

M a substantial deterioration.

e process of assessment was necessarily a subjective one. The principle adopted to obtain a easure of consistency in assessment was to try to visualize the majority opinion at 18 hr of a mber of weather observers supposed to have watched the weather closely in the previous 48 hr. sessments were made before the pressure data were computed.

Readings of barometers located at Kallang Airport (Singapore), Sitiawan (260 n.mi from gapore, bearing 310°), Trengganu (240/350°), Kuching (390/090°) and Palembang (270/170°) re used. To reduce the effects of meaningless random and periodic pressure variations, the ssure assumed at the outer stations was the mean of the 13 and 19-hr observations, while the ntral, most critical, pressure (p_0) was the mean of the 10 to 21-hr (inclusive) hourly pressures at allang. The larger distance between Kuching and Kallang was allowed for by reducing the uching pressure by one third of its excess over that of Kallang. No correction was made for the satisfactory position of Sitiawan.

The number $4p_0 - \Sigma p$, in tenths of millibars, where Σp is the sum (after the correction to uching's pressure) of the pressures at the outer stations, was computed daily. The 24-hr ndencies were obtained by subtracting previous days' values from current ones. A tendency value 5, operating constantly over 24 hr, may be shown to be enough to change the horizontal vergence by 10^{-5} sec^{-1} approximately; this value, according to Kindle (quoted by Forsdyke 49), if negative, is the upper limit for a condition of light rain. This figure is given here to show at the classes in Table 1 are not physically unrealistic.

3. Result

The frequencies of the various types of change, occurring after tendencies within three nges, are given in Table 1. There was some evidence that, after a substantial positive tendency, relapse occurred which did not have weather significance. This is shown by the subdivision the third column, where the figures after the positive signs are the frequencies of the various ange-types after negative tendencies which followed positive values $\geqslant 26$, but which did not ceed numerically the value of the positive tendencies they followed.

TABLE 1. Frequency table of 24-hr pressure tendencies and weather trends

	Pressure-tendency parameter			
Weather-change type	$\geqslant 6$	5 to -1	$\leqslant -2$	Total change-type. frequency
B	63	18	24 + 4	109
D	27	13	32 + 7	79
S	23	16	25 + 1	65
M	16	26	69	111
Total tendency-class frequency	129	73	162	364

Table 1 is of interest because it shows that it was possible to isolate a class of positive ndencies a considerable proportion of which were followed by *B* and *D* days and a class of egative tendencies showing a pronounced bias towards *M* days to follow, these two classes eing separated by one of moderate width showing no significant bias. The facts that the pressure oints used are separated by about 250 n.mi and the linear dimensions of Singapore island are f the order of 15 n.mi, and that, on the whole, fine-weather areas are much larger than those f bad weather can give an explanation of the superiority of the first-column results over those f the third column. A deterioration within the central part of the area between the pressure tations is more likely to miss Singapore than is an improvement there. Similar though poorer esults were obtained using two other sets of tendencies obtained from less-smoothed pressure ata, the results improving with increased smoothing.

4. Conclusion

The contents of Table 1 suggest that the almost universal meteorological opinion that the quatorial pressure-field is valueless to local forecasting is a pessimistic one. The positive-

tendency class for the period examined would have anticipated an improvement of weather, the persistence of dry weather, correctly on 90 occasions out of 129; a result which makes appear likely that further refinement of the pressure data might provide a class of positive par meter useful by itself to short-range forecasting. There appears less likelihood of a negativ tendency class performing a similar service, though many of the large number of *D* days followi the negative-tendency class might turn out to be foreshadowable from considerations of th humidity in the lowest few thousand feet of the atmosphere.

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551.521.31 : 551.576 : 551.578.4

The brightness distribution of the overcast sky when the ground is snow-covered

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(Manuscript received 4 March 1955)

Summary

Measurements have been made of the brightness distribution of a densely overcast sky when the groun was covered with snow. The distribution proved to be more uniform than has been reported for norm ground conditions.

The distribution was found to be somewhat asymmetric, which may have been due to differences i cloud thickness or to the local influence of inter-reflections between the base of the cloud and the groun below.

The averaged data were found to be represented with reasonable accuracy by the empirical formula :

$$B_\theta = \tfrac{1}{2} B_Z (1 + \cos\theta)$$

where B_θ = brightness of the sky at zenith angle θ

and B_Z = brightness of the sky at the zenith.

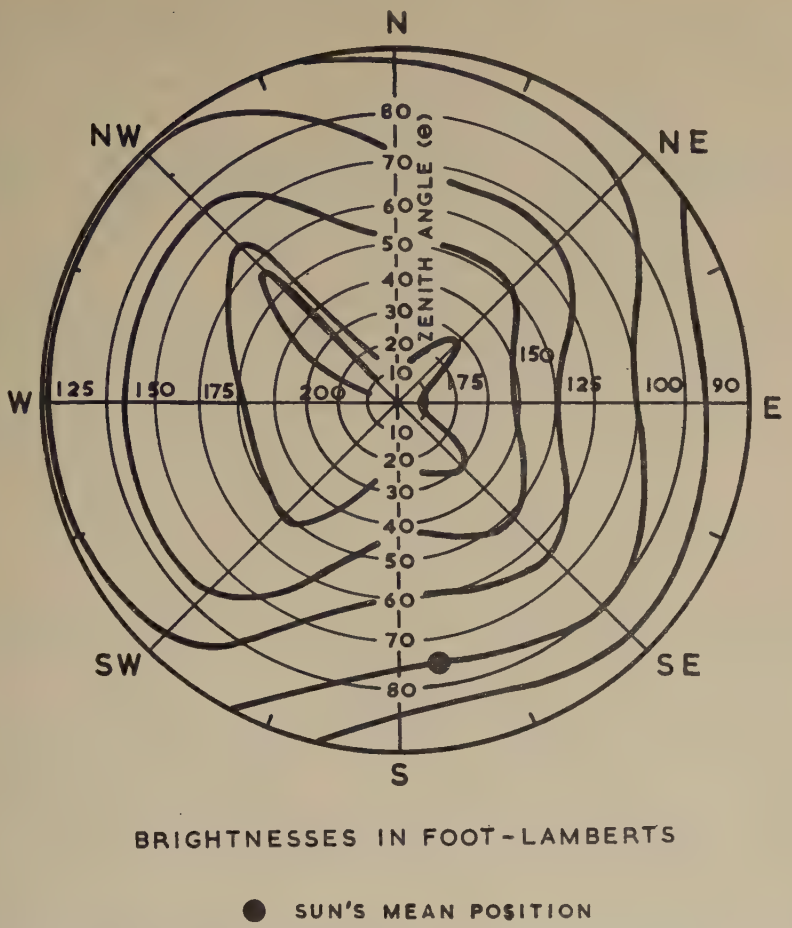
The reflection factor of the snow was measured and found to be 80 per cent.

The opportunity was taken after the snowfall on 4 Jan. 1955 to measure the distributio of brightness of the densely overcast sky which occurred at Garston, Hertfordshire (51° 42′N 0°23′W) on the following day. The measuring technique employed was similar to one alread reported (Hopkinson 1954).

The ground to the west of the measuring position, at the Building Research Station, i mostly open fields whereas to the east it is predominantly woodland. On the occasion of th measurements the whole terrain, except for the woodland, was covered with snow to a dept of 4 in. The reflection factor of the snow was measured under conditions of diffuse incide illumination and 45° viewing, and found to be 80 per cent.

The sky appeared visually to be completely overcast with no detectable unevenness in th cloud cover. Observations at two neighbouring meteorological stations showed 8/8 stratocumulus having a cloud base at 300-400 ft and a thickness of about 2,500 ft. Fig. 1 shows the brightnes distribution over the sky vault which was obtained.

The distribution is seen to be asymmetric about the zenith, there being slight irregularitie in the western sky. It is not known whether these irregularities were due to differences in clou thickness or to the local influence of inter-reflections between the base of the cloud and the ground below.



gure 1. Equal-brightness contours corresponding to a densely overcast sky and snow-covered ground.

It can be shown that the averaged data of Fig. 1 fit a formula of the same type as one derived om sky-brightness measurements with normal ground conditions (Moon and Spencer 1942). he Moon and Spencer formula can be expressed in the form :

$$B_\theta = B_Z (1 + k \cos\theta)/(1 + k) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

ee, for example, Middleton 1952)

here B_θ = brightness of the sky at zenith angle θ.

B_Z = brightness of the sky at the zenith.

d k has the value 2·0.

The averaged data of Fig. 1 for an overcast sky with snow-covered ground fit the same formula k is given the value 1·0

$$B_\theta = \tfrac{1}{2} B_Z (1 + \cos\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In other words, the sky-brightness distribution is more uniform with snow-covered ground an with normal ground conditions, the ratio of zenith to horizon brightness being reduced om 3 : 1 to 2 : 1 by the presence of the snow.

Clearly it is necessary to make a more extended series of measurements on a number of casions, if possible from a measuring position in completely open country, if a general formula the brightness distribution of the overcast sky under snow conditions is to be obtained.

Unfortunately the coincidence of conditions and opportunities occurs only rarely in southe England and so it was thought that the present series of measurements was not entirely witho value.

Acknowledgments

This work was carried out at the Building Research Station, D.S.I.R., and is published permission of the Director. Mr. J. Longmore assisted with the observational work.

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551.574.1 : 536.

Ice-crystal counts and the freezing of water drops

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(Manuscript received 24 February 1955)

Summary

An analysis of the concentrations of ice crystals observed in a cold box by Smith and Heffernan (195 in the course of 155 measurements is shown to provide support for values calculated for a given temperatu from an equation used by Bigg (1953), thus verifying the analogy between freezing of airborne droplets an freezing of drops suspended in an insoluble liquid.

In a previous paper (Bigg 1953) an attempt was made to compare the glaciation of clouds i cold boxes, expansion chambers and the atmosphere, using some experimentally derived equation for the freezing of water drops. The equations were deduced from observations on the freezir of drops suspended between insoluble liquids, and their application to cloud problems coul only be justified by the success of the explanations which they provided.

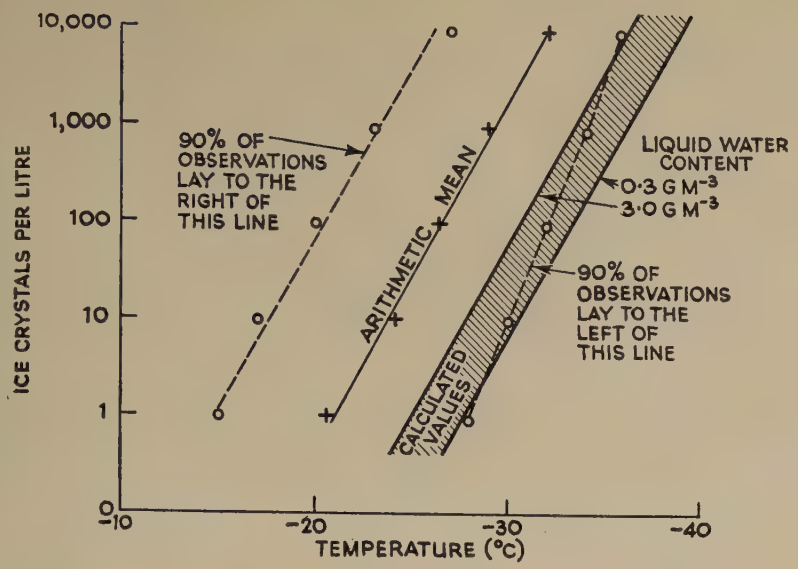
The importance of having equations which can predict glaciation under all condition in the absence of special freezing nuclei, lies in the elucidation of the nature of freezing nucl and the extent to which they influence rainfall, and it consequently seems desirable to call attentio to some results which provide verification of the form of the freezing equations.

It is important to show in this connection that the increase of probability of freezing wit temperature, as well as the absolute value, should be the same for airborne as for liquid-bor drops. The results of experiments by Smith and Heffernan (1954) in which ice-crystal coun were made in a cold box, mounted in an aircraft and cooled at 4°C/min, although not ide for the above purpose, are nevertheless capable of indicating if important differences exist.

In their experiments there were variations in ice-crystal counts from day to day whic may have been partly due to observational error and partly to the presence of freezing nucle As a result of the bias towards higher temperatures for a given ice-crystal count caused by th latter, the number of crystals calculated from drop-freezing experience should probably l somewhere between the mean and the lower limit found in the cold-box experiments. The slop of the mean curve of ice-crystal count as a function of temperature should be the same as tha calculated from the equations.

Eq. (2) from the 1953 paper can be modified for a constant rate of cooling to

$$\ln (1 - P) = - \frac{KV}{\alpha} (\exp aT_s - 1)$$



gure 1. Comparison of the concentration of ice crystals as a function of temperature observed by Smith and Heffernan and calculated by Bigg.

here P is the probability that a drop of volume V will freeze at a temperature T_s degrees below °C when cooled at α°/sec, and a and K are empirically determined constants, 0·82 and $2{\cdot}9\times10^{-8}$ spectively. For n drops, all of volume V, the most probable number frozen at a given tempera-ıre T_s is (for small P)

$$nP = \frac{Kw}{\alpha}\exp aT_s \quad \text{since} \quad \exp aT_s \gg 1,$$

being the liquid-water content.

The liquid-water content was not measured in Smith and Heffernan's experiments, but robably lay within the range 0·3 to 3·0 g m^{-3}, and, using this range, the shaded region in Fig. 1 nows the number of drops calculated to have frozen as a function of temperature. The observed urves, derived from cooling 155 samples of air, are also shown, and are seen to be consistent in orm with the calculated values which lie in the expected region. The slowing of the cooling ıte at temperatures lower than — 33°C would lead to a curvature of the calculated values not nown in the figure.

Assessing the possible errors in a cold box is difficult, for they depend on the ability of the bserver to reproduce the conditions of water content and visibility in the box and to estimate orrectly crystal concentrations, but it seems certain, from the wide spread to higher tempera-ures, that freezing nuclei have often played an important part in the formation of ice crystals. laving regard to the sources of experimental error, the facts that few of the results lie below the alculated values and that there is a close similarity between the slopes of the calculated and bserved curves seem to be a satisfactory vindication of the application of the drop-freezing quations to cloud problems.

Instead of regarding the Smith and Heffernan experiments as a verification of the freezing quation applied to laboratory clouds, suppose that we assume the equation to be valid. Then ıe experiments would indicate that there is a ' background count ' due to frozen drops uncon-ıminated by particles which materially affect their freezing, together with some ice crystals ccasioned by special nuclei. Alternatively, the extra ice crystals may have arisen by some ultiplication process such as fragmentation of a drop during progressive freezing. The caution ecessary in deducing the properties of ice-forming nuclei is emphasized by these considerations.

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551.509.317 : 551.557 : 519

The evaluation of some regression coefficients for estimating the 150-mb wind at Liverpool

By T. H. ELLISON and C. D. WALSHAW
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(Manuscript received 24 May 1955)

SUMMARY

The regression coefficients of the 150-mb wind at Liverpool in winter on (a) the 300-mb wind and th 500-mb wind jointly, (b) the 300-mb wind alone, (c) the 150-mb wind 12 and 24 hr previously, (d) the 150-m wind 12 hr previously alone and (e) 24 hr previously alone, have been computed from data for January an February 1950 to 1954 with the aid of an electronic digital computer. The single regression equations a found to be practically as useful as the double ones. 12-hour forecasts using the regression equation compa favourably with those made by orthodox methods.

1. INTRODUCTION

In an earlier paper (Ellison 1954), it was shown how regression equations could be develope for a random vector depending on two others or on one other only. The regression coefficient Ξ_{ij} in the latter case, H_{ij} and Z_{ij} in the former, are tensors which were found to depend in rather complicated way on the variance and covariance tensors of the vectors concerned. Th best estimate (assuming linear regression) of the vector deviation of the wind $\boldsymbol{\upsilon}'$, given only th deviation $\boldsymbol{\Psi}'$ at one other time or place is $\Xi_{ij}\,\psi_j'$; given the deviations, $\boldsymbol{\Psi}'$, $\boldsymbol{\omega}'$, at two times o places it is $H_{ij}\,\psi_j' + Z_{ij}\,\omega_j'$. It has now been possible to test the usefulness of these equation The upper winds at Liverpool, as reported in the *Daily Aerological Record* for the months Januar and February in the years 1950 to 1954 were used, and the regression coefficients for the 150-m wind on

(a) the 300-mb and 500-mb winds jointly,

(b) the 300-mb wind alone,

(c) the 150-mb wind 12 and 24 hr previously,

(d) the 150-mb wind 12 hr previously alone,

(e) the 150-mb wind 24 hr previously alone, were evaluated.

2. THE CALCULATION

Since the aim of the calculation was primarily to explore the practicability and usefulnes of the method rather than to determine definitive values of the quantities concerned, only moderate number of data for one station were treated. Even so, the calculation would hav been very formidable if done by hand. Fortunately it was possible to use the Cambridge electron digital computer EDSAC and this made the task very light.

The design of the computing programme was influenced by the form of the primary da and by the limited storage capacity of EDSAC, combined with the desirability of short run on the machine. A number of missing observations inevitably broke the primary data int sequences of varying length. As only fairly long sequences could conveniently be used fo computing the relating between the wind and its values 12 and 24 hr previously, the sequence were divided somewhat arbitrarily into long (' L ') sequences with more than about 12 consecutiv observations and short (' S ') sequences. Only the ' L ' sequences were used for the calculatio of covariances involving a time lag (autocovariances), while both L and S sequences were use for the relations between the winds at different heights.

The first part of the calculation (Programme A) consisted of reading the primary data fro punched tape, converting from polar to cartesian co-ordinates and accumulating the require sums and cross-products (60 quantities) in the high-speed store. So that the autocovariance

buld be computed, the value of the 150-mb wind at each observation time was retained in the ore by the machine until those for 12 and 24 hr later had been fed in. Programme A was so onstructed that any number of sequences of data could be dealt with in succession and the results rinted as required. The output tapes (' A '-tapes) were retained for subsequent use.

The second part of the calculation (Programme B) was the trivial but necessary operation f reading a number of A-tapes, summing the corresponding numbers on them, and punching new A-tape for the totals.

The last part of the calculation (Programme C) involved two stages : in the first, the sums f products were adjusted to give the variances and covariances about the means, which were rinted; in the second, the matrix algebra needed to find the regression tensors was performed. or this last purpose an interpretative subroutine (Wilkes, Wheeler and Gill 1951) was constructed o perform the operations of matrix addition, subtraction, reciprocation, multiplication and con- raction. A programme for the matrix algebra could then be formed very simply.

One slight complication concerning the autocovariances must be mentioned. When dealing with an L-tape of n observations, programme A treats the autocovariances in the same way as he covariances between the winds at the different levels, and so computes $\sum_{r=1}^{n} v_i^{(r)}$, $\sum_{r=1}^{n-1} v_i^{(r)}$ nd $\sum_{r=1}^{n-2} v_i^{(r)}$. But the quantities required for correcting the autocovariances to the means are $\sum_{=3}^{n} v_i^{(r)}$, $\sum_{r=2}^{n-1} v_i^{(r)}$ and $\sum_{r=1}^{n-2} v_i^{(r)}$. In order to have these sums available with the minimum omplication of the programme, it was arranged to print $v_i^{(1)}$ and $v_i^{(2)}$ at the beginning of ach A-tape. Programme C was thus able to make the correction to the means in the ppropriate manner.

The time taken for programme A to calculate the sums and products for a typical set of data or 20 times of observation was 3½ min, approximately equally divided between reading the

TABLE 1. REGRESSION COEFFICIENTS FOR 150-mb WIND

The coefficients refer to deviations from the means given in the lower part of the Table.

			i, j				number of observations	total* correlation number
			1, 1	1, 2	2, 1	2, 2		
(a) Partial regression coefficients	on 300-mb wind	H_{ij}	0·33	− 0·01	0·02	0·35	416	0·81
	on 500-mb wind	Z_{ij}	0·30	0·02	− 0·01	0·23		
(b) Regression coefficient on 300-mb wind alone		Ξ_{ij}	0·53	0·00	0·03	0·50	416	0·80
(c) Partial regression coefficients on 150-mb wind	12 hr previously	H_{ij}	0·78	0·09	− 0·09	1·09	190	0·82
	24 hr previously	Z_{ij}	0·02	− 0·07	0·06	− 0·32		
(d) Regression coefficient on 150-mb wind, 12 hr previously, alone		Ξ_{ij}	0·79	0·03	− 0·05	0·83	190	0·81
(e) Regression coefficient on 150-mb wind, 24 hr previously, alone		Ξ_{ij}	0·64	0·03	− 0·03	0·58	190	0·60

Means for (a) and (b) : 500-mb wind = (− 15·1, 10·3) = 304°, 18·3 kt
300-mb wind = (− 20·1, 17·2) = 311°, 26·5 kt
150-mb wind = (− 20·7, 16·3) = 308°, 26·3 kt
for (c), (d) and (e) : 150-mb wind = (− 18·6, 17·9) = 314°, 25·8 kt

* See Ellison (1954). The definitions of the *partial* correlation numbers given in Ellison (1954) are not correct, since $H_{ij}\,\rho_{ij}$ nd $Z_{ij}\,\tau_{ij}$ are not both necessarily positive. A more correct definition would be

$$[\mu\,(\upsilon\,;\,\psi \mid \omega)]^s = \frac{(H_{ij}\,\rho_{ij} + Z_{ij}\,\tau_{ij})\,H_{pq}\,\gamma_{qm}\,H_{pm}}{\phi_{nn}\,(H_{rs}\,\gamma_{st}\,H_{rt} + Z_{rs}\,\varkappa_{st}\,Z_{rt})}$$

This change does not affect the definition of the *total* correlation number.

numbers and doing the calculation, and punching the output tape. Programme C took just over 4 min; 120 sec for the input of the programme, 35 sec for reading numbers, 15 sec for matrix algebra, and 80 sec for punching results.

3. Results

The most striking feature of the results, which are set out in Table 1, is the very slight difference between the total correlation numbers for the regressions with two independent variables, and the best of those for the regressions with one independent variable. It is clear that in the case of the relations treated here, a single regression equation is scarcely inferior to a double one. Of course, it does not follow that this would be the case if a different problem had been considered (e.g., the prediction of the wind at Liverpool given that at Aldergrove and that at Hemsby).

It is difficult to estimate the probable errors in the regression coefficients, as this depends on several unknown factors; it seems likely that the last figures in the values given above may be in error by a few units.

The values of the autocorrelation numbers are in good agreement with those found by Durst (1954) using a rather different approach.

In order to test the practical use of the single regression equations (b) and (d), they were employed to estimate the 150-mb wind at 15 hr on each Wednesday in January and February 1955 (no data from 1955 had been used in calculating the coefficients). The results of this test are set out in Table 2. The figures in brackets are the vector differences between the winds to which they relate and the observed 150-mb wind at 15 hr. The last row sets out the root-mean-square values of these differences which would be expected on the basis of the 1950-54 data. The agreement between the observed and expected RMS deviations is excellent.

TABLE 2. Results of tests of regression of 150-mb wind on 300-mb wind and also on its own value 12 hr previously.

Winds are given in degrees and knots. The figures in brackets are the differences between the wind to which they relate and the observed 150-mb wind at 15 hr. The last row sets out the RMS values of these differences which would be expected on the basis of the 1950-1954 data.

Date	150-mb wind at 15 hr	Difference from mean	300-mb wind at 15 hr	Estimate of 150-mb wind at 15 hr based on 300-mb wind	150-mb wind at 03 hr	Forecast of 150-mb wind at 15 hr based on value at 03 hr
5.1.55	260 10	(22)	130 33 (40)	122 03 (12)	270 09 (2)	282 12 (5)
12.1.55	270 75	(55)	280 116 (44)	281 73 (15)	280 84 (16)	285 72 (19)
19.1.55	300 50	(23)	300 24 (26)	304 27 (22)	300 56 (6)	301 49 (2)
26.1.55	210 41	(52)	210 55 (14)	238 30 (20)	220 23 (19)	236 21 (24)
2.2.55	220 13	(29)	230 22 (9)	178 20 (16)	180 27 (19)	195 21 (11)
9.2.55	310 52	(25)	290 62 (22)	293 45 (16)	290 55 (19)	294 49 (13)
16.2.55	330 60	(37)	330 56 (4)	323 41 (20)	010 47 (39)	005 41 (35)
23.2.55	240 23	(28)	250 46 (24)	272 34 (17)	270 23 (12)	278 23 (15)
RMS deviations		36	27	18	19·5	18
RMS deviations expected on basis of 1950-54 data		30	29	18	18	17

The RMS error of the 12 hr forecasts given in the right-hand column of Table 2 (18 kt) is roughly equal to the apparent geostrophic departure on working charts (Murray 1954) which indicates the minimum error likely to be obtained by orthodox methods and so forecasts made by the regression method compare favourably. The fact that the autocorrelation falls off rapidly as the time interval is increased (and it must be remembered that it is its square which is significant), in this case from 0·81 at 12 hr to 0·60 at 24 hr, means that the regression equation is unlikely to be of value for forecasts for periods longer than 12 hr. However, a forecast using a regression equation can be prepared, perhaps with the help of suitable graphical aids, in a few seconds after the relevant data become available and without the need to plot any charts. This gives the method some initial advantages over orthodox techniques. In using regression equations for forecasting, their limitations must be borne in mind : they always predict that the wind will change towards its mean value and cannot predict the onset of extreme conditions.

As a by-product of computations of variance and covariance tensors, some valuable information could be obtained about the general circulation of the atmosphere and the meridional exchange of angular momentum (Starr 1954). While it would not be proper to draw conclusions from the small amount of data treated in this investigation, it is perhaps of interest to list the variance tensors which were found; they are given in Table 3.

TABLE 3. VARIANCE TENSORS AND MEAN WINDS AT LIVERPOOL FOR JANUARY AND FEBRUARY, 1950-1954

	150 mb	300 mb	500 mb
Variance tensors (kt^2)	$\begin{bmatrix} 446 & 75 \\ 75 & 437 \end{bmatrix}$	$\begin{bmatrix} 881 & 243 \\ 243 & 1214 \end{bmatrix}$	$\begin{bmatrix} 537 & 171 \\ 171 & 657 \end{bmatrix}$
Components of mean winds	(− 21·7, 16·3)	(− 20·1, 17·2)	(− 15·1, 10·3)

4. CONCLUSIONS

The evaluation of variance and covariance tensors from upper-wind observations can be performed with great ease on an electronic digital computer. Such a machine will also readily handle the matrix algebra necessary to determine the regression coefficients for the regression of one vector on two others. However, in the particular examples examined, it was found that the use of two independent variables had negligible practical advantage over that of only one.

Forecasts of the 150-mb wind based on its value 12 hr previously can be prepared extremely rapidly and are of comparable accuracy with those prepared by orthodox techniques.

Much valuable information about the statistics of winds and the general circulation of the atmosphere would be obtained if variance and covariance tensors were computed in quantity, as could perhaps be done by the official meteorological services.

5. ACKNOWLEDGMENT

The authors are grateful to the Director of the Mathematical Laboratory, Cambridge University, for granting the facilities for the calculation on EDSAC, without which the work would not have been possible.

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Correspondence

551.54

'Gravity waves in the atmosphere'

By Earl Gossard and Walter Munk

Mr. I. J. W. Pothecary's interesting article (Pothecary 1954) anticipated by a month some of our observations and conclusions (Gossard and Munk 1954).

There are apparently three methods for obtaining the phase velocity of coherent internal waves : (a) by a network of stations, (b) by what we have called the impedance relation (our Eq. 1) and (c) by internal wave theory. Mr. Pothecary used (a) and (b) and we used (a), (b) and (c). We wish, therefore, to point out the close agreement of Mr. Pothecary's observations with (c). He estimates the average height of the stable layer to be 1650 m, and the difference of potential temperature across this layer is about 7°C. Shallow 'water' internal wave theory (our Eq. 16b) then yields a phase velocity of 20 m sec^{-1}. The other two methods gave 21 and 22 m sec^{-1}.

We have also pointed out that the direction as well as speed of propagation can be obtained from records of a single station by simple vector techniques. Using the first waves of the Abingdon record we find them to be propagating from 240° in good agreement with the orientation of the isochrones obtained from Pothecary's grid of stations.

In our paper we suggested that the observed periods of the oscillations were slightly lower than Brunt's period (1927) for the lapse rate above the inversion, though Martyn (1950) suggested that it should be equal to Brunt's 'just trapped' period. Examination of the initial-value problem offers a reasonable explanation of the observed shorter periods, however. In spite of the large ratio of wave length to depth, dispersion is found to be important when stability above the inversion is considered, and a maximum group velocity is found which is associated with a wave front (Eckart 1948) where the amplitudes are a maximum.* Let T_F designate the period at this front and T_B Brunt's period. For a stable lapse, T_B/T_F approaches $(3/2)^{\frac{1}{2}}$ as a limit; for smaller stability the ratio is less, and for an adiabatic lapse it goes to unity as both T_F and T_B become infinite.

It is difficult to apply these considerations to Pothecary's Larkhill sounding because the lapse is so nearly adiabatic. Our estimate for the Larkhill sounding is 0·9 of the adiabatic lapse. This gives $T_B = 20$ min, and $T_F = T_B/1{\cdot}15 = 17{\cdot}5$ min. The observed values were 17, 12·5 and 20 min.

Mr. Pothecary mentions the possibility of gravity waves having the effect of forming or dispersing stratus clouds in air near saturation. An oscillation of solar radiation closely correlated with the oscillation of wind and pressure has been observed in Southern California (Fig. 1). This oscillation is presumably due to oscillations in the thickness of the stratus deck.

We have also found that the oscillations can usually be classed as one or the other of two types depending on duration, character and, apparently, origin. Fig. 2 shows oscillations occurring in fair weather but associated with a storm off shore. Their dying out appeared to be related to the destruction of the Southern California inversion as the storm developed. Apparently this type of oscillation requires storm activity for generation, and a low inversion for propagation. Here, in California, this is a rare combination of events, and so is the occurrence of good examples of these complex oscillations. In the absence of an inversion we seem to find, only, simple pressure surges. Coherence of individual waves seems to extend over about 30 mi, but coherence of wave groups has been noted well over 100 mi.

The other type of oscillation is quite regular and of short duration (Fig. 3). It is a fair-weather phenomenon often associated with a föhn condition which in this area is called a 'Santa Ana.' The oscillations often occur during the night, and usually in the early morning hours as the Santa Ana regime begins to break up. The inversion is low and well developed, leading to low phase velocities and thus large wind oscillations. The neat trace is suggestive of a single, simple source such as a night convection-cell, perhaps triggered by simultaneous föhn heating below and radiational cooling aloft. However, vertical shear across the stable layer is common in these cases and should not be ruled out as a possible source. In Fig. 3 the recording station was located at about the base of the inversion, so that the oscillations are seen in the temperature and humidity traces also.

* This wave 'front' is often very abrupt, and in the case of instruments with pen-drag may give the appearance of a pressure jump, as in Pothecary's Farnborough and Abingdon traces.

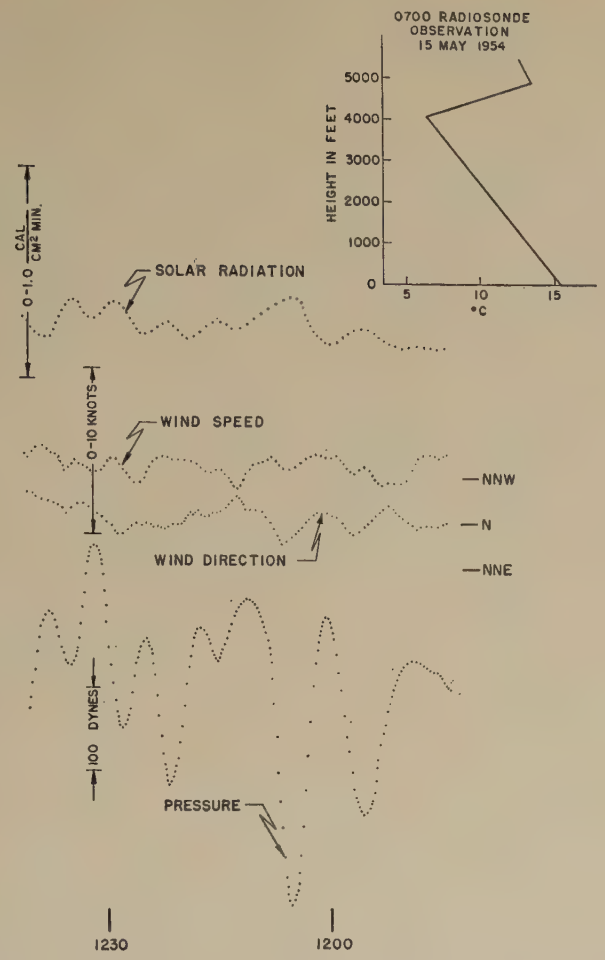

Figure 1. Meteorological record of gravity waves showing also fluctuations of solar radiation through a thin stratus deck.

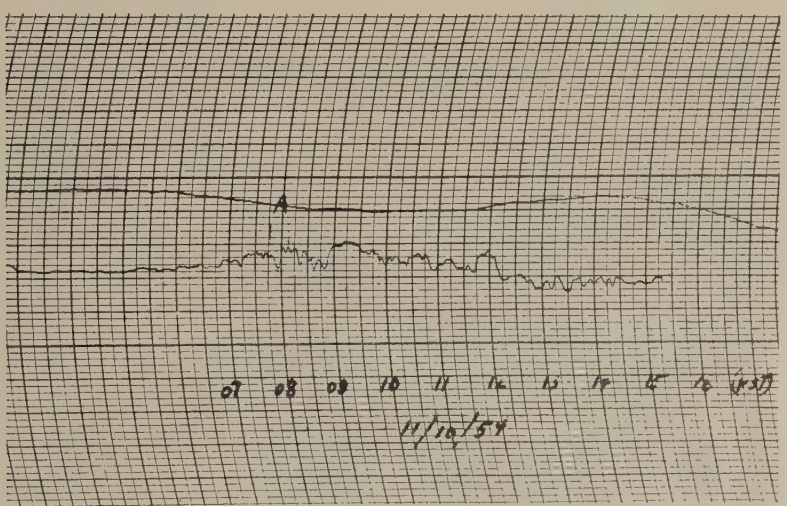

Figure 2. Oscillations associated with a storm centred some 500 mi WSW. of San Diego.

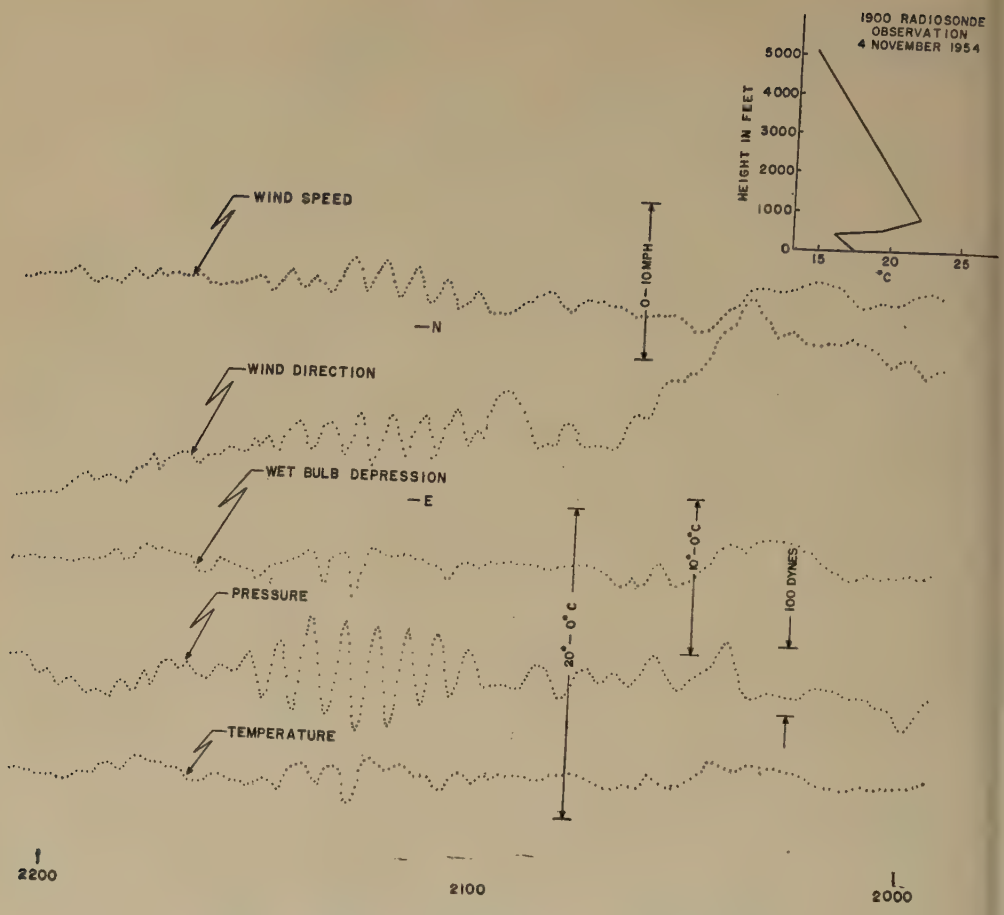

Figure 3. Oscillations associated with a local föhn condition.

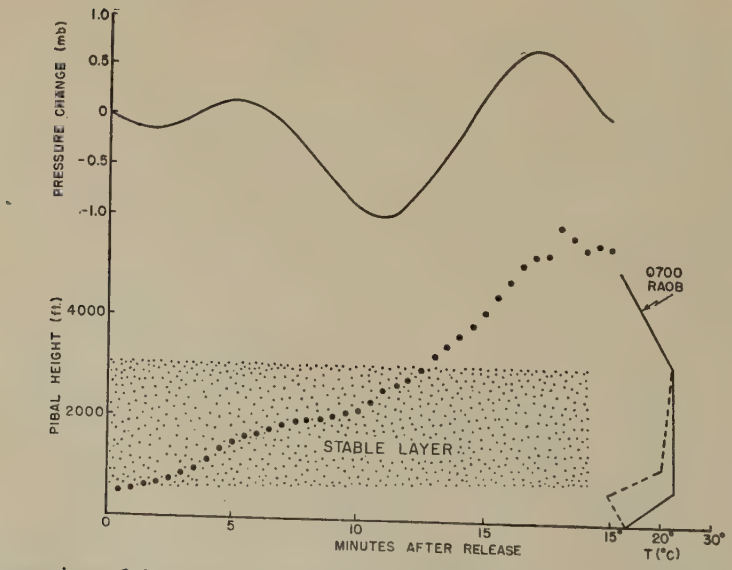

Figure 4. Observations of slow-ascent pilot-balloon through wave 'A' of Fig. 2, together with pressure record. The dashed 'RAOB' curve is the best estimate of the temperature distribution at the time the pibal was taken. The inversion was rising rapidly and its base passed the release point at about 0745 hr.

A slow-ascent pilot-balloon record, taken by a method described by Bellaire (1955), was btained through wave A of Fig. 2. The plot of balloon height vs. time, shown in Fig. 4, indicates at the oscillation was largely in the fundamental mode.

dded in proof:

While the above note was in press we had the good fortune to observe the waves visually. On May 1955 the temperature and humidity records showed a striking front of marine air travelling inland, llowed by a wake of regular wind and pressure oscillations of 7-min period. At the same time a fog bank 0 ft above the sea level and silhouetted against the Ysidro mountains revealed a series of travelling cillations. The largest oscillation had amplitudes of the order of 200 ft. The outstanding feature was e peakedness of the waves. Their sharp crests separated by long, flat troughs gave the appearance of a ng swell in shallow water near the breaking point. The period could be estimated directly and was accord with the recorded period.

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cripps Institution of Oceanography,
a Jolla, California, U.S.A.
8 February 1955.

Reply

By I. J. W. Pothecary

In their original paper (Gossard and Munk 1954) the authors drew attention to a general neasure of agreement between the observed period and a period given by $2\pi/s$ where s, the funda-nental mode of the longest trapped waves, is derived from $s^2 = g\dfrac{d(\ln\theta)}{dz}$ which is an index f the stability above the inversion.

At 03 hr, 6 July 1952, the lapse rate at Larkhill through about 75 mb above the inversion at 30 mb was 16·3°F/km. This corresponds to a value for $\dfrac{d(\ln\theta)}{dz}$ of $3{\cdot}59 \times 10^{-8}$, and gives for s^2 value of $3{\cdot}52 \times 10^{-5}$.

The derived period of the longest trapped waves is then 1058 sec, which is in good agreement vith the values of 1020, 750 and 1,200 sec measured at Farnborough, Abingdon and Croydon espectively.

Meteorological Office,
Dunstable, Beds.
April 1955.

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Discussion meeting on ' Airflow over mountains,' 19 January 1955

As already reported on p. 296 of this volume, a discussion meeting on ' Airflow ove mountains,' opened by Mr. G. A. Corby, took place in the Society's rooms on 19 January 1955. A summary of Dr. J. Förchtgott's paper and the general discussion are given below.

The measurement of airflow deformation behind mountains

By J. FÖRCHTGOTT

Aeroklub Republiky Ceskoslovenske, Prague

(Presented by A. H. Yates)

SUMMARY OF MAIN POINTS

More systematic observations of airflow in the lee of mountain ridges are required for th appraisal of theoretical work, for forecasters to gain experience of the relation of the flow to th synoptic situation for the inclusion of information in flight forecasts, for the planning of flight in mountainous regions, and for the planning of research in chosen areas.

To assist in the recording of phenomena, waves are classified into three types :

(i) Downwind wave : a series of lee waves whose intensity decreases with altitude, quasi stationary regions of strong turbulence depending on the mountain height, often with a periodi regeneration described below; produced by the lee slope.

(ii) Obstacle wave : a smooth deformation above the obstacle, common when there is pronounced inversion with wind increasing with height above; no turbulence; often accompanie by continual release of thermals from the windward slope which produces it.

(iii) Composite wave : combination of (i) and (ii). Conditional instability at higher level seems important for this; more frequent in regions of high mountains.

Clouds are good indicators of wave crests but, if air is saturated throughout, or if there ar no clouds, observations must be made using a variometer. In a downwind wave, the low-leve rotor-like clouds and the upper bands of billow clouds often change their position over fairl short periods. They move slowly downwind and then make a quick leap against the wind, on account of the periodic release of eddies with a nearly stable flow-pattern in between. At th same time periodic changes of surface wind and pressure are observed at places in the downwin region.

These changes can be thought of as the periodic release of circulations from the lee slope each eddy passing through alternate stages of motion and rotation. The motion becomes insigni ficant at about one wavelength from the ridge, where the rotation may be seen in rotor clouds The rotation then suddenly weakens and the eddy increases its horizontal motion, the rotatio becoming dominant again, for a short time, in the position of the second wave, and this may be repeated several times. Sometimes successive eddies maintain a quasi-stationary pattern o flow in the lee of the ridge.

The temperature is less in the wave crest and greater in the wave trough than in between where it is equal to that of the undisturbed stream. These temperature anomalies can be measure by glider if the system is stationary, if the flights are planned to make measurements in ascendin and horizontally moving air during the ascent, and in the downcurrent during a rapid descen from the greatest height reached. They can also be measured by radiosondes released either simultaneously or in quick succession from suitably chosen sites downwind of the ridge. The amplitude of the wave can be deduced from the temperatures measured.

The use of specially designed blank forms for the reporting of wave phenomena would help This is particularly true of soaring flights which have been and will be the main source of in formation provided that pilots have suitable means for recording the phenomena. Since soaring flights exploit the conditions in question it is better to obtain as much information as possible from them as they occur, rather than by short-period investigations.

Mr. C. E. Wallington: I was fortunate enough to accompany Mr. G. O. Smith on a glider flight in a lee wave ahead of a warm front at Camphill, Derbyshire, on a dull summer morning last July. We found that flight conditions were extremely smooth despite a fresh westerly wind blowing across the steep escarpment forming the western edge of the flying field. A steady updraught took us to 6,000 ft in 10 min and from this height we were able to make some useful observations of the broken low cloud beneath us and the cover of altostratus above.

These observations supported Dr. Scorer's theory of lee waves, but perhaps some of the features arising out of the theoretical investigation may be of more interest than the simple comparison between observations and theory. For example, the method of investigation underlined the need for considering the air flow as a composite wave pattern in regions where no single ridge or valley dominates the surrounding terrain. Bearing this in mind, we must not be surprised to find wave clouds or apparent waves changing their shape or position quite erratically; such changes can easily be attributed to gradual changes in wavelength. This view may also suggest that waves are damped more by mutual interference than by turbulent dissipation.

Considering the general nature of lee waves ahead of a well-defined warm front from a theoretical standpoint it seems that waves are likely in two zones. Some 400 to 700 mi ahead of the front, the nature of these waves may well contribute to turbulence at medium or high levels (see p. 251 of this volume).

Mr. H. H. Rosenbrock: The work with which I have been associated at the Electrical Research Association is concerned with a different aspect of airflow from that dealt with by other speakers. We are interested in the flow within some 200 ft of the summit of hills mainly less than 1,000 ft high, with the object of finding how much energy might be produced by wind-driven electric generators. The hills which have been studied are generally smooth in contour with moderate slopes. A great deal of practical experience of wind flow under these conditions has been obtained by measurements taken in the field.

On the theoretical side little work has been done, but we are aware that two mechanisms are at work in speeding up the airflow over the summit of a hill. In the case of a long ridge lying across the wind, the streamlines near the summit will be compressed, indicating an increase of speed. With an isolated hill, air from the lower levels over the surrounding plain may go round the hill rather than over it, and the air at, say, 100 ft above the summit may have come from several hundred feet above the plain, and therefore have a greater speed. These two effects are mutually exclusive: if we compel air to flow over a hill in order to obtain the first effect, we automatically get air from lower layers and so lose the second effect. It is found in practice that long ridges lying across the prevailing wind are not necessarily better sites for windmills than isolated hills.

It would be interesting to have the views of other speakers on whether the flow conditions disclosed in Mr. Corby's paper are likely to affect greatly the conditions near the summit of the hill.

Dr. R. S. Scorer gave a short summary of his paper published in this issue (p. 340).

Capt. F. Foster: I will describe two examples of waves met in normal airline flying and one in sailplane flight.

In my experience, waves in the Auvergne district of Central France produce vertical components in excess of 1,500 ft/min. In this area they can be dangerous if flight is parallel to the mountain range.

A second example of wave formation of short wavelength, (*c.* ½ mi), has been experienced in NW. winds between Elba and Nice. Wavelet clouds show that the steady wave-flow becomes turbulent in places. This is borne out by outbreaks of roughness during flight through these areas.

As an example of vertical components found in a wave train during sailplane flight from Great Hucklow, Derbyshire, the third wave, (Sheffield) in the train was found to produce 2½ ft/sec at 8,500 ft, and the eighth (Brough Airfield on the Humber) 2½ ft/sec at 2,000 ft.

Mr. J. S. Sawyer: Clearly the understanding of the nature of airflow over mountains is of the greatest importance to the interpretation of the distribution of orographic rainfall. However, the period over which rainfall measurements are usually made is at least 12 hr (more often 24 hr) and over such periods many changes are likely to take place in the strength, direction, stability and wind profile in an air current crossing a range of hills. It is therefore only on special occasions that the standing waves which have been described by Mr. Corby will hold their positions long enough to be shown in the distribution of measured rainfall despite their potential influence on it. Since this scale is small, 2-10 km, say, a small shift in position will blurr their effect and, in any case, a very dense network of raingauges would be needed to observe it. The

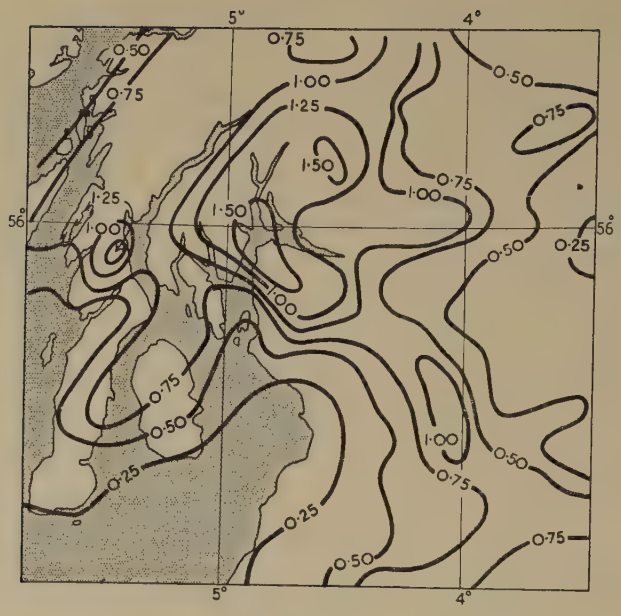

Figure 1.

only rainfall patterns which I have seen clearly suggesting standing wave effects are those published by Prof. Bergeron (*Tellus*, **1**, p. 15, 1949) which are suggestive of the effect of a standing wave set up by the coast of Holland on a steady south-west airstream.

In a paper which I read to the Society some time ago (*Quart. J. R. Met. Soc.*, **78**, p. 231, 1952) I discussed the orographic rainfall of two synoptic situations and obtained a general agreement between the rainfall in west Wales and in Devon, and values calculated on the assumption that the air was lifted at all levels and that the lift fell off upwards to zero at a level between 700 and 500 mb. Since most of the water vapour is condensed from the lower layers the calculations are not very sensitive to the level at which it is assumed the vertical lifting ceases; even if upward motion extends only to 850 mb almost half as much water is condensed as if the lifting extended to 500 mb. It is probably for this reason that rough agreement was achieved because even if we ignore the rather small-scale pattern of standing waves and consider the movement of air over a wide range – wide enough to ignore the horizontal air accelerations – the vertical pattern of the flow and depth of the layer of upward motion are still sensitive to the wind and temperature profile and in particular to Dr. Scorer's parameter, l^2.

In one of the cases which I studied the rainfall pattern over Scotland showed an extraordinary complication which is inexplicable on the basis of the synoptic situation. This is illustrated by the detailed rainfall map for the 24-hr period ending 09 hr, 15 Feb. 1950 (Fig. 1) and must have been due primarily to orographic effects. As the maximum is not over either the highest ground or the windward slope, but extends to low ground in the lee, it could only be explained by a detailed understanding of the dynamics of the flow over the mountains.

One further point is of interest. During the course of a study of the rainfall from depressions on various tracks across the British Isles, I extracted the rainfall at some high-level stations both north and south of the track. Whereas the rainfall at the high-level stations south of the track was on the average nearly three times greater than that of neighbouring low-level stations, that at the high-level stations north of the track showed very little increase over low ground. This difference may in part be due to the lower wind-speeds on the north side of a depression, but it is greater than I would have expected from this cause. Dr. Scorer has pointed out to me that it may be a dynamical consequence of the fact that the easterlies north of the depression usually decrease with height, producing an air current in which lifting by the hills is confined to a much shallower layer than in the westerlies to the south of the centre.

Prof. G. Manley : With reference to the statement that no example of a ' rotor ' or ' rotating bar ' cloud had been reported in Britain, I should like to emphasize that the ' helm bar '

ociated with the Crossfell escarpment (*Q. J.*, 1945, pp. 197-219) makes at times a most con-cuous example of this sort. (*Added later* : I have since been informed by Dr. Jane Meiklejohn t, from an airfield on which she was stationed in Northumberland, she observed, on a single asion, during the war, a ' bar ' with a south wind blowing across the Cheviots). Whether ation is always fully developed is difficult to say from ground level; I have observed the most finite rotation when the depth of the stream over Crossfell, as indicated by the height of the elm ' cloud was relatively small, so that the bar stood within 3 mi of the crest of the range.

Förchtgott's (Corby, *Q.J.*, p. 517, 1954) case (*a*) is illustrated in my 1945 paper, Fig. 9. I nk his case (b) can often be observed if the lee slope is relatively steep by watching the wavelets tarns in well-defined corries when a strong wind is blowing directly over the crest. I have t seen it on Crossfell; I do not think the lee slope is steep enough. I find it noteworthy that he nts a wind of 12 m/sec over a ridge 900 m high to provide his ' wave streaming.' For Crossfell, out 600 m above the ground to leeward, my suggestion of 9 m/sec (1945, p. 207), appears to ree quite well with his scheme. Whether the case of the single helm bar, with no cloud further wnstream, is analogous to his ' rotor streaming ' I am not quite sure; one does not see a ' bar ' ing to a higher level than the ' helm ' in the manner depicted by the drawing of his case (d).

With regard to side eddies, I think these frequently develop when the wind is blowing a critical angle over a range such as Crossfell, in which short stretches may lie almost at right gles to the wind direction at times when the main trend of the ridge lies at 60° or less; in such ses a short stretch of ' standing wave ' is found while nearby no such wave develops. With east wind a rather confused situation of this sort developed in the lee of the summit of rossfell (map, 1945, p. 199) on 5 January this year. On 3 January, it might be mentioned at overriding lenticulars were visible from 50 mi to the southward.

It may not be known to Members that there is now excellent opportunity for further research. n top of Dun Fell there is now a radio-direction-finding station with abundant warmth and mfort at 2,780 ft, accessible by road to the summit. As far as I know little or no advantage as yet been taken of this remarkable meteorological opportunity.

Mr. H. S. Turner : There has been discussion of waves in frontal conditions, and, to void misunderstanding, I should like to emphasize that in the majority of the hundred cases have examined the conditions were non-frontal. The average pressure associated with waves ver the United Kingdom during the winter of 1953-54 was 1020 mb. Mother-of-pearl clouds ver Norway are related with an ozone content which also indicates ridge conditions. Previous scussion of evening waves may have given an impression that there is a marked diurnal variation wave frequency. This is not borne out by the cases I have examined.

I would like to refer to a possible connection between high ground and high-level turbulence. uring the forenoon of 3 Dec. 1954 twelve cases of turbulence were reported. I suggest that ese positions, all of which are over or close to the lee of high ground, are not accidental. The Vorcester ' Thum ' for 09 GMT which should represent the air concerned in most of the reports ows a stable layer at about 900 mb but this is less marked than is usual in wave cases and, ough the increase of wind with height is typical, I think that the air was not quite suitable r wave formation and that incipient waves may have broken up into turbulent flow.

If the relation between high ground and high-level turbulence is real, it follows that turbulence less common over oceans than over continents. This may be important in determining the tigue life of aircraft engaged on particular routes.

(*Contributed later*) : Professor Manley mentioned in the discussion that the helm bar was isible on 5 Jan. 1955. That afternoon the easterly winds at Aldergrove and Liverpool decreased y 5-10 kt between 950 and 500 mb. This condition suggests Förchtgott's rotor streaming nd may perhaps be typical of the helm wind. Further reports would be valuable

Mr. R. K. Pilsbury : I have had the opportunity of examining the reports of standing aves received at Northolt Airport, thanks to the excellent co-operation of pilots of British uropean Airways and Aer Lingus. I think the meeting will be interested in some statistics om these reports. The investigation was conducted at Northolt from October 1953 to April 954 and over 100 pro-formae were received from pilots, of which 87 were concerned with ights over the United Kingdom. The figures I shall quote relate only to these 87 reports, of hich 66 deal with waves and 21 with no waves experienced.

As regards the 67 cases of waves found, it would appear from a preliminary investigation hat certain upper-air conditions were present in practically every case, namely :

(a) The presence of an unstable layer from the surface to at least 2,000 ft; in some cases this ayer was 7,000 ft deep. We have four cases of no waves found when this layer was only 1,000 ft eep but other upper-air conditions were conducive to waves.

(b) The presence of a second layer above (a) where the lapse rate was less than that of th saturated adiabatic. The thickness of this stable layer varied from 800 to over 10,000 ft.

(c) In 62 cases there was a third layer, above (b) but below 18,000 ft, where the lapse ra was greater than the saturated adiabatic.

(d) The wind at 2,000 ft was at least 15 kt and in 61 cases greater than 20 kt.

(e) Wind direction was almost constant with height throughout the first and second layer

(f) In every case but one the wind speed increased with height at least to the top of th stable layer.

Waves were experienced at all flying heights at one time or another; for civil aircraft thes are normally 4,000-10,000 ft with a few cases over Wales at about 18,000 ft. During the perio Nov. 1953-Jan. 1954, waves were experienced on one day in three in November and Janua and one day in eight in December; they may of course have occurred on other days in area where our aircraft were not flying. They were widespread on six days. Reported wavelengt were of the order of 3-8 mi with the shortest 2 mi. (As the orientation of the track of the ai craft to the wave train is not known, these figures cannot be regarded as the shortest distan from crest to crest). Wave trains do not appear to persist for long distances, in general, as w have many reports of waves found over Wales but not experienced by pilots on the route Lichfie to Chester, although these latter pilots often found waves later in flight when over the Lak district.

In support of Dr. Scorer's theory it would appear that a pilot flying near the top of th stable layer experienced stronger vertical currents than pilots flying at lower altitudes. Upwar currents were generally stronger than downward ones. We received ten cases of vertical curren stronger than 800 ft/min; the strongest was 2,200 ft/min, one night off Gt. Orme's Head, whil another pilot found an up current of 1,900 ft/min near Dollar Law, south of Edinburgh on morning. In every one of these ten cases of strong currents, there was an inversion prese in the upper air profile whilst in six cases there was a double inversion. In 8 of the cases, th wind increased rapidly with height in the stable layer, to two or three times its value at 2,000 f They occurred by day and by night.

Turbulence was reported in 20 cases of the 66 but only two reached moderate values. examined these two cases and it looks as though the waves were either beginning to form or to d out, in the light of earlier or later reports of waves.

I also looked at the cases of no waves. Of these 21 cases, upper-air conditions were no conducive to waves on 6 occasions, whilst on three occasions the aircraft concerned probabl did not fly near to the lee of high ground causing waves. In 8 other cases variations taking plac in the upper air could possibly account for lack of waves. Thus I was left with two cases whe waves could have been expected but were not found. These were all the more peculiar as bot cases occurred when waves were widespread and found by other pilots.

Capt. K. D. G. Mitchell : I would like to add a few words based on my own experienc I always associate areas of lift and sink with flights over stratocumulus, never having encountere them at other times. As I have, on several occasions, had pronounced lift and sink with a strom NW. wind over the entire London to Belfast route – thus showing that waves can persist u to 60-70 mi to the lee of high ground – I am wondering if, in the instance previously quote of reports of lift over the Welsh Hills not being noticed by an aircraft over Lichfield en rout for Scotland, the wind was NW. such that the Scotland-bound aircraft was soon clear of th train from the Welsh Hills.

I have also noticed variations in temperature : one recent case was 20 Dec. 1954, agai London-Belfast with the normal 8,500 ft temperature of — 8°C. There were several marke rises above this to — 2°C and — 1°C but never did the temperature fall below the normal.

Finally I have often observed marked lift and sink over stratocumulus having an even clou top, yet found it absent over stratocumulus which has a marked and evenly undulating or creste top.

Mr. K. N. V. Townsend : As a forester I am particularly interested in the behaviou of the wind within 150 ft of the ground. The behaviour of trees in mountainous areas may be o assistance in investigating ' Airflow over mountains ' so this contribution will present the genera case as given by forest text-books, papers and personal observation.

The visible signs of exposure on trees certainly confirm some of the empirical informatio concerning the behaviour of winds in hilly areas.

1. As a class, trees are seldom blown down when less than 30 ft high; a few suffer thi between 30 ft and 45 ft in height but the greatest number of blown-down trees occurs when the are over 45 ft. This accords with Bilham's graph of wind speed against height above the ground

2. Wind velocity is usually decisive only when it is exceptional. High wind-speed areas enerally suffer no more damage than regions of much lower wind-speeds.

3. Trees on ridge tops are not particularly liable to wind-throw as there is an altitude ffect on exposed trees whereby height growth is less and other inherent stability factors come to the picture. If any of these trees are felled, however, taller trees a little lower down the lee ope sometimes fall.

4. Damage on lee slopes is nearly always worse than on windward slopes especially opposite a ap or pass. This is NW. American experience. In the January 1953 NE. Scotland gale, which was f an exceptional nature, most damage occurred on the windward slopes, but the majority of the lee opes in this case were greater than 8°. Damage is usually worse on lee slopes less than 8°.

5. Lee slopes nearly always show a sheltered area just below the summit.

On windward slopes wind-throw is common in the lower part of the slope but there are me examples of damage extending from foot to summit due to exceptional winds.

6. Isolated hills show damage on the lower part of the flanks rather than anywhere else.

General cases for gale damage liability in mountain forests can be made but exceptional inds are too devastating to reveal the pattern of wind flow and force using trees as indicators.

The whole subject is most complex.

Mr. G. A. Corby (*in reply, communicated*) : Mr. Wallington's calculations by perturbation heory of the airflow over a section of the Derbyshire countryside in an actual case show what an be done by way of approximating to real topography and real airstreams. His example emonstrates the great variations in the lee wave-length and amplitude which can occur owing to ariations in the airstream chacteristics and to interference between the wave trains from separate rographic features; this provides plausible reasons for the erratic behaviour of lee waves.

The observational evidence, arising from the research into the possibility of exploiting wind ower, that the increase in wind speed over the crest of an isolated hill is similar to that appro- riate to a long ridge is interesting and at first sight somewhat surprising. However, Mr. Rosenbrock's explanation is convincing and probably correct, although theory has not as et been taken far enough to confirm this. For this purpose it may be useful to note that the flow ver inland hills would often be affected by interference caused by other hills upwind. This would ot be so in the case of hills facing seawards towards the prevailing wind direction.

The importance of separation has in the past only been partially understood, although it as been appreciated that separation must have important repercussions on the flow at higher evels. Dr. Scorer (see this *Journal* p. 340) has done a useful service in elucidating further the ffect of separation and in suggesting the factors determining its occurrence. Furthermore, he explanation, in terms of periodic separation, of Förchtgott's observation of the slow down- tream movement of lee waves followed by rapid upstream movements, is ingenious and revealing.

If the tentative explanation mentioned by Mr. Sawyer for the increased orographic rainfall o the south of depressions compared with that to the north is correct, this is a remarkable indica- ion of the effect of the wind profile on the liability to orographic disturbances of an airstream.

Captain Mitchell's observation that he has never encountered waves above stratocumulus cloud having an undulating upper surface is curious. The explanation may be that such cloud would commonly be just beneath an inversion at which it is known the amplitude of any lee waves has pronounced maximum. The amplitude falls off rapidly above the inversion and the waves might thus be negligible in an aircraft flying well above the cloud.

551.579.1 : 551.577.51 : 551.583.14 : 551.509.67

Meteorology in a large water-engineering project

As already reported on p. 297 of this volume, Dr. E. B. Kraus gave a talk on ' Meteorology n a large water-engineering project ' on 16 February 1955. The discussion following his talk s given below.

Mr. G. Bell : After such an engrossing and ably presented lecture, I hesitate to ask a irst question rather indirectly related to it.

However, Dr. Kraus described ' race lines ' drawing water from a wider catchment than he natural drainage. Has the possibility been considered that a lessened water-supply to land elow these areas could affect local flora and hence, indirectly, animal and human population ?

Dr. H. L. Penman : I have four queries or comments. First, is it not somewhat unfortunat that the main collecting reservoir is on the east side of the catchment, where föhn effect will b most marked and evaporation losses greatest ? Secondly, can Dr. Kraus give a reference t or supply a copy of, an account of the frost-hollow in which trees cannot be re-established This is an extremely interesting piece of micro-meteorology about which I would like to kno more. Thirdly, I agree that flooding of valleys can have little effect on local weather. Afte all, it is merely replacing one kind of evaporating surface by another, and the increase i evaporation will not be very great. Finally, has anyone calculated which will give the greate energy return, the falling water or the increased yield of irrigated crops regarded simply a fuel ?

Mr. F. A. Sharman : Is it possible for Dr. Kraus to give any further information on th estimation of evaporation from the new reservoirs ? Have any field studies or experiment been undertaken ? Has the possibility of evaporation control been considered in connectio with the project ? Can Dr. Kraus say what actual figure has been taken for mean annual evapora tion in estimating output, and within what limits of accuracy would he expect this estimat to lie with, say, a 95 per cent probability ?

Mr. D. J. Schove : Dr. Kraus mentioned the need for seasonal forecasting in the Snow Mountains. I know that, until recently, Australian farmers relied on Crohamhurst Observatory whose prognostications were based on the positions of the planets. I should be interested t know whether scientific methods such as those of Sir Gilbert Walker have proved valid for Soutl Australia ?

Dr. Kraus mentioned how the selection of the ' normal ' period 1902-1947 happened t give minimum values for mean rainfall in the Snowy Mountains. The situation is similar in thi country, where water engineers often still have to rely on the means for the very dry perio 1881-1915. I should be interested to know what the official policy is to be in Australia with regar to a normal 30-year period.

Mr. J. M. Craddock : The fact that the estimated yield of water using the basic perio 1902-1948 is less than that using a basic period extending into the 1890's and 1880's suggest that the rainfall will be less still in future. Does Dr. Kraus agree ?

Dr. J. Glasspoole : I should like to know whether there are any local interests which hav to be provided for in this scheme, or whether the whole of the water can be used for hydro- electric or irrigation purposes, even during dry periods. In this country many local interest have to be considered. Thus, in one hydro-electric scheme, the open channel, or catchwater constructed along a flank of Ben Nevis allowed one stream to pass underneath because tha water was used in making whisky.

Mr. G. Derrington : Is the radar to be used merely for storm-warning, or can refinement of technique render it a suitable substitute for a dense raingauge network in measurement o precipitation ? It is my opinion that the amount of rain falling as snow would prevent its use as a measuring instrument, even if the inaccuracies due to variation of drop-size distributio in rainstorms were within acceptable limits.

Mr. A. J. Whiten : Climatic change due to deforestation has been mentioned, but I wonder whether there is not another factor in artificial climatic change in this work. Australia, I believe, is one of the places with highest evaporation rates in the world. I want to differentiate between potential and actual evaporation. Potential evaporation is a function solely of saturation deficit, but, given a high s.d., actual evaporation will, I take it, become a function of exposed water surface. In this work you have been providing massive new surfaces for evaporation. Will these have any significant effect on the relative humidity of your prevailing winds, or are these effects so small as to be undetectable ?

Mr. E. Gold : I think the outstanding result which has emerged from Dr. Kraus's lecture is the great variety of applications of the meteorologists' knowledge, even in a hydro-electric scheme. It is sometimes thought that only rainfall and hydrological measurements come into consideration, but Dr. Kraus has indicated quite unexpected applications. Similar results no doubt arise in connexion with any large undertaking – or ought to arise – to prevent waste and minimize error. I should like to ask Dr. Kraus whether consideration has been given to the effect of a possible reduction in the rainfall – already small, 19 in. – in the area east of the mountains, if the release of silver iodide – or other measures – brings down more of the available rain, over the mountain area. A reduction of, say, 3 in. in the area with only 19 in. might be very serious for agricultural production there. Old rainfall measurements are sometimes faulty. Forty years ago the totals for two stations in Dunkirk, at one of which the observer had used, with a 20 cm diameter gauge, a measuring glass for a 400 cm^2 gauge, for a period of 20 years,

were 12,787 mm and 16,342 mm. These lead to $\pi = 3{\cdot}13$; the gauges were good though the measure was wrong. The error was only discovered when I was having maps of rainfall prepared for the Army in France and had been furnished with the smaller value only.

Mr. H. H. Lamb : It has been suggested that, if further rainfall were extracted from the westerlies by artificial means for the Snowy Mountains Project, the districts on the eastern side of the mountains might have their meagre rainfall depleted by a serious proportion. Can Dr. Kraus say what proportion of the annual rainfall in these eastern districts comes not from the westerlies but from totally different meteorological situations ?

Dr. E. B. Kraus (*in reply*) : In reply to Mr. Bell, I would like to say that the race lines in the Snowy Mountains area are closed pipes with openings at minor rills and creeks along the slopes. This has been done to prevent choking by snow, litter and sediment. The reduction of water in the gullies below will undoubtedly affect local flora. On the ridges and slopes between, the effect is expected to be small. Human habitation begins only below levels where the water would have reached the main streams in any case.

Dr. Penman wonders whether the situation of the main collecting reservoirs on the east side of the mountains will not cause great evaporation losses. Unfortunately, the highly variable flow requires large storages, and the terrain is such that these can only be established on the eastern side. Work on frost-hollows has been carried out Dr. Muller of the Authority Soil Conservation Section, and also by the Plant Industry Division of the C.S.I.R.O. As regards the question of energy return, this will certainly be larger from falling water under the particular condition of the Snowy Mountains, where fuel crops could not easily be grown from irrigation because of the steep terrain and rocky soil. Where possible, afforestation is planned and may be carried out in a way which will not compete with the energy available from running streams.

Mr. Sharman asks about evaporation. In the absence of any records this was computed, in the first instance, from estimated winds and temperatures above the future water surfaces. A number of evaporimeters have been established, but it is doubtful to what extent readings from them can be related to the as yet non-existing lake surfaces. The possibility of evaporation control with cetyl alcohol films is being considered. It is not possible to give a figure for mean evaporation for a region which extends in height from 1,000 ft to 7,000 ft, and where the rainfall varies from 18 in. to about 120 in. annually. Conditions will vary greatly from storage to storage. The economically important figure is not total evaporation, but the excess evaporation from an open water surface as compared with vegetation, swamps, etc., which existed there before.

Mr. Schove's question about the feasibility of Sir Gilbert Walker's statistical methods of seasonal forecasting is of great interest. These methods would probably be of value to forecast the monsoon of Northern Australia and, perhaps, also the summer rainfall of SE. Australia. I doubt whether they could be applied to winter precipitation in the Snowy Mountains, which is mainly of a frontal nature, and which supplies the largest amount of water.

With very considerable changes in the Australian rainfall regimes, which have in fact occurred during historic times, any 30-yr period will necessarily be arbitrary. The Commonwealth Meteorological Branch has chosen the period 1910-1940 as their normal period mainly because there were fewer stations available between 1900 and 1910. This made it disadvantageous to conform to the international period of 1900 to 1930.

Mr. Craddock asks whether the rainfall is still decreasing. I do not know the answer. The driest periods occurred actually early in the century and again in the late 1930's and early 1940's in agreement with world-wide effects in the tropics and sub-tropics. The last six or seven years were rather wet on the average, but it is not possible to say whether these conditions will last or not.

The local interests referred to by Dr. Glasspoole are practically non-existent in the wild country of the Snowy Mountains.

Mr. Derrington's comments, on the possible use of radar are very justified.

The effects of the new lakes, to be established, upon the relative humidity of the air and the rainfall are likely to be extremely small, as Mr. Whiten and Dr. Penman have already suggested.

Mr. Gold and Mr. Lamb both query the effect which artificially increased precipitation in the mountains would have upon the low-rainfall regions which lie to the lee of the hills in the east. In reply, I would like to say that the latter regions receive much of the rainfall during the warm seasons from thunderstorms and east-coast cyclones. Most of the moisture in the westerlies appears to be extracted in any case by the hills, and little of it becomes available in the east. The question is very interesting as one of general principle, but I do not know of any work that could provide a reply for the general case in an incontestable fashion.

551.524.7 : 551.547 : 551.578.4

Two-way relationship between the snow or ice limit and 1,000-500 mb thicknesses in the overlying atmosphere

By H. H. LAMB

(Read 16 March 1955. See *Q.J.*, **81**, p. 172)

Mr. R. Murray : Mr. Lamb has referred to some work which I did, a few years ago, in connection with the likelihood of rain or snow in relation to the 1,000-500 mb thicknesses. I would like to point out that there is an important difference between Mr. Lamb and myself in the grouping of the precipitation forms. Based on some 1,700 observations, during three winter seasons, I found that about 17,150 ft was the value of the 1,000-500 mb thickness at which snow (including snow showers) and rain (including rain showers and drizzle) appeared equally likely on low-lying ground over the British Isles. At this critical value in my data 100 observations of precipitation would be made up very nearly as follows : snow 40, rain and drizzle 40, sleet 5, hail 15 – total 100. I then say that snow and rain are equally likely. Mr. Lamb groups these 100 observations thus : frozen forms (snow plus sleet plus hail) 60, rain 35, drizzle 5 (say). So there is now a much greater likelihood of frozen precipitation (snow and sleet and hail) than of rain (excluding drizzle) at the level which I call the critical value. It is clear that Mr. Lamb's critical thickness is appreciably higher than mine, for we are not dealing with precisely the same thing. Nevertheless, I agree with Mr. Lamb that there is a significant difference between the British and Continental stations as regards the value of the critical thickness, whether the precipitation grouping be his or mine. However, I myself would not describe the change-over from snow to rain at the so-called critical value of the 1,000-500 mb thickness as ' sharp.'

Mr. J. M. Craddock : I should like to describe some unfinished work which is going on in the Forecasting Research Division following up a suggestion made by Mr. Lamb that the establishment of an early snow-cover extending from Asia into Central Europe may be a necessary prelude to a severe winter.

First I looked at 47 years' observations of snow cover at Potsdam to see whether the earliness or lateness of the first snowfall was connected with the severity of the winter at Berlin, but could find no useful relation. I then passed the problem to Mr. C. V. Smith, who reached the conclusion that snow at Berlin was a good indicator of cold weather at the time, but that such cold weather was no more persistent than equally cold weather unaccompanied by snow. Later Mr. A. Gilchrist looked at the radiation balance and concluded that since, in these latitudes, the atmosphere in winter is losing much more heat by radiation at high levels than it is by heat transfer to the ground surface, the presence or absence of snow on the ground is unlikely to alter the balance significantly.

Finally, Mr. F. E. Dinsdale examined the synoptic situations during the winters, since 1893, with most snow at Potsdam during December, and compared them with the winters in which there was least snow. He found that in the snowy winters there was a complete absence of the westerly weather type during January and February, whereas after snow-free Decembers the westerly weather type recurred later in the winter.

Now, if there is a tendency in some years for general-circulation types to persist or recur throughout a season, and a good deal of synoptic experience is in line with this hypothesis, a relation between early snow and cold winters may arise as follows. If the recurrent circulation pattern, in a particular winter, involves the penetration of westerly winds into Europe, then the early winter will be snow-free and the late winter mild. If, however, the recurrent circulation pattern does not involve the penetration of westerly winds into Europe, the winter will be cold, and snow once fallen is likely to last. At present we incline to look on the general-circulation pattern as the controlling feature, with the snow-cover acting mainly as an indicator, rather than accepting Mr. Lamb's view and regarding the snow-sheet as imposing a modification on the general circulation.

Dr. R. C. Sutcliffe : The use of the 1,000-500 mb thickness as a parameter in synoptic meteorology and forecasting is now well established. Forecasters have, of course, to go into greater detail, but it is essential to simplify the problem when preparing forecast charts. It is interesting to note that the technique which was earlier adopted by synoptic forecasters has been adopted also for numerical forecasting by electronic calculation at Dunstable. Even if, however, we can predict the future patterns of pressure and thickness, we still have to put in the weather, and Mr. Lamb's study of the critical thickness where rain and snow are equally probable is very

ractical and useful. It could be applied to the forecast for tomorrow, 17 March, for cold air is oming down from the north and the critical-thickness line is likely to extend across the country, viding snow showers in the north from rain showers in the south.

Mr. Lamb uses snow already lying as a guide to whether further precipitation will be snow rain. I suppose, however, that there is no direct connection : it is merely that snow lying dicates low surface temperatures permitting further snow to reach the ground when the thickess (that is the general upper air temperature) is higher than the average critical thickness.

Mr. J. S. Sawyer : Examination of Table 7 in Mr. Lamb's paper might suggest that an r mass travelling at 30 kt suffered a cooling such that the 1,000-500 mb thickness was reduced y 400 ft. in a distance of 800 mi, i.e. between 200 and 1,000 mi from the snow edge. This presents an average fall of temperature in the air-column of 10°F in 27 hr, and is more than should have expected as a result of the downward transfer of heat by turbulence. I think that we ust be cautious in interpreting these figures as representing solely the cooling by a snow surface. ynamical factors may have been producing an equally important cooling by vertical motion many of the cases studied.

Mr. R. F. M. Hay (*communicated*) : Mr. Lamb says ' the low mean thickness and small tandard deviation at the snow limit, wherever this may happen to be, point to a real association etween the snow limit and the 1,000-500 mb thickness over it,' and later ' the average 1,000-500 mb hickness at the windward limit of extensive snow and ice is found to be just 17,300 ft with a s.d. nly about two thirds of that characteristic for specific points normally far away from the snow r ice margin.'

I do not consider that this inference is justified from the data without further qualification. Mr. Lamb makes a comparison between the s.d. of 1,000-500 mb thicknesses measured at points ear the snow or ice margin and the s.d. of 1,000-500 mb thicknesses measured at other points n different parts of the region in January to March. In the former case, the temperatures near he surface will be almost entirely in the range 28°-40°F, whereas in the latter, they will clearly xtend through a much wider range, say between 25° and 50°F at the surface, as is indicated y the data for the thickness of 17,400 ft in Table 2 of the paper. In these circumstances it is ardly surprising that the s.d. of the 1,000-500 mb thicknesses in the former cases is only two-thirds f that found at other points of the region in January to March; the remarkable thing is that it s not even smaller. It would have been preferable to have made a comparison between the s.d. f the 1,000-500 mb thicknesses measured at points near the snow or ice margin and the s.d. f the 1,000-500 mb thicknesses at other points in different parts of the region *when the surface emperature was between say* 28° *and* 40°F (or a similar range).

The fair inference to be drawn from the data presented in Lamb's paper, and supported y his Table 2, is that, while the association between the snow limit and the 17,300 ft thickness s a real one, there is an equally real association between every other thickness line and the surface emperature, provided that equal and specified ranges of surface air temperature are considered i.e. the s.d.s of the 1,000-500 mb thicknesses are virtually equal for successive ranges of observed urface temperatures 28-30°F, 30-32°F, 32-34°F, etc.).

The importance of the position of the snow or ice limit lies not in any closer association f the thickness pattern with these particular surface air temperatures than with any other range f surface air temperatures. It lies rather in the evidence that there is perhaps a closer association etween the snow or ice limit and the surface air temperatures than between any other surface hysical feature (e.g., land or sea surface temperature) and air temperature.

More decisive evidence to support this conclusion may yet be needed. The importance of he snow or ice limit on account of its conservative properties is already fully stressed by Mr. Lamb.

Mr. H. H. Lamb (*in reply*) : I am glad Mr. Murray has drawn attention to the fact, which had missed, that a small apparent discrepancy between his results and mine must arise from he different grouping under ' frozen forms.' His figures and mine represent separate, fair-sized ample collections of observations belonging to the same problem under study. It is gratifying o see how far they do agree and lend support to the pattern which I found of rather lower critical hicknesses at the more oceanic stations.

I am not satisfied that Mr. Craddock's Potsdam tests are really relevant to the point under iscussion. Presence or absence of snow at Potsdam at any given date tells us nothing about the road distribution of snow or the position of the western limit of the main Eurasian snowsheet. otsdam is far enough west to be liable to get snowfall with, for example, northwesterly winds t any stage of the winter and still be separated from the main continental snowsheet by hundreds f miles of bare ground. The essence of the facts I have uncovered in this investigation is that hey have to do with thicknesses at the windward limit of *a vast snowfield* and thickness

modification in the air overrunning the first thousand miles or more of the snow. The effect is, c course, related to the *contemporary* position of the snow limit, and forecasts based upon assumin no great retreat of that limit during the winter will go wrong if the snow is in fact got rid c over wide areas.

Where Mr. Craddock speaks of the general circulation as producing patterns over the Atlanti and Europe which tend to persist or recur throughout a season, I find myself in agreement wit him (but are not these very patterns related to the position of cold outbreaks over America ? Whatever importance we ascribe to the contemporary position and lay-out of the Eurasian snow cover as an influence upon the general circulation, it is quite certain that this must be reconcile with the succession of circulation systems generated far away and coming in with varying fre quency from the west.

It seems, too, that the actual tracks followed by disturbances pressing into Europe from th west may make a great difference. My results suggest that we should recognize the concep of a ' *well-directed warm-air invasion,*' for it stands out clearly that, in cases where the Europea continent is largely snow-covered, a thrust of warm air which happens to be directed (either b way of the Channel or North Sea) towards the Baltic gap will make far greater inroads than thrust which comes up against the mountain barriers.

With regard to Dr. Sutcliffe's reservations about quoting a different critical thickness fo snow or rain when there is snow lying, the observed fact that some snow may then fall with highe thickness values than at other times is of course merely an expression of low temperatures an a stable layer in the lowest kilometre or so of the atmosphere. The forecaster, concerned wit how far he can use the 1,000-500 mb thickness map to give a generally illuminating synopsis will still be glad to know the critical-thickness range applicable to the case of snow already lying

Mr. Sawyer is quite right in stressing that dynamical cooling must come in in the more rapi cases of thickness modification in invading warm-air masses. This is implied in the paper and i actually emphasized in connection with the mountain barrier cases. I believe that the statistica results in the paper do suggest an element of uplift (dynamical) cooling in the general run o cases where the gradient winds in the invading warm airstream are moderate or light.

I think Mr. Hay and I are basically in agreement that a lower s.d. of thickness is to be expecte at the snow limit or over the (moving) position of any surface isotherm than at some fixed poin where wide variations of surface temperature bring in an additional variability. The specia interest of the snow or ice limit lies in the fact that it is much less mobile than any surface isotherm Nevertheless, there may well be a possibility of finding some useful relationships between 1,000 500 mb thickness and certain sea-surface isotherms, especially where the mobility of these surfac isotherms is limited by some geographical feature.

551.574.

The concentration, size distribution and production rate of large salt nuclei over the oceans

By D. J. MOORE and B. J. MASON

(Read 16 March 1955. See *Q.J.*, **80**, p. 583)

Dr. T. W. WORMELL : I should like to congratulate the authors on a most interesting anc valuable paper. I believe that Professor Dessens found evidence that when a droplet evaporates and ultimately crystallizes the process of crystallization can, on occasion, be of an almost explosive character, thus producing numerous smaller salt particles. Does the possibility of such a process affect Mr. Mason's estimates of the number of small salt nuclei available ?

Mr. B. J. MASON (*in reply*) : The possibility that the crystallization of a solution droplet may produce several separate crystals was *one* of the reasons for my believing that Simpson's estimate for the required rate of production of sea-salt nuclei was too high – perhaps by a factor of 5 or 10. However, I doubt whether this effect is of major importance, since Dessens reports that this fragmentation occurs only rather rarely and usually with only large nuclei. Also one has to be cautious in deducing what may happen to airborne droplets from the behaviour of droplets supported on threads or solid surfaces. A recent laboratory investigation by Lodge and Baer (*J. Met.*, **11**, p. 420, 1954) indicated that desiccation of sodium chloride droplets, of dry radius 0·5 μ to 4 μ in the free air, was rarely accompanied by shattering of the nucleus.

Discussion meeting on ' The weather of 1954,' 20 April 1955

As reported on p. 525 of this volume, a discussion meeting on ' The weather of 1954 ' ok place in the Society's rooms on 20 April 1955. The opening papers and general discussion e given below.

551.506.2 (41/2)

The weather of 1954 over the United Kingdom

By J. GLASSPOOLE
Meteorological Office, Harrow, Middlesex

The main features of the weather of 1954 are brought out by the monthly values of rainfall, mperature and sunshine set out in the table below.

TABLE 1. GENERAL MONTHLY VALUES, 1954

	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.	Year
Rainfall (inches)													
ngland and Wales	2·6	3·4	3·1	0·6	3·0	3·5	3·5	4·6	3·5	4·8	6·6	3·5	42·7
cotland	5·0	4·1	3·6	2·0	4·8	4·1	4·0	4·7	6·3	9·4	7·9	8·1	64·0
orthern Ireland	3·1	4·4	3·6	0·9	4·4	3·1	4·4	3·1	4·8	6·9	5·7	5·2	49·6
Percentage of average 1881-1915													
ngland and Wales	87	131	117	29	131	142	124	139	139	122	189	89	121
cotland	101	97	89	65	157	143	106	102	156	190	149	137	127
orthern Ireland	93	149	117	35	163	111	129	74	162	191	149	124	125
Temperature (°F) at sea level													
ngland and Wales	38·5	37·5	43·5	46·2	52·9	56·7	58·1	59·0	56·1	54·6	45·9	45·3	49·6
cotland	39·0	36·8	40·8	45·4	50·9	54·1	55·8	55·9	51·5	49·4	42·7	42·3	47·0
orthern Ireland	39·4	38·2	42·5	45·7	51·1	54·4	55·8	56·0	52·4	50·9	42·6	43·9	47·7
Departure from average 1921-50													
ngland and Wales	− 1·9	− 3·1	− 0·1	− 1·3	+ 0·1	− 1·5	− 3·5	− 2·3	− 1·5	+ 3·2	+ 0·9	+ 3·7	− 0·5
cotland	− 0·1	− 2·7	− 1·0	+ 0·5	+ 1·2	− 0·6	− 2·3	− 1·5	− 2·2	+ 1·0	− 0·1	+ 1·9	− 0·5
orthern Ireland	− 0·3	− 2·1	− 0·2	− 0·1	+ 0·5	− 1·3	− 2·7	− 1·9	− 2·0	+ 1·9	− 0·6	+ 3·2	− 0·5
Sunshine (mean hr/day)													
ngland and Wales	1·77	2·03	3·27	6·07	4·99	4·69	4·36	3·99	5·28	2·67	1·93	1·45	3·52
cotland	1·32	1·81	3·17	5·83	4·95	4·52	3·63	3·36	3·78	2·26	1·49	0·72	3·05
orthern Ireland	1·13	1·45	3·21	5·68	5·14	4·15	2·92	3·77	4·19	1·74	1·77	0·69	2·99
Percentage of average 1921-50													
ngland and Wales	117	88	87	121	82	70	75	73	120	84	102	108	89
cotland	112	84	97	127	87	78	80	77	105	89	94	80	91
orthern Ireland	78	65	94	117	86	71	66	87	112	62	89	58	85

(*a*) *Rainfall*

The first 4 months of 1954 gave less than the average. The last 3 months of 1953 had also een relatively dry, so that by the end of April there was some concern about maintaining full ater supplies, especially as this was the second consecutive dry winter half-year. Subsequently ne rainfall exceeded the average for 7 consecutive months over England and Wales and 8 over cotland. This long sequence of wet months is one of the outstanding features of the rainfall f 1954. The number of any groups of 7 consecutive wet months over England and Wales in 00 years had been 6, and of 8 consecutive wet months 2. The rainfall of the 7 months May

to November 1954 amounted to 29·5 in., or 8·5 in. more than the average, over England and Wales, while over Scotland the total was 41·2 in., or 12·9 in. more than the average. Since 1869 similar periods were wetter over England and Wales only in 1903 and 1872, while there were no wetter similar periods over Scotland, the next wettest being May to November 1938 with 40·8 in.

The last year wetter than 1954 over Wales was 1872 (1877 being about as wet as 1954) over Scotland 1903, over Northern Ireland 1928, and over England 1951. Over England and Wales since 1869, 7 years were wetter than 1954 and over Scotland only 3 years, 1903, 1877 and 1872. Taking England, Wales and Scotland together, 1954 ranks as the wettest year since 1903.

The only part of the country to record less than the average in 1954 was in the neighbourhood of the Thames Estuary, (see Fig. 1). The percentage values increased towards the west, where in the mountainous districts between South Wales and the Southern Uplands of Scotland considerable areas recorded more than 140 per cent. Rather more than 150 per cent was reported from Rhondda, Preston, Kendal and Thornhill, in the Southern Uplands. At Keswick (Cumberland), in parts of central Lancashire and at Ystalyfera (Glamorgan), 1954 ranks as the wettest year on record since before 1869.

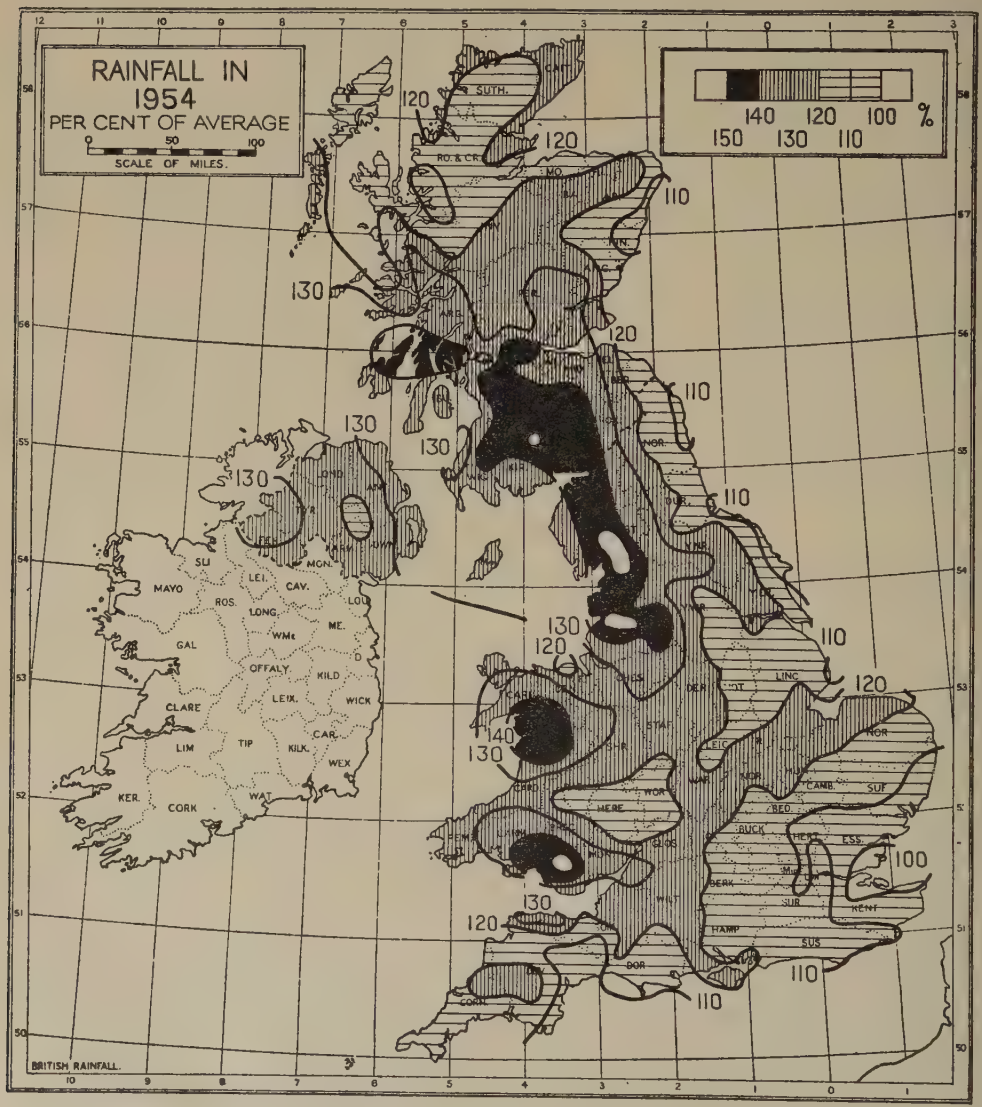

Figure 1.

The unusual fall of 10·37 in. for 17-18 December at Cruadhach to the west of Glenquoich in Inverness-shire warrants special mention, although a few larger amounts for two consecutive days are on record (Hawke 1942).

Over England and Wales, in the series since 1919 the number of rain-days in 1954 was exceeded in 1930, 1927 and 1923 and the number of wet-days also in 1951 and 1924. In Scotland the only year with more rain-days and wet-days was 1923. This is all the more striking in that 1954 gave over England and Wales the average number of rain-days for January to July and over Scotland from January to June. The general number of rain-days (102) over Scotland for September to December exceeded that of any year since 1919, the next largest in a similar period being 97 in 1923. The general number of wet-days was 87 in 1954, the next largest being 82 in 1923.

(*b*) *Temperature*

Temperatures were most outstanding during the four months June to September, each of which was appreciably cooler than the average. The last three months of the year were mild and the mean temperature of 1954 over the country was only 0·5°F. below average. Over England

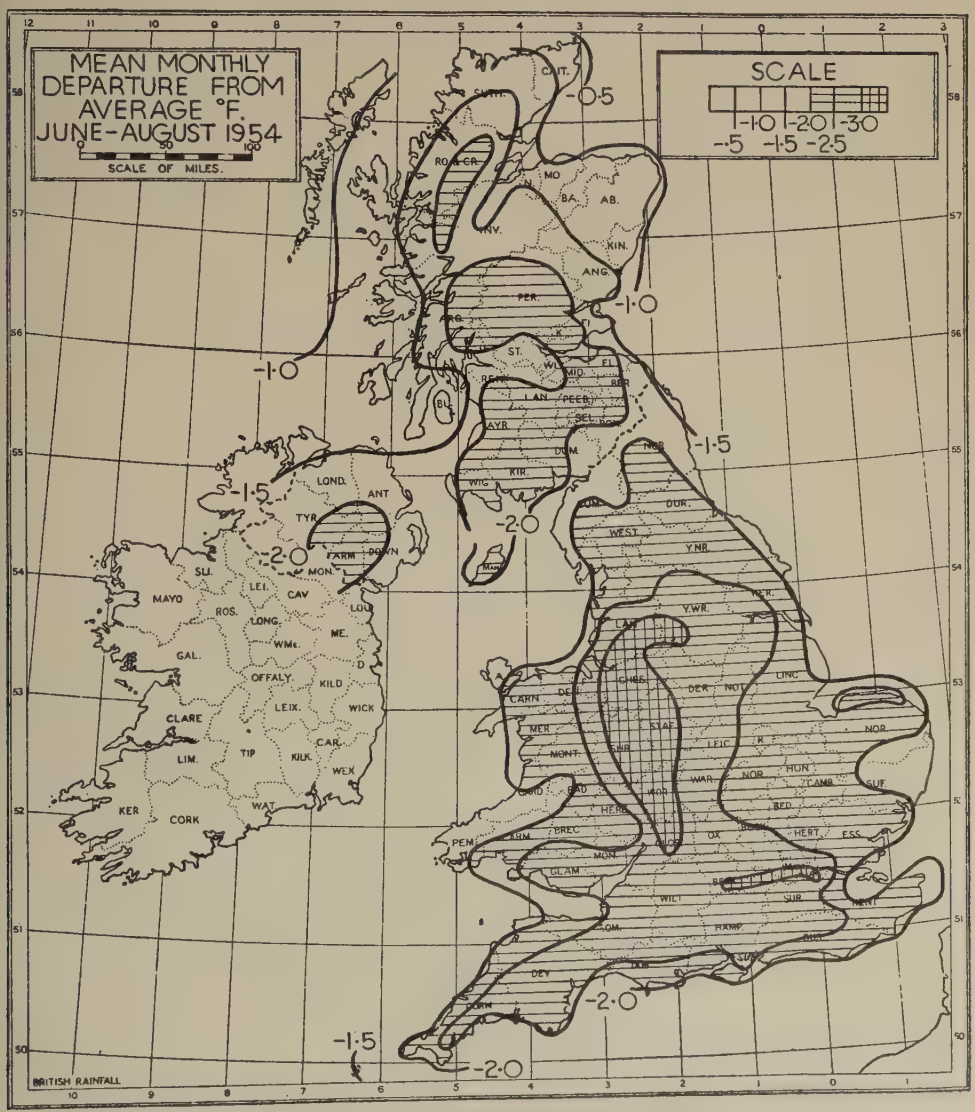

Figure 2.

and Wales there were 20 cooler years since 1901, of which 19 occurred within the 41 years 1902-42 but only one, 1952, within the 11 years 1943-53. Over Scotland the recent years 1952, 1951, 1942, 1941 and 1940 were all colder.

Over England and Wales, June to September 1954 was colder than any similar period since 1901, apart only from those in 1922, 1912 and 1909. The temperature of the summer three months, June to August, was perhaps more striking, because there were only two cooler summers than that of 1954 over England and Wales, and eight over Scotland. The coldest summers over England and Wales were those of 1922 and 1907 (each being 0°4°F colder than 1954). Although July and August followed the usual pattern of being the warmest months of the year, the mean temperature of the summer of 1954 only reached the normal temperature of June. The low mean temperatures were due more especially to cooler days, and this is generally the feature of cool summers. During the summer the mean maximum temperature over England and Wales was 3·7°F below average, and the mean minimum 1·2°F below. When compared with the warm summers which have prevailed since 1932 the cool summer of 1954 is outstanding, but when compared with the cooler summers of 1902 to 1931 it is not remarkable. Indeed in this earlier period there were, over England and Wales, 12 summers out of 30 colder, or nearly as cold as that of 1954 (i.e., within 1°F) compared with none in the last 23 years.

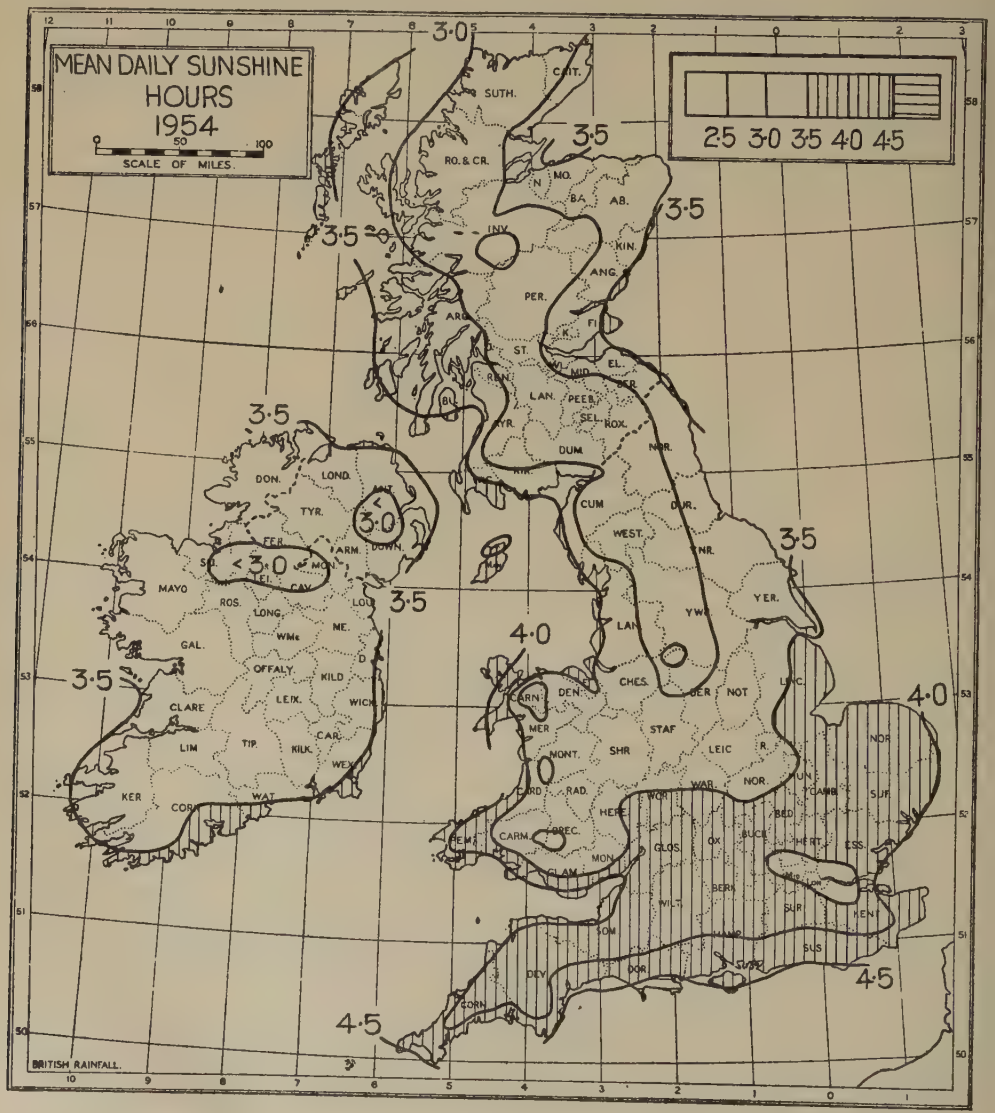

Figure 3.

The temperature of June to August 1954 was everywhere below the 1921-50 average. mean monthly deficiency exceeded 2°F over parts of southern Scotland and over most ngland and Wales and exceeded 3°F in parts of the Midlands and London area. The map, . 2), shows that the deficiencies were least in the north and in coastal districts. The n temperature was 2·4°F below the 1921-50 average over England and Wales and 1·5°F Scotland.

(*c*) *Sunshine*

Sunshine during 1954 was less than in any year since 1906, apart from 1936 and 1912 over land and Wales, and 1944, 1916 and 1912 over Scotland. Fig. 3 shows that the distribution ean daily sunshine during 1954 was very similar to that on the average annual map, although values were smaller, the mean daily amounts being less than average by 0·4 hr over England Wales and 0·3 hr over Scotland. London stations reported about ½ hr/day less than average. interesting to compare this with the difference on the average between London and south t stations which amounts to about 1·0 hr/day.

The mean sunshine experienced during the summer three months, June to August 1954, less than we expect during the shorter days of April or September, and appreciably less than of April 1954 which remained the sunniest month of the year over Great Britain. April also the sunniest month in Northern Ireland. Since 1906 the only summer with less sunshine that of 1912 (also a wet summer), which gave 0·2 hr/day less than in 1954 over England Wales and 0·6 hr/day less over Scotland. Taking the four months May to August then n over England and Wales only 1912 gave a less sunny period, but over Scotland both 1912 1907 were duller.

A study of the running 10-year means of seasonal sunshine for England and Wales shows the effect of the inclusion of the values for 1954 is a decrease in the spring and in summer. s general trend for a decrease in the spring and summer sunshine had already persisted for e years. The 10-year means for the autumn and winter have remained fairly steady, the umn value being not much more than the lowest in the series and the winter value remaining bout the highest value. The mean annual sunshine for the last 10 years has exceeded the rage, owing to the relatively sunny springs and winters, while the summers and autumns e been less sunny than usual (Glasspoole 1955).

(*d*) *Pressure*

The mean pressure at 9 hr for 1954 varied from 1015 mb in the south to 1008 mb in the th of Scotland, rather less than the average in the south and some 2·5 mb below average he north. The pressure gradient was thus greater than usual. The pressure distribution is refore in accord with the rainfall distribution already given, since 1954 was a wet year and the fall exceeded the average more especially in the west. It was shown (Salter and Glasspoole 3), that the correlation coefficient of general pressure over the British Isles for each year the general rainfall is $r = -0{\cdot}83$ ($e = 0{\cdot}04$), while the correlation of pressure gradient ween Dungeness and Stornoway with the percentage rainfall at selected stations in the west us those in the east is $r = +0{\cdot}69$ ($e = 0{\cdot}07$).

The general run of the isobars of mean pressure for 1954 is similar to that on the average . This presumably results from the two types of abnormal weather experienced during the r. In the first the depressions approached the British Isles from the north-west, as at the of June and in the beginning of July, while in the second the depressions approached from south-west, as at the end of July and in the first half of August.

(*e*) *Wind*

Considering wind records for the last 20 years at representative stations over the country, clear that 1954 was not in general the windiest year or the year of record gusts. There were, vever, a few outstanding values in 1954. Thus Tiree in 1954 reported the greatest number abulated hours with more than 33 kt and Scilly the highest gust on record there. It is also ed that Oxford in 1954 reported the largest run of wind since 1903.

The incidence of prolonged outstanding weather is reflected by the papers published by the val Meteorological Society. These include : ' The exceptional summer of 1933,' ' The wet mmer of 1924,' ' The drought of 1921,' ' The cold period December 1916-April 1917,' ' The English winter of 1914-15 ' and ' The abnormal summer of 1911.'

In view of the discussion in the Press about the possible causes of the poor summer of 195 it is of interest to recall that, after the wet summers of 1924 and 1927, the B.B.C. approach the Society for their views, ' in order to lift from the shoulders of wireless the stigma that broa casting was responsible for the adverse weather.' The Society contributed an article to t *Radio Times* of 28 June 1928, which closed with the paragraph :

> From time immemorial our weather has had its vagaries, and from time immemorial the people ha demanded a cause for each successive vagary. Comets are out of fashion now, so in 1916 and 1917 the ra was caused by the war, in 1921 the drought was caused by the coal strike, and now rain is caused by broa casting. It is as if a butterfly watched a mighty pendulum, and of each successive swing remarked : ' A That was because I flapped my wings – this time I caused it by blowing my nose – and then I stamped r foot.'

The permission of the Director of the Meteorological Office to publish these details gratefully acknowledged.

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Hawke, E. L.	1942	*Quart. J. R. Met., Soc.*, **68**, p. 297.
Royal Meteorological Society	1928	' Wireless and weather,' *Quart. J. R. Met. Soc.*, **54**, p. 260
Salter, C. and Glasspoole, J.	1923	*Ibid.*, **49**, p. 207.

551.506.2 : 551.547

Weather in 1954 in relation to northern-hemisphere circulation patterns

By A. G. FORSDYKE
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1. Introduction

We are naturally led to enquire whether the exceptional weather of the British Isles in 195 had its counterparts, either good or bad, in other parts of the world. One way of doing this woul be to catalogue noteworthy meteorological events of 1954. From time to time reports of thes appeared in the Press; the catastrophic hurricanes which affected the eastern parts of U.S.A. an Canada in the autumn come especially to mind. Nevertheless my impression is that 1954 wa not outstanding as regards the number of meteorological news items and it may well be tha the year is regarded as noteworthy solely on account of the bad summer in the British Isle In fact, the theme which I hope to develop is that in 1954 the weather of our area wa exceptional as compared with many other parts of the northern hemisphere, and this becaus certain features of the large-scale circulation, by their recurrence and persistence near the Britis Isles, affected this part of the hemisphere to a markedly greater extent than most others.

2. Weather phases of 1954

To begin with, it would help our understanding if we could divide the year 1954 into phase or seasons, determined by the broad characteristics of the weather. Ideally this division woul be on a world-wide or, at least, a hemispherical basis, but experience has shown the extrem difficulty of classifying weather situations on such a scale. More for convenience therefore tha anything else, we consider how the year can be divided according to the type of weather in th British Isles. There were four phases as follows :

(i)	January to March	Changeable; no settled type; cold on the whole with wintr spells (late January was the most severe for seven years
(ii)	April	Dry, sunny, rather cold; anticyclonic with easterly winds
(iii)	May to September	The dismal summer.
(iv)	October to December	Mild, stormy, wet on the whole.

During each of these phases there were certain recurrent or persistent features of the large-scale circulation, which may be typified by selected monthly-mean charts. In the Meteorological Office data are received, from world-wide sources, from which are constructed monthly-mean charts of pressure, temperature, rainfall, humidity, upper-air thickness and contours – the so-called CLIMAT data. Certain monthly-mean charts are also published in Germany, in *Die Grosswetterlagen Mitteleuropas*. For consideration this evening I have selected just two sets of charts; monthly charts of pressure anomaly and monthly charts of 1,000-500 mb thickness anomaly.

The thickness charts were selected as combining an indication of the temperature in the lower troposphere with the broad pattern of the upper flow. The area considered is that from the Rockies eastward to the Urals, the portion of the northern hemisphere for which data are most readily available in this country.

It is apparent from these charts that, in all the months selected, the British Isles area was situated either near an area of marked anomaly of pressure or thickness, or within a region of abnormally strong gradient of pressure or thickness – factors which tend to be associated with abnormal weather. Examination of the charts for other months shows that similar characteristics were present to a greater or less degree. Moreover, these features were present near the British Isles more frequently and persistently than in most other areas of the hemisphere.

The charts, which are described in the next paragraph but not reproduced in this report of the discussion, were selected in relation to the four phases of weather.

3. Description of the pressure and thickness anomaly patterns

Phase (*i*) (January charts). The largest positive anomaly was situated near Ireland and Iceland and the principal negative anomaly covered the Mediterranean, the Black Sea and the Caspian, indicating an increased northly component of wind over the British Isles. The thickness chart shows negative anomalies covering Canada and the whole of Europe with positive anomalies over the Atlantic and most of U.S.A. Over the British Isles the thickness was slightly below normal.

Phase (*ii*) (April charts). The largest positive pressure anomaly was situated centrally over the British Isles and covered much of the Atlantic and Europe, and the principal negative anomaly covered Greenland and the polar regions. In the thickness pattern there was a large positive anomaly over northern Greenland; otherwise the pattern was much as for January.

Phase (*iii*) (July and September charts). These charts are completely different from those of January and April. Both in July and September, a large area of negative pressure anomaly lay immediately north and north-east of the British Isles. The Azores anticyclone was rather more intense than normal. In both months the pressure gradient over the British Isles was stronger than normal, the increased component being north-westerly in July and westerly in September. In both months also there was a positive anomaly over most of Canada; in July this was joined to the Azores centre by a ridge across Davis Strait and the west Atlantic, but in September the

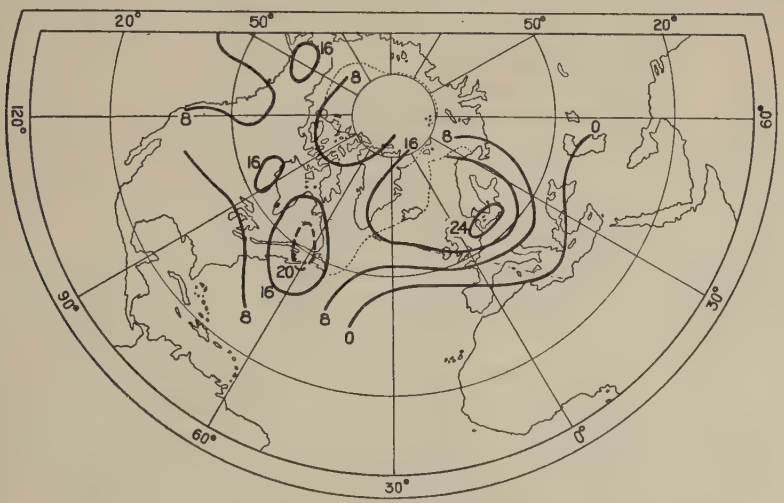

Figure 1. Number of cyclonic days in July 1950

two areas of positive anomaly were separated by a trough from the British Isles across Newfoundland to the Great Lakes. The July thickness pattern showed a marked negative anomaly covering western Europe and the British Isles, and centred over the Low Countries – a clear association with the cool weather of July in this country. By contrast, European Russia and U.S.A. both had marked positive thickness anomalies associated with warm and dry weather on the whole. Despite the similarities in the pressure patterns the thickness anomaly chart for September, however, bears no resemblance to that for July. A large area of negative anomaly centred between Scotland and Iceland covered the British Isles, Greenland and north-west Europe, and extended in weakened form across Canada and the northern part of U.S.A.

Fig. 1 has been prepared to illustrate the way in which the weather of the summer near the British Isles was notably unsettled as compared with other areas. It is a chart showing the number of days on which cyclonic weather was experienced in July 1954. The data were taken from the chart for 12 GMT. in the *Daily Weather Report* of the Meteorological Office. The isopleths were drawn in relation to a grid of points situated at the centres of 10° latitude and longitude squares. On a given day a grid point was classed as cyclonic if the corresponding square was covered by a closed area of low pressure, or a trough, or if a substantial part of the square was affected by a front. The number of cyclonic days during July 1954 was plotted at each grid point and the isopleths drawn as in Fig. 1. The maximum, east of the British Isles, is outstanding; inspection of the daily charts shows that it was associated with the frequent passage of depressions from west or north-west across the British Isles and their slowing down and deepening over the North Sea. It is noteworthy that the British Isles were situated in the sector with cool north to north-west winds to the west of this region of cyclonic activity.

There was a secondary cyclonic maximum over Labrador and Newfoundland. It is tempting to ascribe this longitudinal spacing of cyclonic maxima as associated with the long-wave pattern. This may be so, but it does not appear to give rise to any useful relationship since, as will be seen later, the correspondence was not consistently in evidence for other months.

Phase (*iv*) (December charts). A large area of negative pressure anomaly extended from north-west Canada across the polar basin to Scandinavia and central Europe. The anomaly was positive over Siberia, western U.S.A. and from Hudson Bay to the Azores region. This

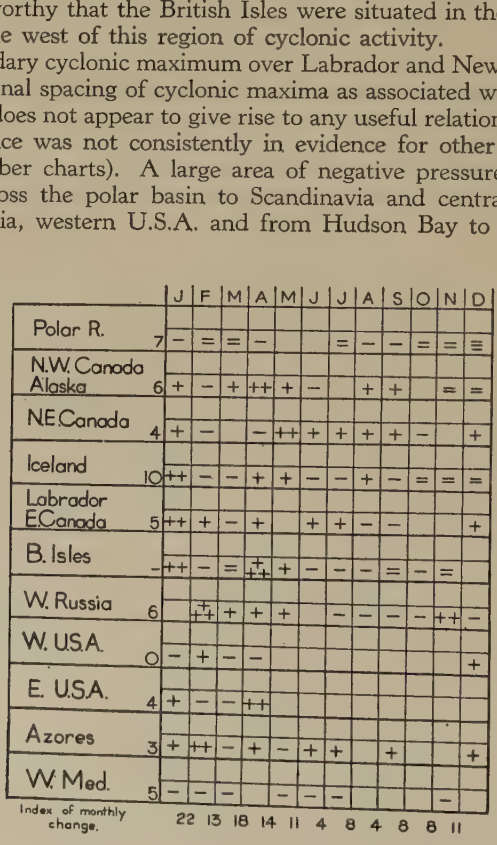
J F M A M J J A S O N D
Polar R. 7
N.W. Canada Alaska 6
N.E. Canada 4
Iceland 10
Labrador E. Canada 5
B. Isles –
W. Russia 6
W. U.S.A. 0
E. U.S.A. 4
Azores 3
W. Med. 5
Index of monthly change.
22 13 18 14 11 4 8 4 8 8 11

Figure 2. Anomalies of monthly mean sea-level pressure in 1954.

Anomaly (to nearest 2 mb)	Key Positive	Negative
⩾ 10 mb	+ + +	≡
6-8 mb	+ +	=
2-4 mb	+	–
< 2 mb	Blank	Blank

istribution indicates an increased westerly circulation about a spread-out but lop-sided polar ortex; over the British Isles the increased component of wind was from the north-west. The ositive thickness anomalies, noted over Russia and U.S.A. in September, were still in evidence, nd a narrow tongue of positive anomaly extended from western Europe to Newfoundland. negative anomaly covered Baffin Land, Greenland and Iceland. The result was an increased esterly component of thermal wind in a belt across the whole Atlantic north of about latitude 0°N. The British Isles lay near the eastern end of this belt, that is in a situation which on dyna-ical grounds is liable to disturbed weather.

4. Inter-relationship between different areas of the northern hemisphere

Fig. 2 has been prepared with the object of examining possible inter-relationships between ifferent parts of the northern-hemisphere circulation. It shows the monthly anomalies of ea-level pressure for the areas named.

It will be observed from the diagram as a whole that positive anomalies were most frequent n the first half of the year, and negative in the second. In the polar regions the anomaly was lmost consistently negative, pointing to an increase in the general westerly circumpolar flow. n the British Isles only one month, December, had near-normal pressure.

The figures to the left of the column for January show, for each area, the number of months n which the pressure anomaly was of the same sign as that for the British Isles. Only for the celand area is there anything like consistent agreement and this is probably due to its proximity o the British Isles. From the point of view of pressure anomaly, there appears to be no useful ssociation between the British Isles and other parts of the hemisphere.

The figures in the bottom row of the Table (Fig. 2) are total indices of the month-to-month hange of pressure anomaly (taken without regard to sign). For this purpose, the indices for the ndividual areas were converted to the following arbitrary scale :

$$\pm \geqslant 10 \text{ mb}, \quad \pm 3; \quad \pm 6\text{-}8 \text{ mb}, \quad \pm 2; \quad \pm 2\text{-}4 \text{ mb}, \quad \pm 1; \quad \pm < 2 \text{ mb}, \quad 0$$

Thus, for example, in the British Isles the index for March is -2, for April it is $+3$ and for May it is $+1$. The March-April change is taken as 5 and the April-May change as 2. Totalling for all the areas, we obtain the figures at the foot of the diagram. The index provides a measure of the steadiness of the pressure pattern. In the first four months the index was high, indicating great changeability, but from June to November the index was small, indicating comparative steadiness of the pattern over a wide area – particularly from June to July and from August to September. This suggests that the uniformly bad summer weather in the British Isles was associated with the settling down into a fairly steady state of the circulation pattern over at least half the hemisphere.

551.506.2 : 551.586 : 631.5

Weather and farming, 1954

By H. L. PENMAN
Rothamsted Experimental Station

1. General survey

As it is impossible to go into every field on every farm on every day in 1954, any brief survey of weather and farming must be broad. Reduced to an intelligible minimum the summary in Table 1 applies to England and Wales as a whole, and refers to grass and a few arable crops only, i.e., it ignores Scotland, and omits all reference to fruit, vegetables, and animals.

Most of the table is self-explanatory. On the whole, 1954 was a wretched year for field work. t delayed spring seed-bed preparations, badly interfered with hay and corn harvest, and, partly because of delay in getting land clear, and partly because of unfit land, autumn ploughings and drillings were restricted. Some of the effects of 1954 weather will appear in the 1955 crops.

The 1954 story of the grains has three main weather aspects. First, growth was slower than usual, and by the end of July the crop was two to three weeks behind. Second, the prevailing high soil humidity helped to maintain a micro-environment in the crop favourable for development

TABLE 1. WEATHER AND FARMING, 1954

	Weather	Cultivation and field work	Grain	Grass	Potatoes	Sugar beet
J	Mild – frost	Good progress	Checked by frost		Keeping well : some blight	
F	Snow : rain	Unfit to work		Little growth		
M	Snow – dull, cold	Delayed	Wheat : fair Barley : checked Oats : good	Backward	Frost damage in store Planting preparations in arrears	
A	Frost, drought	Good progress	Fair	Retarded by cold and drought	Frost damage on earlies	Fair sowing progress
M	Cold		Backward	Retarded by cold and drought	Backward	Sowing complete Germination poor
J	Cold, changeable		Average yield expected	Little haymaking Plenty grass	E. fair : being lifted M.C. backward	Backward
J	Dull, cold, wet		Fair : some lodging 2 to 3 weeks late	Hay difficult : yield and quality reduced Grass good	Earlies lifted : good yield > average	Fair : weed and aphid trouble
A	Cold, wet	Harvest protracted and difficult	Late : lodging and fungus disease	Hay difficult : yield and quality reduced Grass good	Good growth : widespread blight	Healthy : average yield expected
S	Cool, changeable	Some harvesting – nearly complete in E. Little cultivation	Quality poor	Some hayfields abandoned Quality still lower Grass good	Slow start to lifting Widespread blight	Lifting started : virus disease widespread
O	Mild : some floods	Ploughing in E. and SE.	Some fields abandoned Quality poor	Hay abandoned Grass good	Lifting delayed in N. and W.	Lifting
N	Wet	Much in arrears	Early sown crops good	Too wet for grazing	Lifting in E. Blight	Slow harvest
D	Wet – mild	Some progress	Backward	Fields waterlogged	Still some to be lifted	Sugar content low

of fungus diseases, and thence or otherwise made the crops susceptible to lodging – the flattening of the crop under the impact of wind-driven rain. Third, harvesting weather was bad, and in some places cornfields had to be abandoned. The same bad autumn weather also interfered with drilling of winter grain for 1955, and even where drilling had been early enough, the crops started 1955 in a retarded state because of early winter weather.

There are two important forms of grass. First, there is that cut and conserved, referred to here as ' hay ' : second, there is that eaten on the field by cattle, referred to here as ' grass.' This crop will be discussed in a little more detail with Table 3, but the main history of the 1954 crops is clear from Table 1. In the early part of the year growth was retarded by cold, and in April by drought. As commentary on this, the newly-planted grass plots on the irrigation experiment at Woburn had to be irrigated in May to save the crop. From June onward, the grass grew well, and for several months there was abundant grazing. Opportunities for cutting were, however, few, and outdoor drying almost impossible so that hay stood long in the fields, quality deteriorating all the while, and eventually a lot had to be abandoned. Hay prices rose fantastically toward the end of the year. Grass was good until the end of the year when heavy rain made the ground too wet for grazing.

The potato story, too, shows several aspects of weather effects on crops. Early in 1954, the 1953 crop in store suffered some frost damage. Frost also caused some damage to earlies, delaying development. When lifted, the crop was fairly good, with a yield a little above average. Growth of maincrop (M.C. in Table) was retarded by poor weather, and the humid weather was favourable for potato blight. The wet autumn delayed lifting and there were still some potatoes in the ground at the end of the year.

Sugar beet had a similar history of delayed growth early in the summer, excessive weed petition because of wet weather, and aphis attack (probably a weather effect at second hand). of the objection to aphis infestation is that the aphids act as vectors for virus diseases and reports show widespread virus disease, leading to reduced yields. Harvesting was slow use of wet weather at the end of the year.

2. Crop yields in 1954

On the whole, Table 1 is a story of delay and disease. It shows nothing of the effects of an skill and tenacity in attempting to overcome the handicaps imposed by weather, nor he ability of the plant to make the best of bad weather.

Table 2 shows the effect of the weather on some of the more important crops in terms of d per acre. Three figures are given, including a ten-year average. For the grains, the 1954 ds were above average, though below the 1953 values. The same is true of the root crops. e it should be noted that the 1953 potato and sugar beet yields were record yields, and the 4 potato yield is stated to be equal to the pre-1953 record. In the next group, hops have been uded for the benefit of those who will regard this as a major crop ! The results for grass are only ones in which the area is of possible relevance in this survey. The 1954 area given is hat actually cut and harvested, and assuming some permanence in the area of permanent ss it appears that about 10 per cent of the hay crop had to be abandoned because of harvesting iculties. Yields were down on 1953, and quality too.

TABLE 2. Estimates of production 1953, 1954 and 1944-53

Crop	Area (1 June) (Thousands of acres) 1953	1954	Estimated yield/acre 1944-53 (10-year mean)	cwt 1953	1954
eat	2,143	2,376	20·5	24·0	22 6
ley	2,025	1,874	19·2	22·6	21·4
s	1,663	1,469	17·9	21·2	19·2
ns for stock	140	119	15·3	17·8	15·8
				tons	
atoes	672	649	7·3	8·7	7·9
ar Beet (roots)	401	420	10·2	12·8	10·5
rnips and Swedes	304	287	13·7	17·8	16·0
ngolds	207	197	21·1	25·7	22·0
				cwt	
ps	22	20	13·3	12·5	12·0
from temp. grass	1,890	1,880*	—	31·9	29·1
from perm. grass	2,571	2,293*	—	22·5	21·3

* Area cut and actually harvested.

TABLE 3. Grass Yields : Grassland research station

Experiment H9. Each block sown the year before the first for which a figure is given.

	Yields in cwt of dry matter/acre			
Block	1951	1952	1953	1954
1	66·5	54·0	56·0	—
2		54·0	54·5	69·0
3			54·5	70·0
4				78·0

3. Grass and hay

In discussing Table 1 it was stated that late in the summer the grass grew well enoug As an indication of this, Mr. W. Stiles of the Grassland Research Station has supplied yiel from one of their experiments on four-year leys. The figures are yields in the second, third a fourth years of the experiment, and there is no doubt about the excellence of the late summ and autumn growth. Grass however, is not much use unless it can be cut or grazed at the rig time, and Mr. A. S. Barker of I.C.I. has supplied a commentary based on experience on I.C farms that is probably valid for the whole country. Summarized it is :

The onset of spring growth was unusually late, which upset the whole programme of grassland manag ment during the first half of the season on most farms. Many swards were over-grazed at that time and fie which would normally be set aside for conservation later had to be grazed. There was a sudden spate of grow in June, which occurred too quickly for most hay and silage crops to be cut at the best time. This in tu prevented aftermaths being available at the usual period and, of course, the dull and wet weather duri the normal conservation season impeded progress and reduced quality. Although from this time onwar to the end of the season there was abundant grass available, the herbage and land were far too wet for effici utilization or for herbage of good feeding quality.

TABLE 4. Sugar yields (cwt/acre)

Year	Average national yield (B.S.C.)	Site	Experimental irrigation Control	Best irrign.
1943	30·5			
1944	24·0			
1945	31·3			
1946	34·7			
1947	28·7	Milford	77	99
1948	35·4	Milford	66	67
1949	30·2	Milford	34	44
		Kesgrave	31	49
1950	42·3	Kesgrave	52	63
1951	37·1	Woburn	50	54
1952	35·8	Woburn	52	62
1953	42·7	Woburn	84	86
1954	34·0	Woburn	44	44

4. Sugar beet

Sugar beet is one of the few crops for which accurate yields are available. Table 4 show the average yield of sugar per acre for the last twelve years in the column sub-headed B.S.C To these figures, supplied by Mr. O. Rose of the British Sugar Corporation, have been adde those for sites of irrigation experiments wholly or partly controlled from Rothamsted. Th B.S.C. column demonstrates that sugar yield is not a good climatic index : it would show 194 and 1949 as worse years than 1954. Weather modification can be important as shown by benef from irrigation in five years out of eight, but plant hygiene could do more, e.g. compare Milfor results in 1948 and 1949, or Woburn in 1953 and 1954. Both 1949 and 1954 were years of sever virus infection. Better farming skill in growing the crop would also help : note that the exper mental sites have given better, sometimes very much better, yields than the national averag

5. Salt-damaged land

So much for 1954 and everyday obvious farming. There is a not-so-obvious farming proble where the weather of 1954 might have been very important. On the east coast there are larg areas of salt-damaged land slowly recovering from the effects of the North Sea floods of January 1953. Part of the reclamation process is the washing out of the salt by rainwater. Where field are under-drained heavy rain is useful, and Mr. P. J. O. Trist, who is responsible for some of th work of reclamation, states that where there was a fair rainfall and adequate drainage the summe of 1954 was beneficial. Much of his area however is marsh, with very little under-drainage

Here, for more than one reason, his technical problem is to try to prevent the surface layer of soil from standing waterlogged, which is possible with a system of marsh dykes and surface water furrows, provided that rainfall distribution is not too unkind. For the most part the 1954 rainfall was helpfully distributed. Broadly, the 1954 summer did no harm anywhere, and in some places did some good, so that the general reduction in salt levels during 1954 continued more or less according to plan.

Summary

Summarizing, 1954 weather was most marked in its grievous interference with soil management and crop husbandry. Farming skill and energy succeeded in doing much to counteract this, so much so that final yields were generally above average. The weather generally retarded growth and encouraged fungus diseases, so reducing yields and quality. Some of the effects of 1954 weather may appear in 1955 crops.

Acknowledgments

The material for Table 1 is taken from the ' *Monthly Agricultural Report for England and Wales*,' and, for Table 2, from the ' *Agricultural Returns of England and Wales*, 1954,' both issued by the Ministry of Agriculture and Fisheries. For the remaining data and comments I gratefully acknowledge my indebtedness to Mr. W. Stiles of the Grassland Research Station, Mr. A. S. Barker of I.C.I., Mr. O. Rose of the British Sugar Corporation, and Mr. P. J. O. Trist of the East Suffolk Agricultural Executive Committee.

Discussion

Mr. R. F. M. Hay : Before showing two slides of seasonal anomalies of air and sea temperatures, during 1954, over the NE. Atlantic, I would like to draw attention to the distribution of sea temperature, at the end of the winter, in the surface layers in this region. The obvious inference, that the storminess of this region would bring about an isothermal distribution of sea temperature prevailing to a considerable depth by the end of the winter, is confirmed by recent bathythermograph observations down to a depth of 450 ft made at the British Ocean Weather Stations. Such an isothermal distribution can reasonably be assumed to exist at the end of each winter.

Late-winter sea-temperature anomalies around the British Isles thus imply the existence of an excess (or deficiency) of warmth to a considerable depth. In two summers when the amounts of solar insolation penetrating the surface are assumed to be equal, the sea temperatures will thus be higher after the late winter which had the higher sea temperature, other factors being equal.

The slides, which are based entirely upon observations made by British selected ships, show that a large area of the ocean around the British Isles had positive air- and sea-temperature anomalies in the winter (December 1953-February 1954) and spring (March-May 1954). It follows that the large negative anomalies (up to 3°F locally) of air and sea temperature which occurred in the summer indicate that the deficiency of solar heat-energy penetrating the surface throughout the summer in these regions was even more marked than the summer data would suggest, when considered in isolation. Similar data for the N. Atlantic ocean weather stations broadly confirm the pattern of distribution of anomalies of air and sea temperatures shown by the selected ship observations. In computing anomalies, the normals used were based upon the data in the *Climatological Atlases* of the Marine Branch for the periods 1887-99 and 1921-38. However, in the case of some parts of the north-west of the area and of the ocean weather stations Able and Baker (in the Denmark and Davis Straits respectively) means based on 5-yr records from the U.S. Weather Ships manning stations ' A ' and ' B ' were the only data available.

Mr. D. J. Schove : I am particularly interested in Dr. Glasspoole's observation that 1954 may indicate the turn of the tide. He states that, since the 1920's the mean pressure in England has steadily risen and that, in the 1950's, the pressure gradient has begun to increase. This situation is thus similar to that of the 1890's and, according to the principle of south-steering (*Quart. J. R. Met. Soc.*, 1950, **76**, p. 147, Table III) this should indicate a fall in pressure in the next decade or so. The reason for the motion of pressure anomalies from N. to S. may be

that small variations in the frequency of occurrence of offshore winds cause a persistent difference in the temperature of the cold water upwelling on the western coasts of Europe. Colder water means a very slight rise of pressure and perhaps drier conditions; the increase of upwelling since the 1920's may have helped to cause the rise of pressure, and drier conditions, noted in this country up to *c.* 1950.

I should be interested to know of some method by which this upwelling theory of south-steering can be tested, and perhaps the methods used by Mr. Hay will elucidate the mechanism. Meanwhile it looks as if a dry phase affecting S. and E. England (1954 excepted) will be replaced in the next decade or so by a wet phase already beginning to affect the W. and N.

The correlation of icebergs and world weather – mentioned by some speakers – is significant and has been discussed in recent papers by I. I. Schell.

Mr. H. H. Lamb : I think there is a serious difficulty in interpreting climatic behaviour from averages and totals which provide a welter of figures but, for all that, treat months (or longer periods of the year) as a block. The range of variability of day-by-day patterns and events is also important. Hindrance to human activities, caused by the weather of 1954, was materially increased by an unusual persistence of spells and the repetitiveness of weather patterns within the individual months. Thus the flooding in the autumn visited the same afflicted localities in the N. and W. of Britain again and again, partly because of the closely similar tracks followed by different disturbances.

I believe that the position and alignment of the stronger frontogenetic cols (marked *inter alia* by a surface temperature-contrast of at least 5°C) can provide a useful indicator of how the individual daily circulation-patterns compare with each other and with the resultant average pattern for the month, as well as with the normal for that month. These cols are significant, because they represent the zone of sharpening thermal contrast, ahead of the major cold troughs in the upper flow, and are, on the whole, favourable regions for the origins of developing frontal waves. Moreover, the orientation of the col corresponds to the steering current in the upper troposphere and the track along which developing wave disturbances will be initially directed.

Maps of the daily position of the stronger frontogenetic cols in various months of 1954 show considerable displacements from the normal positions. In both January and February, when much of the North American continent was unusually cold (as shown by the 1,000-500 mb thickness anomalies presented by Dr. Forsdyke), the Florida-Bermuda col was for the most part displaced far east towards the Azores and so orientated in January that cyclonic disturbances were steered north towards Greenland. This was a time when anticyclones were centred near Ireland. In February, however, the alignment of the frontogenetic cols changed and developing cyclonic disturbances were so directed that they crossed England. In April, northern Canada was still intensely cold and depressions again travelled on tracks near Greenland, avoiding the eastern Atlantic, whilst Britain enjoyed anticyclonic weather. In June and July 1954, the frontogenetic col, normally found between S. Hudson Bay and the Great Lakes, was entirely replaced by a concentration of daily occurrences a little north of Bermuda, near the limit of the Labrador stream. The orientation of these cols directed developing disturbances towards the British Isles.

These observations suggest that the coldness of much of North America and Eurasia in the early part of 1954 and their unusual warmth in the summer – in the latter season the configuration was combined with unusually cold conditions over the Labrador stream (icebergs reached 40°N in 48°W.) – were of importance for weather in the British Isles.

Mr. F. J. Parsons : I would like to ask whether Dr. Glasspoole can give any information on the quantity of ice present in the N. Atlantic during the spring of 1954, especially as he has made reference to the cool wet summer of 1912, and I recall we had, that year, a very fine dry bright April during which the amount of Atlantic ice was abnormal – in fact the *Titanic* disaster resulted from it.

Mr. R. F. M. Hay (*in reply to Mr. Parsons*) : Cool, wet summers in the British Isles are not considered to be related to the numbers of icebergs which are carried south by the Labrador current to disintegrate off the Grand Banks of Newfoundland (in the cool, rainy summer of 1924, 11 bergs reached the Tail of the Bank; in the relatively warm and dry summer of 1929 the number of bergs was 1351, to take just two examples). On an average, about 400 such bergs are carried down in a single season, the average for May alone being 130. Making reasonable assumptions about the number and size of the bergs, the area of sea in which they melt and the depth through which the melt-water is mixed with its environment, it can readily be shown that the fall of sea-surface temperature over an area of any size (say 10^4 km^2) would be less than 1°F, due to the melting of icebergs alone along the region of confluence of the Labrador current

nd the Gulf Stream (locally for short periods, of course, the fall of temperature might be larger ntil mixing took effect). Large fluctuations of temperature off the Grand Banks are more ikely to occur because of shifts in the position of the line of confluence and changes in the tem- peratures of the two main currents the lattter being caused partly by seasonal meteorological factors (including the melting of icebergs at an earlier stage), and partly by changes in surface currents and consequent rate of advection of sea-surface isotherms.

Mr. R. G. VERYARD : I would like to utter a word of warning regarding making a comparison of the weather in one year (or season) with that in another – especially for the British Isles as a whole. The whole country does not constitute a climatological unit – it is certainly necessary to treat Scotland separately from England and Wales; but, in these smaller areas, maritime or continental influences may be detected in one part yet not in the other – especially as between east and west. As a matter of interest I spent some time going through back records to see whether I could find a year comparable with 1954 both in respect of the temperature and rainfall distribu- tion either in Scotland or in England and Wales and I did not succeed. This does *not* mean that 1954 was an exceptional year – it means that, like synoptic charts, the pattern and sequence of weather in any two years are never really alike even in small parts of a country like ours which has such very variable weather.

Dr. R. C. RAINEY (*partly communicated*) : One of the most striking anomalies in the weather of 1954 was the rainfall in Scotland in October, which Dr. Glasspoole has shown to have amounted to 190 per cent of normal. Between the 15th and 18th of the month, in particular, southern Scotland experienced ' its worst rainstorm within living memory ' (*Times*, 19 Oct. 1954); and the exceptionally well-developed southerly airstream which contributed to the intense associated convergence also transported from Africa the first Desert Locusts to reach the British Isles since 1869.

Prior to 15 Oct. the nearest reported locust swarms were on the coast of Spanish Sahara and in southern Algeria. That morning swarms appeared in the Canary Islands (the beginning of the first heavy invasion of the Islands since 1932) and further north over the Atlantic and, by nightfall, groups of flying locusts were seen at sea as far north as the latitude of Gibraltar (log extracts kindly furnished by the Marine Division of the Meteorological Office). At the French ocean weather ship *Le Verrier*, stationed about 700 mi west of Bordeaux, the southerly spell culminated in a full gale from SSW. on the night of 16/17th (M.O. *Daily Weather Reports*); and on the afternoon of the 17th two Desert Locusts were picked up at St. Martin's in the Scilly Islands (subsequently exhibited, still living, at the Royal Entomological Society) and another just offshore at Tramore, Co. Waterford – some 1,700 mi from Spanish Sahara.

Mr. G. A. Corby at the Meteorological Office, Dunstable, has very kindly tracked back the air in which these locusts are likely to have arrived, starting from a position 50°N., 07°W. at 06 GMT. on 17 Oct. A trajectory for a height of the order of 500 ft (obtained by backing the geostrophic winds given by the m.s.l. isobars by 10° and reducing their speed by 10 per cent) passes at the appropriate time within 80 mi of the position of one of the sightings of flying locusts west of Gibraltar, and leads back to the north-west African coast in the vicinity of the Canaries on the night of the 14/15th, – a particularly appropriate time and place, as already indicated, to have provided a source of locusts.

The mere presence of such a source in north-west Africa is not unusual; Desert Locust swarms have, for example, been reported in Algeria during 51 out of the last 65 years. In 1954 they were undoubtedly particularly numerous in this area – the swarms which began to invade southern Morocco for example on 16 Oct. inflicted crop losses on a scale without precedent during the current outbreak – but it is also suggested that the first arrival of Desert Locusts in the British Isles for 85 years may be regarded as additional evidence of the unusual nature of the synoptic situation which was responsible for the Scottish floods.

Mr. L. C. W. BONACINA (*communicated*) : A noteworthy feature of the summer of 1954 in the London area about which there seems to have been little comment was the unusual frequency of severe thunderstorms in each month from May to August. Of those personally experienced by myself, particularly violent on the electrical side were the storms of the afternoon of 12 May in S. Kensington, of the night of 12 June in Hampstead and of the evening of 6 August in Hampstead. Cold summers actually often are thundery showing that when general meteorological conditions will not permit the energy of the high summer sun to raise temperatures it will be available to promote convectional disturbances.

There seems to be little doubt that, for combined cold, gloom and wet, the summer of 1954 was the worst in England this century and probably since 1888. But I think we should be careful to view this fact in proper perspective. After all, every season departs in some manner from that

convenient standard of reference which we call the normal, and every season has its own unique meteorological topography – never repeated – as regards the weather elements distributed in space and time. Therefore I cannot understand why a summer like 1954 which departs very widely from the normal and so only occurs rarely should be regarded as in any way more wonderful or deserving of special explanation than more ordinary summers. The major swings of the pendulum, so to speak, are as much part of the natural rhythm of climate as the minor ones, and for my part I cannot think of the summer of 1954 in any other light than as a normal abnormality. I feel strongly that the term 'normal abnormality' is sound climatological philosophy, and not a facetious comment on the fact that no season is ever really normal. A true abnormality would be a complete dislocation in order of magnitude such as apparently never does happen in meteorological experience. 'Records' are only such by narrow margins, as a general rule.

Miss C. M. Botley (*communicated*): In view of Dr. Glasspoole's closing remarks about some of the popular reasons for bad weather recently and in the twenties, it is interesting to compare the remarks of Mr. Peter Opie in a paper 'England the great undiscovered' which he read before the Folklore Society two years ago. Speaking of the idea that unusual weather since the war has been due to atomic disturbances he said 'This belief follows a well-established folk-pattern continually recurring in different forms' (*Folklore*, **65**, p. 162, 1954). He cites the fairly well-known instance of the question in Parliament in 1911 about the advisability of gunnery practice during the harvest.

Report of the Council

for the year 1954

(Submitted to the Annual General Meeting, 20 April 1955)

General

The Council are glad to report yet another year of progress. For the fourth year in succession there has been an increase over the previous year in the number of Fellows and Foreign Members of the Society, so that for the second consecutive year the total number of Members has exceeded 2,000.

A new local centre was formed at Manchester towards the end of the year.

The Proceedings of the Toronto Meteorological Conference 1953 were published during the year, in addition to the *Quarterly Journal*.

The first two-day Summer Meeting, consisting of scientific symposia, was held at Edinburgh University. Fuller details are given later in the report.

The fifth vacation course held at Malham Tarn Field Centre, in conjunction with the Council for the Promotion of Field Studies, was very successful and 17 students were subsequently elected to Membership of the Society.

Council

From 1 January to 30 September the constitution of the Council was as recorded in the 1953 Annual Report.

From 1 October, 1954 the following Council held office :

President

Oliver Graham Sutton, C.B.E., D.Sc., F.R.S.

Vice-Presidents

Professor Percy Robert Crowe, Ph.D.
Sir Charles Normand, C.I.E., M.A., D.Sc.
Cameron Darrell Ovey, M.A., B.Sc., F.G.S.
Professor Percival Albert Sheppard, B.Sc., F.Inst.P.

Vice-President for Canada

Frank Waldemar Benum, M.A.

Vice-President for Scotland

Alexander Ebenezer Mclean Geddes, O.B.E., M.A., D.Sc.

Treasurer

James Strachan Farquharson, M.A., D.Sc.

Secretaries

Howard Latimer Penman, M.Sc., Ph.D., F.Inst.P.
George David Robinson, Ph.D., F.Inst.P. (Librarian)
Richard George Veryard, B.Sc.
Thomas Wilson Wormell, M.A., Ph.D. (Editor)

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Charles James Boyden, B.A.
George Arthur Bull, B.Sc.
Henry Charnock, M.Sc., D.I.C., A.Inst.P.
James Marston Craddock, M.A.
Ronald Frith, O.B.E., Ph.D.
Ralph Edwin Lacy, B.Sc.
Frank Henry Ludlam, D.I.C.
Instr.Lt. Thomas Arthur Marshall, B.Sc., R.N.
Frank Pasquill, D.Sc.
Professor Norman Pye, B.A.
Richard Segar Scorer, M.A., Ph.D.
Mrs. Ann Courtenay Welch, M.B.E.

The Council met 9 times during the year and there were also held 33 meetings of the various Committees of the Council.

Meetings

Eight Ordinary meetings were held in the Society's rooms during the year, including two discussion meetings with invited experts. The first, in January, was on ' Smog.' The second, in November, was on ' Water supply ' and after it there was an informal dinner at Ayrton Hall, Imperial College, at which 50 Members and guests were present.

In February Mr. J. S. Sawyer repeated his Popular Lecture on ' Weather forecasting.'

The Symons Memorial Lecture on 7 April was given by Dr. G. K. Batchelor on ' Heat convection and buoyancy effects in fluids.'

The Annual General Meeting was held in the Society's rooms on 28 April, 1954, when the Report of the Council and the Annual Accounts for 1953 were adopted. Dr. O. G. Sutton, C.B.E., F.R.S. then gave his Presidential Address on the subject, ' The development of meteorology as an exact science.'

The Ordinary Summer Meeting took place on 7 July at the National Institute of Oceanography, Wormley, Surrey by kind invitation of the Director. Council wish to record their appreciation of a most enjoyable visit.

Two-day Summer Meeting, Edinburgh, 15-17 July 1954.

The first two-day Summer Meeting to be arranged by the Society was held in the Department of Natural Philosophy, Edinburgh University. The meeting consisted of two scientific symposia, with an attendance at each of some 100 Members and visitors.

The first on ' The upper atmosphere,' with the Society's President, Dr. O. G. Sutton, C.B.E., F.R.S., in the Chair, was opened by Sir Edward Appleton, F.R.S. The second symposium was on ' Meteorology and Scottish agriculture,' with Dr. A. E. M. Geddes, O.B.E., M.A., Vice-President for Scotland, in the Chair. At each symposium there were six opening speakers and then a discussion.

In addition a very enjoyable excursion to Eskdalemuir Observatory was arranged, attended by the President and some 70 Members and guests.

Council wish to record their sincere gratitude to the Principal of Edinburgh University and the Professor of Natural Philosophy for the use of the University facilities, to the Director of the Meteorological Office for the opportunity to visit the Observatory and to the Secretary of the Scottish Centre, Mr. J. Paton, who was responsible for making the local arrangements.

Publications

The inflow of scientific communications for the *Quarterly Journal* and *Weather* was maintained at approximately the same level as in the previous year. 82 papers, including 8 read at the Edinburgh meeting, were submitted for publication in the *Quarterly Journal* during 1954 as against 72 in 1953. 50 of the papers have already been accepted for publication. The average time between receipt of approved copy and publication in the *Journal* was about $3\frac{1}{2}$ months.

The four numbers of the 1954 *Journal* together contained 662 pages, and included three contributions in the series ' Reviews of modern meteorology.'

By arrangement with the American Meteorological Society the *Proceedings of the Toronto Meteorological Conference 1953* were published in the format of the *Quarterly Journal* in a volume of 294 pages, containing 45 papers dealing with 9 symposia.

Council are greatly indebted to the Editor, Dr. T. W. Wormell, the Editing Committee and the proof readers, Mr. E. L. Hawke and Cdr. C. R. Burgess, for their services in the production of these publications.

The Society's monthly magazine *Weather* has maintained a high standard of production. Council are indebted to the *Weather* Editing Committee, and particularly to Mr. D. D. Clark and Mr. D. Chilton, who have now resigned after long service.

Membership

Table A

	Fellows and Foreign Members			Associates	Students	Honorary Members	Total
	Annual	Life	Total				
31 Dec. 1953	1,370	181	1,551	343	97	18	2,009
Since elected	107	—	107	—	19	1	127
Readmitted	4	—	4	—	—	—	4
Transferred	6	2	8	—	− 8	—	—
Deceased	− 9	− 3	− 12	—	—	—	− 12
Resigned, etc.	− 61	—	− 61	− 39	− 6	—	− 106
31 Dec. 1954	1,417	180	1,597	304	102	19	2,022

Deaths

The following deaths are recorded with regret, the dates of election being appended to the names: Wg. Cdr. D. M. Egan (1947), C. Griffiths (1950), Sir Nelson K. Johnson (1920), R. M. Ker (Life Fellow, 1919), T. R. Ker (Life Fellow, 1905), L. G. H. Lee (1904), T. L. Ormiston (1930), C. H. Plant (1920), W. C. Reynolds (1942), A. W. Talbert (1952), R. S. Whipple (Life Fellow, elected 1920), C. N. Wynn (1944).

Library

The author catalogue of the books was completed during the year and a start was made on cataloguing the collection of pamphlets. During the year 189 books and other material were issued on loan and 69 enquiries dealt with. Visitors to the library included the librarians from the South African, American and Irish Meteorological Services; to the first was given a list of the Society's publications relating to the Antarctic for inclusion in a bibliography to be published in South Africa.

Council wish to record their thanks to the Librarian, Dr. G. D. Robinson, for his valuable work during the year.

Staff

At the close of the year the staff consisted of Mr. M. E. Shoenberg, B.A., Assistant Secretary; Mr. R. S. Read, I.S.O., M.A., B.Sc., A.R.C.S., F.Inst.P., Assistant Librarian; Mrs. A. Wallis, Accounting Assistant and Miss G. C. Jones, *Weather* Clerk; together with the part-time services of Mrs. W. A. Steward, Publications Clerk. Miss J. S. Sampson resigned her position as Secretarial Assistant and was replaced by Miss A. F. Hayward on a wholetime basis as from 1 November. Miss A. Dent, Junior Clerk, resigned and was replaced by Miss Y. M. Cullum as from 1 October.

The thanks of Council are offered to the Assistant Secretary and his colleagues for the high standard of efficiency maintained in an exceptionally busy year.

Branch and Centres

Canadian Branch

Eight Meetings were held during the year at Toronto, one being the Annual Business Meeting at which the Presidential Address was delivered. The remaining Meetings consisted of lectures, presentation of papers, or panel discussions. The December meeting took the form of a public lecture, and was called ' The Toronto Weather Festival.' The meeting was held in the theatre of the Royal Ontario Museum, and was well received by the general public.

Regular Meetings were also held at the newly-formed Montreal Centre.

Membership and interest in the Canadian Branch continued at a high level, with the total membership at the end of the year at 331.

The members of the committee were : President, D. P. McIntyre; Vice-President, F. D. Thompson ; Vice-President for Canada, F. W. Benum ; Treasurer, J. P. Potter; Secretary, K. T. McLeod; Councillors-at-large : D. Strachan, C. E. Thompson, J. A. McPherson, B. A. Power, R. E. Munn.

Montreal Centre. The newly-formed Montreal Centre met 7 times during the year at the Faculty Club, McGill University. The Officers were : President, R. H. Douglas; Secretary, W. S. Harley; Treasurer, W. Hitschfeld.

Scottish Centre

Four Meetings were held during the year at Edinburgh University. In addition to its normal activities, the centre acted as host to the Society at its two-day summer meeting held in Edinburgh in July. The summer excursion to Eskdalemuir Observatory was arranged to take place during the two-day summer meeting. The Officers were : Chairman and Vice-President for Scotland, Dr. A. E. M. Geddes; Secretary and Treasurer, Mr. J. Paton.

Manchester Centre

In response to numerous requests from Members in the area, Council agreed to the formation of a local Centre based on Manchester. The inaugural meeting was held on 3 December and was addressed by the President, Dr. O. G. Sutton, C.B.E., F.R.S., on ' Some current problems in meteorology.' The following committee was elected : Chairman, Professor P. R. Crowe, Secretary and Treasurer, Dr. Ruth Rogers, both of Manchester University, and 11 Fellows mainly from the Universities and the Meteorological Office in the area.

Representatives

The Society's representatives on other scientific bodies were :

Royal Society Committees

Gassiot Committee, President; *Scientific Centre Accommodation Committee*: President; *British National Committee for Geodesy and Geophysics*, E. Gold (1949/54), O. G. Sutton (1952/57); *British National Committee for Physics*, F. J. Scrase (1949/54).

Institute of Physics Committees

Journal of Scientific Instruments Advisory Committee, F. Pasquill; *British Journal of Applied Physics Advisory Committee*, J. S. Forrest (January-September), G. D. Robinson (October-December).

Cambridge University Meteorological Committee

Sir Charles Normand

Royal Astronomical Society

Geophysical Committee, G. D. Robinson

Advisory Committee for the Meteorological Office, Edinburgh

Sir Charles Normand

Awards

Hugh Robert Mill Medal and Prize for 1954.

To C. K. M. Douglas, O.B.E., A.F.C., for his outstanding contributions to the science of meteorology with particular reference to rainfall.

Buchan Prize for 1949-53.

To R. S. Scorer, M.A., Ph.D., for his outstanding contributions to dynamical meteorology published in the *Quarterly Journal*.

Darton Prizes for 1953.

Home award : Messrs. B. J. Mason and R. Ramanadham for their paper ' A photoelectric raindrop spectrometer ' (*Q.J.*, **79**, p. 490).

Canadian awards :

1st Prize : Dr. W. L. Godson, for his paper ' The evaluation of infra-red radiative fluxes due to atmospheric water vapour ' (*Q.J.*, **78**, p. 490).

2nd Prize : Mr. G. W. Robertson, for his paper ' Some agro-meteorological problems in Canada ' (*Canadian Branch Papers*, 1953, No. 2).

The awards were presented at the Annual General Meeting on 28 April.

Napier Shaw Memorial Prize

The first competition was announced in March, 1954; a prize of not less than £100 being offered for the best essay or monograph on ' The energetics of the general circulation.' Full details were published in the *Quarterly Journal* (p. 288, 1954). The closing date for entries was given as 4 March 1956.

Scientific Activities Fund

During the year the following grants were made :

Dr. R. S. Scorer in respect of attending the 3rd Conference on Alpine Meteorology at Davos in April.

Mr. A. R. Curtis, Mr. J. T. Houghton, Dr. G. F. Walton for attendance at the U.G.G.I. Conference at Rome in September.

Mr. J. K. Rothwell towards the purchase of a Stevenson screen.

In addition a grant was approved, for payment in 1955, towards the expenses of the newly-instituted vacation course in elementary meteorology for sixth-form students, organized jointly with the C.P.F.S., to be held in September 1955 at their Juniper Hall Field Centre, Surrey.

Bequests

Council gratefully acknowledge the bequest by the late Mr. E. C. Rutherfurd, of a collection of some 70 slides, mainly of cloud forms.

Finance

Attention is drawn to the modified form of presentation of the Accounts, under which the basis of accounting has been changed from Receipts and Payments to Income and Expenditure. This more realistic method enables a clear picture of the year's working to be given, as all transactions for the year are recorded in the Accounts, whether paid during the year or not. In this way the excess (or otherwise) of income over expenditure for the year is clearly shown and it will be seen from the balance sheet in the Accounts, which follow, that there was a small surplus on the year's working.

The continued record membership of the Society and the flourishing conditions of its publications as indicated elsewhere in the Report are reflected in the satisfactory financial position.

£523 (as compared with £525 in 1953) has been received as a refund from the Inland Revenue in respect of deeds of covenant signed by Fellows in relation to their subscriptions. In view of the substantial benefits accruing to the Society under this scheme it is hoped that more Fellows will be encouraged to sign a seven-year deed for their subscriptions.

Council record with gratitude an allocation to the Society by the Royal Society Committee of £1,000 from the Government Grant-in-aid of Publications Fund, with an addition of £600 for the Library.

GENERAL

INCOME AND EXPENDITURE ACCOUNT

EXPENDITURE

	£	£	£
ESTABLISHMENT—			
Architect's Fee		21	
Rates		12	
Cleaning Wages and Materials		265	
Insurance		22	
Lighting and Heating		177	
Repairs and Maintenance		190	
Depreciation (including instruments wholly written off £16)		123	
			810
ADMINISTRATION—			
Salaries		2,275	
Contribution to Staff Pension Fund		390	
Printing		148	
Stationery		109	
Postage, Carriage and Packing		654	
Telephone		79	
Audit and Accountancy Fees – Royal Meteorological Society		79	
– *Weather* Magazine		21	
Sundries		26	
			3,781
PUBLICATIONS, ETC.—			
Quarterly Journal		3,419	
Illustrations		272	
Proceedings Toronto Meteorological Conference 1953—			
Printing	1,425		
Reprints	146		
Illustrations	200		
Postage and Packing	38		
		1,809	
Reprints of Papers		206	
Reviews		7	
Monthly Weather Report		100	
Members' copies of *Weather*		670	
Second-hand *Journals*		102	
			6,585
LIBRARY—			
Salary		442	
Books, Bindings, etc.		148	
			590
MEETINGS' EXPENSES			137
PROVISION FOR BAD AND DOUBTFUL DEBTS			100
GRANTS AND PRIZES—			
Buchan Prize		16	
Grant to British Gliding Association		20	
Grant to Society for Promotion of Field Studies		25	
			61
BALANCE BEING EXCESS OF INCOME OVER EXPENDITURE			116
			£12,180

FUND

FOR THE YEAR ENDING 31 DECEMBER 1954

INCOME

	£	£	£
SUBSCRIPTIONS, ETC.—			
Subscriptions		5,086	
Income Tax recovered from Covenanted Subscriptions		523	
Entrance Fees		105	
Donations		13	
			5,727
PUBLICATIONS—			
Sale of Publications		2,759	
Advertisements		262	
Proceedings Toronto Meteorological Conference 1953			
Sales and Advertisements	1,266		
Donations, etc.	18		
		1,284	
		4,305	
Grant from Royal Society (excluding Library)		1,000	
			5,305
INVESTMENT INCOME—			
Interest on Government Stocks (gross)		195	
Rent of Maisonettes	236		
Less: Rates £89, Repairs £15	104		
		132	
			327
PHOTOGRAPHS—			
Sale of Cloud Postcards		18	
Clarke and Cave Copyright Fees	74		
Less: Reproduction Costs	3		
		71	
			89
LIBRARY—			
Grant from The Royal Society		600	
Sale of old Library Books		21	
			621
AMOUNT RECEIVED FROM OTHER FUNDS—			
Life Composition Fund		96	
British Rainfall Organization Fund (for administration)		15	
			111
			£12,180

GENERAL FUND

BALANCE SHEET AT 31 DECEMBER 1954

	£	£
ACCUMULATED FUND—		
Excess of Assets over Liabilities at 1st January 1954	11,836	
Adjustment owing to change in basis of accounts	564	
		12,400
Add : Excess of Income over Expenditure		116
CURRENT LIABILITIES		
Creditors and Accrued Charges—		
Proceedings Toronto Meteorological Conference 1953	150	
Others	1,428	
	1,578	
Amounts received in advance—		
Publications	412	
Subscriptions	240	
	652	
		2,230
		£14,746

We have examined the above Balance Sheet and annexed Income and Expenditure Account which in our opinion respectively give a true and fair view of the state of the Society's affairs as at 31st December 1954 and of its income and expenditure for that year.

Spencer House,
South Place,
LONDON, E. C. 2.
6 *April* 1955.

BALL, BAKER & Co.,
Chartered Accountants.

	£	£
FIXED ASSETS—		
Freehold Property – Estimated value (in 1925) of the Society's house, 49 Cromwell Road		5,375
Furniture and Equipment—		

	At 1.1.54	Additions during year	Depreciation	Net
General	1,000	192	90	1,102
Library	209	20	17	212
	£1,209	£212	£107	1,314

(No amount has been included for the value of the Library and the stock of publications.)

	£	£
INVESTMENTS—		
Valued at market price 31.12.54—		
Government of Australia 4% Registered Stock 1955-70 £649.1.7.	645	
4% Funding Stock 1960-90 £200	206	
3½% Conversion Loan 1961 or after £900	796	
2½% Consolidated Stock £1,400	924	
Government of Australia 4% Registered Stock 1966-68 £2,000	2,030	
4% Consolidated Stock 1957 or after £1,250	1,206	
		5,807
CURRENT ASSETS—		
Debtors—	£	
Proceedings Toronto Meteorological Conference 1953	650	
Subscriptions	267	
Publications	490	
	1,407	
Less : Provision for bad and doubtful debts	100	
	1,307	
Payments in advance	23	
Deposit at Post Office, Balance at Bank and Cash in Hand	920	
		2,250
		£14,746

RECEIPTS AND PAYMENTS OF SPECIAL FUNDS FOR 1954

	Balance at 31.12.53	Income for 1954	Expenditure 1954	Balance at 31.12.54
	£	£	£	£
Awards Fund	17	–	5	12
British Rainfall Organization Fund	192	156	202	146
Life Composition Fund	25	(a) 249	96	178
Napier Shaw Memorial Fund	43	63	–	106
Royal Society Reprints Scheme	20	12	10	22
Scientific Activities Fund	312	86	189	209
Staff Pension Fund	50	(b) 487	(c) 492	45
Symons Memorial Fund	39	31	16	54
	£698	£1,084	£1,010	£772

(a) Includes £150 received on Redemption of 3% Defence Bonds; (b) Includes £390 received from General Fund, being 15% of salaries paid; (c) Cost of Purchase of £500 4% Trinidad Stock 1973-76.

Examined and found correct.

BALL, BAKER & Co.,
Chartered Accountants.

6 April 1955

VALUATION OF INVESTMENTS AT 31 DECEMBER 1954

Market Value 31.12.53			*Market Value 31.12.54*	
£	£		£	£
		*General Fund**		
	643	£649.1.7 Government of Commonwealth of Australia 4% Registered Stock 1955-70	645	
	203	£200 4% Funding Stock 1960-90	206	
	774	£900 3% Conversion Loan 1961 or after	796	
	903	£1,400 2½% Consols	924	
	1,990	£2,000 4% Government of Commonwealth of Australia 4% Registered Stock 1966-88	2,030	
	–	(a) £1,250 4% Consols	1,206	
4,513				5,807
		Scientific Activities Fund		
	212	£250 3½% War Stock 1952 or after	218	
	86	£100 3½% Conversion Stock 1961 or after	88	
	935	(a) £1,000 4% Consolidated Stock 1952 or after	–	
	–	£1,000 4% Trinidad Government Stock 1973-76	1,030	
1,233				1,336
		Staff Pensions Fund		
	678	£800 3½% War Stock 1952 or after	708	
	258	£300 3½% Conversion Loan 1961 or after	265	
	742	£1,150 2½% Consolidated Stock	759	
	841	£900 4% Consolidated Stock 1957 or after	868	
	–	(b) £500 4% Trinidad Government Stock 1973-76	515	
2,519				3,115
		Life Composition Fund		
	516	£800 2½% Consolidated Stock	528	
	688	£800 3½% Conversion Loan 1961 or after	708	
	1,100	(c) £750 3% Defence Bonds	750	
	327	£350 4% Consolidated Stock 1957 or after	338	
	–	(c) £200 3½% Defence Bonds Conversion Issue	200	
2,631				2,524
		*British Rainfall Organization Fund***		
	636	£750 3½% War Stock 1952 or after	654	
	2,617	£3,097.10.0 British Transport 3% Guaranteed Stock 1978-88	2,710	
	93	£100 4% Consolidated Stock 1957 or after	96	
3,346				3,460
		Napier Shaw Memorial Fund		
	301	£350 3½% Conversion Loan 1961 or after	310	
	374	£400 4% Consolidated Stock 1957 or after	386	
675				696
		Symons Memorial Fund		
	559	£650 3½% Conversion Loan 1961 or after	575	
	187	£200 4% Consolidated Stock 1957 or after	193	
746				768
£15,663				£17,706

(a) Exchange with Scientific Activities Fund for £1,000 4% Trinidad Government 1973-76 Stock purchased during year; £250 4% Consols purchased during year. (b) £500 4% Trinidad Government Stock 1973-76 purchased during year. (c) £150 3% Defence Bonds Redeemed, £200 3% Defence Bonds converted to 3½% Defence Bonds.

* *General Fund* also holds the Freehold of 49 Cromwell Road, S.W.7.

** *British Rainfall Organization Fund* also holds the freehold of 62 Camden Square, N.W.1.

Examined and securities seen.

BALL, BAKER & Co.,
Chartered Accountants.

6 April 1955

Proceedings at meetings of the Society

Annual General Meeting, 20 April 1955

Professor P. A. Sheppard, Vice-President, in the Chair. The chairman conveyed to the meeting the President's apologies for his unavoidable absence, owing to an international meeting abroad.

The minutes of the Annual General Meeting held on 28 April 1954 were confirmed and signed.

In presenting the Report of the Council for the year 1954 the chairman referred to the main items of interest, and in particular to the success of the first two-day Summer Meeting, consisting of scientific symposia, held at Edinburgh, and also to the publication of the *Proceedings of the Toronto Meteorological Conference, 1953*, including which there was a record total of 956 pages printed in the *Quarterly Journal*.

He also referred to the formation of new local Centres at Manchester and Montreal and to the record membership of the Society, which had again exceeded 2,000.

In the absence of the Hon. Treasurer, the Hon. Secretary, Mr. R. G. Veryard, reviewed the financial section, drawing attention to the very satisfactory position, and to the modified form of presentation of Accounts, giving a clearer picture of the year's working.

On questions being invited, the following discussion ensued :

Mr. E. L. Hawke : May we have some information about an item under the heading ' Income ' on p. 9 of the Report – ' sale of old Library books, £21 ' ? Were these books among those listed in the catalogue circulated by the booksellers Messrs. Lange, Maxwell & Springer to the Society's membership last autumn ? It is stated on the cover of the catalogue that the books offered for sale are ' mainly from the Library of the Royal Meteorological Society.' The prices quoted by Lange, Maxwell & Springer for the meteorological books (some 300), listed in this catalogue, amount in all to just over £500, while the prices asked for various sets of serial publications relating to meteorology amount to rather more than £1,400, making a total of nearly £2,000. Turning back to the Report of the Council for 1952, I find that the Society had received £235 from the sale of old books from the library. Might we know, please, what was the aggregate sum paid us by Lange, Maxwell & Springer for the meteorological literature which they purchased from us and are now offering to the public ? May I ask, too, whether it would not have been desirable to insist that the firm should make it clear that the books which they advertise as being from the Society's Library were surplus to our needs, or duplicate copies – if this was in fact the case ? Some of us were distressed to see that at least one volume from the Symons Bequest is included in the catalogue. I wonder if it was wise to dispose of the Society's copy of the very rare first edition (1814) of W. C. Wells's famous *Essay on dew*. Lange, Maxwell & Springer ask £21 for this. Was its value realized here ? Can it still justly be claimed, as was done in our *Descriptive Account* up to a few years ago, that the Society's Library contains ' about 30,000 books and pamphlets, and about 1,200 manuscripts, probably unequalled by any similar collection in the world '?

Dr. G. D. Robinson (*in reply*) : The catalogue to which Mr. Hawke refers contains a number of books not originally in the Society's Library. None of the sets of journals advertised in the catalogue came from our library. Since it is not possible briefly to specify the books sold to them it would be misleading to quote the figure received from Messrs. Lange, Maxwell & Springer. It was of course very considerably more than £21. Perhaps Mr. Hawke will be a little reassured if I detail the procedure adopted in disposal of the surplus library books.

The books sold are in four categories – those containing only data on non-meteorological topics such as terrestrial magnetism; those containing only undigested meteorological data from overseas, which were confirmed to be available in the Meteorological Office library, at Harrow; those duplicated in the library; and those which, on the widest interpretation, could only be described as of no meteorological interest. All books for disposal were seen by at least two Fellows, many by three. No lot was sold until two, or in most cases three, independent offers had been obtained. The Assistant Secretary was most enterprising in advertising these sales to the trade, and much of the material was sold to America. I am sure that he obtained the best possible terms for the Society; the receipts from all sales totalling about £2,000. I cannot, of course, claim that no mistakes were made, but I do not see what else could have been done without an expensive and time-consuming professional valuation of the whole library.

In dealing with duplicates, when the interest was mainly historical or antiquarian, the earlier edition was retained; with modern books the later. The question of disposal of books of the Symons and Druce bequests arose in a few cases and was approved by Council. Where two copies of the same edition were held a ' bequest ' copy was retained. (Since the meeting I have confirmed that in the case of Wells's *Essay on dew*, two copies were retained – a first edition (1814) copy from the Druce bequest, and a copy of an edition of 1866 annotated by L. P. Casella and R. Strachan, presented by Mr. Casella.)

I still think the quotation from the *Descriptive Account* is a fair statement. I would add that the Library, thanks to Mr. R. S. Read, is a good deal easier to use than it was a few years ago, and I would like to see more Fellows using it.

The Chairman then moved that the Report be received and adopted and printed in the *Quarterly Journal*. The motion was unanimously carried.

Mr. P. J. Meade proposed the vote of thanks to the President and Council for the year 1954. This was seconded by Mr. B. Mason.

Professor Sheppard then announced the following awards :

(a) *Symons Memorial Gold Medal* for 1955 :

To Dr. Reginald Cockcroft Sutcliffe, O.B.E. for his outstanding work in the field of synoptic and dynamical meteorology, as given in the following citation :

Dr. Sutcliffe's success as a practical weather analyst and forecaster is matched by his achievements as a practical theorist and teacher. To his fundamental researches we owe a great deal of our knowledge of the effect of temperature patterns on development. His two-layer model of atmospheric motion has become the basis of a practical method of numerical weather forecasting. To his ability and enthusiasm is due in large measure the remarkable success of the Napier Shaw Laboratory. His engaging popular style combined with his seriousness of purpose have made him one of the most successful teachers and there can be few meteorologists whose understanding has not been deepened and broadened as a result of Dr. Sutcliffe's efforts.

(b) *Darton Prizes* for 1954 :

Home Awards : (1) Dr. J. T. Houghton and Dr. A. W. Brewer for their paper ' A new radiometer,' (*J. Sci. Instr.*, **31**, p. 184, 1954). (2) Dr. L. G. Smith, for his paper 'An electric field meter of extended frequency range ' (*Rev. Sci. Instr.*, **25**, p. 510, 1954).

Canadian Awards : (1) Dr. D. P. McIntyre and Mr. R. Lee for their paper ' Jet streams in middle and high latitudes ' (*Proc. Toronto Met. Conf. 1953*, p. 172). (2) Dr. J. S. Marshall for his contribution to his joint paper with Mr. T. W. R. East, ' Turbulence in clouds as a factor in precipitation ' (*Q.J.*, **80**, p. 26, 1954).

In place of the Presidential Address, a discussion meeting was held, on ' The weather of 1954.' The opening speakers were : Dr. J. Glasspoole on ' The weather of 1954 over the United Kingdom,' Dr. A. G. Forsdyke on ' Weather in 1954 in relation to northern-hemisphere circulation patterns,' and Dr. H. L. Penman on ' Weather and farming, 1954.'

The meeting was declared closed at 7.10 p.m.

Symons Memorial Lecture, 27 April 1955

Professor A. C. B. Lovell, Director, Jodrell Bank Experimental Station, University of Manchester, delivered the Symons Memorial Lecture, his subject being ' Radio astronomy and the fringe of the atmosphere.'

Ordinary Meeting, 18 May 1955

The following candidates were declared to have been elected on 20 April by the Council :

FELLOWS

DINSDALE, Frederick Edwin, B.Sc., 14 Brewers Hill Road, Dunstable, Beds.
MOORE, David James, Main Road, Pubersea Bay, Wellington, New Zealand. (*transfer from Student Membership*).
NICHOLAS, Keri Rhys, Idris View, Tyddyn Gwyn, Blaenau Ffestiniog, N. Wales.
ROSE, Sir Charles Henry, Bt., Hardwick House, Whitchurch, Nr. Reading, Berks.
SHAW, Albert Norman, M.A., D.Sc., LL.D., F.R.S.C., 2125 Sunset Road, Montreal 16, P.Q., Canada.
TENCH, Pilot William Henry, Bennebroeker Dreef 19, Bennebroek, Holland.
WILSON, Robert Henry, 86 Gainsford Road, Bitterne, Southampton (*transfer from Associate Membership*).

FOREIGN MEMBERS

CHANDRASEKHAR, Prof. Subrahmanyan, F.R.S., University of Chicago, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.
DE PERCIN, Fernand, Quartermaster, Research and Development Center, United States Army, Natick, Massachusetts, U.S.A.

The following candidates were then declared to have been elected that day by the Council :

FELLOWS

GODLEY, John, 31 Brewer Street, Brighton, Sussex.
HUTCHINSON, Walter Charles Andrew, B.Sc., Ph.D., A.Inst.P., Queen's College, University of St. Andrews, Dundee.
MARSLAND, Albert, B.A., 6 Beverley Terrace, Cullercoats, North Shields, Northumberland.
O'SHAUGHNESSY, Miss Edith, M.A., ' Helenslea,' 4 Trinity Avenue, Westcliff-on-Sea, Essex.

FOREIGN MEMBERS

CRUTCHER, Harold, B.A., M.S., U.S. Weather Bureau, National Weather Records Center, Asherville, N.C., U.S.A.
SIMEON, Prof. Giuseppe, Director, Istituto Universitario Navale, Via Ammiraglio Acton 38, Naples, Italy.

STUDENT MEMBER

DANIELS, Jack, 4541 Main Street, Kansas City, Missouri, U.S.A.

The following papers were then read and discussed :

1. (a) ' Electrostatic field-changes due to lightning discharges.'
   (b) ' The development of lightning discharges,' by E. T. Pierce (*Q.J.*, **81**, pp. 211 and 229).
2. ' Radar echoes from lightning,' by R. F. Jones (*Q.J.*, **80**, p. 579).

Reviews

Selected meteorological papers of Sir Napier Shaw, F.R.S. Selected and arranged by R. G. K. Lempfert and E. E. Austin. London (Macdonald), 1955. Pp. 275; 53 Figs., 28 Plates, 17 Tables. 50s.

It is a pleasant task to welcome the appearance of these Selected Papers and to congratulate the compilers on their achievement. A complete ' collected works ' would have been a delight to those able to acquire it but it was more important to produce a volume within the means of many than a set of volumes to be passed only to the libraries.

About half the book is devoted to ' *The life history of surface air currents*,' doubtless Shaw's most considerable publication where the method of trajectories is used more thoroughly and fruitfully than ever before or since. ' *Principia Atmospherica* ' was something of a curiosity in its time and has power still to provoke new thinking. It would provide a basis for an essay on scientific method. ' *The travel of circular depressions and tornadoes and the relation of pressure to wind for circular isobars*,' another famous paper, is well chosen as representing Shaw in his more methodical and mathematical mood. One is also pleased to have attention drawn once more to ' Units for meteorological measurements ' in an essay with that title reproduced from an early edition of the *Observer's Handbook*. Not all his points are yet gained and the conflict of units is still liable to flare up at any international gathering. Anyone entering this conflict, whichever side he takes, might profitably read Shaw and think it over. Although only a round dozen of Shaw's papers are selected, it was wise while selecting substantial papers of real interest and value to show also the breadth of Shaw's meteorological interests.

The reviewer is one of the generation which barely came under the direct influence of Shaw's gracious personality but which was early enough to read his writings when they were vigorous living works more often on the desk than in the bookshelf. His *Forecasting weather* was the only work in English with any claim to be called a text-book on that subject and after more than forty years we find that Britain has produced no-one to make the attempt again. The different volumes of the *Manual* created much stir as they appeared, providing as they did the very first attempt to cover meteorology as a whole in text-book form, at least in Britain.

In Shaw's time as head of the Meteorological Office, meteorology as a science and as a profession was given the essentials of its modern shape, in no small measure due to his own efforts. Looking back over more than fifty years in the knowledge of what has taken place, perhaps one may make the first hesitant attempt to place Shaw in meteorological history; and in this light his figure grows the larger. When he took over the Office ' none of the members of the staff had had any preliminary scientific training ' (see p. 245). Shaw took over the Office, took over the whole science of meteorology at the same time and went ahead ' on the principle of doing the next thing.' To the reviewer and to many others a most endearing quality of this historical Shaw is his acceptance of meteorology as a comprehensive geophysical and geographical science to be studied empirically before it could be comprehended, a science in its own right and not a mere field for exercises in mathematical physics. That weather forecasting at that time was in the hands of the empiricist who had grown up to the trade was no reason to despise or avoid it. The call was to apply the scientific analysis which was missing and Shaw gave the matter his personal attention. Although a Wrangler ' supported by distinction in physics in the Natural Sciences Tripos ' and with an enviable record of researches in classical physical problems Shaw had time for climatology, for synoptic meteorology, for statistics and applied meteorology, as well as for mathematics and physics. If he did not see the subject very clearly or over simply he saw it whole and the profession has much to be grateful to him for.

There is nothing in this collection which is not a pleasure to read and the excellent editing and printing provide in most, if not all, cases a striking improvement on the originals. The complete bibliography, running to 381 items in chronological order, was no doubt a labour of love, but an exacting one for which we must be grateful. It will be put to many uses and will no doubt allow many a speaker and writer to show evidence of liberal scholarship in circles where the name Shaw needs no title to distinguish it.

R.C.S.

The illumination and polarization of the sunlit sky on Rayleigh scattering. By S. Chandrasekhar and Donna D. Elbert. *Trans. Amer. Philos. Soc.*, Philadelphia. New series, **44**, Pt. 6, 1954. Pp. 86; 11 Figs., 12 Tables. $2.00.

In recent years Prof. Chandrasekhar has greatly advanced the theory of radiative transfer in general, and especially for a plane-parallel medium obeying the Rayleigh (molecular) law of scattering. For such a layer, uniformly illuminated by radiation at a specified angle of incidence and having a specified state of polarization, he has derived a formal solution for the transmitted and reflected radiation fields. The theory can be extended to solve the problem of the scattering layer resting on a surface which reflects according to any given law of reflection, and he has obtained the solution for a surface which reflects according to Lambert's law. Thus, for regions of the spectrum where there are no absorption bands, he has given us a method of calculating the radiation fields at the boundaries of a model which, apart from possible effects of non-Rayleigh scattering due to particulate matter, is a good representation of the earth's atmosphere under cloudless conditions.

The calculation of intensities, etc., from the theory is however by no means simple, and the aim of this paper has been to compute tables of the basic functions occurring in the solutions so that the theory can be applied practically. For the derivation of the theory one must turn to Chap. 10 of Prof. Chandrasekhar's

monograph 'Radiative transfer' (O.U.P. 1950), but the relevant equations for computing the intensit and polarization of the transmitted radiation from the tables of basic functions are also given in this pape The tables can also be used to calculate fluxes directly, but the user will have to turn to 'Radiative transfer for the necessary equations.

The tables are extensive, and users will probably find that they are sufficiently detailed to allow the use in all the appropriate atmospheric problems with but little interpolation. For several optical thicknesse and angles of incidence the basic functions have been supplemented by actual calculations of the transmitte intensities and polarizations, and comparisons have been made with observations in the atmosphere at corres ponding wavelengths. Pressure of time and circumstance have restricted the authors from making detaile comparisons, but it is clear that the theory reproduces faithfully the phenomena of the neutral points an lines which provoked much interest in the latter half of the last century, and which were studied so thoroughl by Dorno in the early part of this century.

It is indeed gratifying that Chandrasekhar's theory has now been followed up by these tables, and man will find them of use as a basis for the discussion of radiation measurements in the atmosphere.

D.B.L.

Collected scientific papers : Meteorology : 1919-1949. Academia Sinica, Peking, 1954. Pp. x, 625; Figs; Tables, Charts. 60s.

With the idea of stimulating research the Academia Sinica decided in 1951 to edit and publish a collec tion of papers embodying the more important contributions of Chinese workers over the period 1919-194 It is stated in the preface that there had been a tendency in the original form of the papers to apply th Norwegian concepts of frontal theory and air masses directly to Chinese meteorological problems wit an insufficient attention to factors peculiar to Chinese topography and climatology. This largely ignore the effect of the Tibetan plateau in producing divergence, confluence, shielding and skipping of the atmos pheric circulation in eastern Asia. A selection was made of 25 of the more important papers in meteorolog which had been published previously in Chinese, German and U.S.A. journals, as well as 6 in th *Meteorological Magazine*, mostly since 1935, and these were edited and partly rewritten. They have bee divided into three sections, with 10 papers on synoptic meteorology, 7 on climatology, and 8 on the gener circulation and dynamical meteorology. The first section includes papers dealing with air masses and col waves in China, the streamline pattern at 7,000 ft over China and typhoons. The second section has paper dealing with the climatology of China and its provinces, historical and seasonal changes, and the relatio of Chinese weather to world oscillations. The third section deals with monsoonal changes, macroscopi turbulence, the general circulation over China and the Far East, the formation of semi-permanent centre of action, and dispersion in the atmosphere. The whole series of papers provides a comprehensive stud which will well repay examination by all interested in the meteorology of the Far East.

R.S.R.

An introduction to climate. By Glen T. Trewartha. New York (McGraw-Hill), 3rd Edition, 1954. Pp. vii 402; 276 Figs., 3 Plates. 50s.

The new edition of a well-known and highly-esteemed book provides a completely up-to-date basi text that must rank as one of the major introductory works in climatology. Although the general organizatio of the material is like that of earlier editions there has been a thorough re-writing throughout to reflect ne ideas on the selection of material that have come from the author's experience in teaching climatology an to bring the treatment into harmony with recent research findings and modern views. In Part I of the boo there is welcome emphasis on dynamic processes and, in the systematic treatment of the several element of climate, there is laid the foundation for a genetic approach to climatology. An indication of the effo made to bring the revision as up-to-date as possible is the inclusion of such items as upper-air waves and th jet stream in this background of physical processes. The theme of Part II is the world pattern of climati types. Although a modified Köppen system of classification is the vehicle for describing the distribution c climates, less emphasis is in fact placed on exact definitions and precise location of climatic boundaries tha upon explanatory descriptions of core climates. The author not only succeeds admirably in the difficu task of presenting clearly the major features of these types but also, and most properly, he demonstrate the significance of weather types as features of climate. For those requiring more advanced or fuller treatmer of topics included there are good selected bibliographies at the end of each chapter. Excellently produce in the familiar smart livery of the McGraw-Hill series in Geography, this comprehensive, authoritativ and well-illustrated book is in all respects in keeping with the author's considerable reputation.

N.P.

Climatic change. Edited with a chapter by H. Shapley, Harvard University Press (distributed by Oxfor University Press in G.B.), 1953. Pp. xii, 318; 59 Figs., 16 Tables. 48s.

The book consists of a series of chapters written by distinguished American authorities on the variou aspects of climatic change. It embraces much of our present knowledge of the evidence for changes in climate both secular and long-term; it also gives an outline of some of the interesting problems concerning the effec of these changes on man and civilization both in historic times and before.

From the other end of the story there are chapters dealing with pertinent astronomical problems, the constitution of the sun, probable changes in the sun's output of energy and the radiation balance of the earth; the pros and cons of theories, such as those concerning the position of the earth in relation to the sun, are analysed. Changes in the circulation of the atmosphere are discussed with evidence from recent meteorological data.

The latter part of the book consists mainly of a brief outline of some of the various pieces of terrestrial evidence for climatic change both within Pleistocene and historic time and far back into geological time, including changes in and migration of fossil flora and fauna and communities of organisms which depend so much on climate for their distribution. Many of the difficulties in the interpretation of palaeoecological problems are expressed.

Each chapter is a useful summary of a certain aspect of the subject with a list of references which includes many of the major works. Each aspect as outlined dove-tails into a whole giving the reader a comprehensive view of the immensity of the subject and impressing upon him the necessity for the many scientists in the numerous fields of investigation, bearing on climatic change, to pool their results and to integrate their conclusions. To this end the book is a success and should be recommended to the student as a basic source of information.

C.D.O.

Man and the winds. By E. Aubert de la Rue. Translated by Madge E. Thompson. London (Hutchinson), 1955. Pp. 206; 26 Plates. 18s.

Throughout the pages of history one can find innumerable examples of the way in which winds have affected the activities of man and the progress of civilization. All over the world man has had to protect himself against the winds, he has given them names and personal characters, and to some extent he has made them serve his own ends. Indeed, it is surprising that so little has been written about the effect of winds on human affairs. In *Man and the winds*, the author makes an emotional rather than a scientific approach to the subject, but he covers almost every aspect. The opening chapters relate to the various types of wind, their causes and general effects – the föhn and the hurricane each get a special chapter – and are followed by chapters on wind as a determining factor of climate and its physiological effects. From then on the reader is given a series of chapters showing how the winds enter into the many fields of human activity – building and architecture, agriculture, engineering, shipping, aviation, etc. After an assessment of the benefits and drawbacks of the wind and a discussion of shelter belts, the author concludes with a chapter on the mythology and legend of wind. The scientific expert may find fault with many of the statements in the book – particularly in the early part. For example, the description of the summer monsoon in India as a south-westerly current which produces rain on the Western Ghats at the same time as a north-easterly current produces rain in Bengal and Assam is hardly correct, and is inconsistent with the description of the monsoons in another part of the book. Also the ' föhn effect ' is not necessarily due to the descent of air over high ground after ascent and precipitation on the windward side – the effect may be due to the descent to the surface of air originally at or near the level of the hill tops. Nevertheless, in spite of its technical deficiencies the book is certainly most interesting, particularly the second half in which the author gives vivid pictures of man's struggle with the winds. There is a useful bibliography but, rather surprisingly, no references later than 1939, are given. The book, which has been translated from French, is well illustrated but a meteorologist might have expected to see a picture of the Tower of the Winds in Athens. The translator has preserved an easy style and the printing is good; at 18s. the price represents current value for money.

R.G.V.

Klima und Wetter der Fischereigebiete West- und Südgrönland. By Dr. Martin Rodewald, 1955. (Beiträge zum Wettergeschehen in den nordeuropäischen Gewässern, No. 3). Hamburg 1955. Pp. 99; 69 maps and diagrams, 4 Tables. Np.

The homely cod serves the meteorologist both as food and as a climatic indicator, since the 2°C isotherm (at the level of the fishing banks) sets a limit to his thriving. Northward movement of this isotherm in the Davis Strait since about 1925 has turned the Greenland cod-fishery, which was formerly negligible, into a major industry yielding annually altogether about half as many tons of cod as the year's landings in Great Britain. German trawlers have been on the scene since 1952, and in this very readable little guide Dr. Rodewald presents and interprets the results of their first three years' weather and ice observations for the benefit of the trawler skippers themselves.

The first lesson is, of course, that in a region subject to such large variations from year to year, the dangers of being misled by a small sample of observations are great. This is illustrated aptly by diversities experienced within the three years. By the unavoidable necessity of grouping months into a few main seasons (e.g., April-May, June-September, October-November, December and January – the round of the year is not quite completed) the author manages to produce valuable detailed maps of the distributions of air and sea temperatures, fog frequency, mean cloudiness, mean wind force, frequencies of strong winds and of gales and of the two dominant wind directions off the south and west coasts of Greenland. The gradients of all these entities are pronounced, and by steaming 50 mi a vessel may reach another régime. The discussion

makes the distributions understandable in terms of well-known atmospheric processes and features of the general circulation, giving due weight to the anomalies which occur in these regions in good and bad years. The reviewer knows of no other work which covers the climatology of Greenland waters in this detail.

An interesting curiosity is the reputation of stranded icebergs as cod traps with good fishing in their lee (in the sense of the water current), and Rodewald suggests a plausible explanation.

British trawlermen may well envy their German fellow fishermen the availability of a weather-forecast service on the fishing grounds, aboard the fishery protection boat '*Meerkatze*' on call by long-range radio telephone. Particularly at the end of the season, in mid-winter, a cold front and a NW'ly outbreak in the Davis Strait after thaw weather may suddenly plunge the vessels and their crews into direst danger with temperatures about — 10°C and rapid icing of the rigging.

The wind-force observations show peak frequencies around 1-3, 6 and 10 Beaufort, though it is clear that the west-Greenland fishing grounds enjoy a less stormy climate than farther south or off Iceland. The fierce NE'ly storms of east and south-east Greenland are without doubt the worst hazard of the Arctic fisheries, especially when accompanied by much drifting ice, low temperatures and snow with bad visibility. The reviewer's one criticism concerns these NE'ly storms and the SE. gales off the west coast of Greenland: it might be made clearer that to steam towards the Greenland coast or the coastal ice-belt in these situations is dangerous. This is because of a perverse, and perhaps little known, rule regarding mountainous coasts of glaciated lands : when pressure is relatively high over the inland ice and a gale is blowing along the coast there is no safe shelter near the coastline where the fiercest winds of all are found. This is particularly the case when an encroaching warm front or occlusion lies off shore. Lighter winds are to be sought farther out to sea. This paradox may be a vital one to know in a region where, despite the drastic risks of sudden changes, good fishing weather is commoner than bad and coastal stations enjoy still air at anything from 10 to 55 per cent of all the observation hours.

The future of the climatic amelioration is wrapped in a big question mark, as suitably discussed by Rodewald. The cod has come, and gone, in Greenland waters before!

H.H.L.

Elements of statistical mechanics. By D. ter Haar. New York (Rinehart and Company, Inc.), 1954. Pp. xix + 468; 56 Figs., Tables. 45s.

In this textbook for post-graduate students and research workers the author has on the whole succeeded in his attempt to combine a description of the main theoretical concepts and methods with an account of many of the successful applications of statistical mechanics.

The first seven chapters are devoted to basic theory, starting with the Maxwell velocity-distribution and carrying the reader as far as quantum-mechanical grand ensembles by fairly comfortable stages. In order to achieve this, the author has had to economise in space; those who intend to use the book for reference purposes may experience a little difficulty in discovering the exact physical meanings of the symbols used within any particular section to which they may turn.

Appendix I is also devoted to theory, being a detailed and critical review of the basic principles as they affect the validity of Boltzmann's H-theorem and the ergodic theorem.

Chapters VIII-XV and appendices II-VI deal principally with applications. There are chapters on the equation of state; condensation phenomena; electron theory of metals; semiconductors; cooperative phenomena; nuclear statistics; origin of the elements; rubber elasticity. Appendices deal with : irreversible processes; the third law of thermodynamics; the Darwin-Fowler method and applications; intermolecular forces; relativistic statistics. It is difficult to see why some of these topics have been relegated to appendices.

Every chapter and appendix in the book is followed by very full bibliographical notes on its subject-matter; these should enormously increase the value of the book to research workers. Indeed, since most of the applications are dealt with in a fashion which, although completely adequate for the student, is (so far as the reviewer can judge) probably too cursory for those whose main interest they are, the bibliographies are likely to prove the most valuable feature.

The book is very carefully written; the reviewer has found only two lapses that are at all serious : on p. 108 there are fairly obvious errors of sign, while at the top of p. 368 it is implied that $10^{-7\cdot5\times10^{18}}$ sec (or years; there is some confusion, but the difference is scarcely important) is an 'ordinary time interval'.

A.R.C.

Analog methods in computation and simulation. By Walter W. Soroka. London (McGraw Hill), 1954. Pp. xii, 390; 274 Figs., Tables. 53s. 6d.

This book gives an exceedingly clear and up-to-date account of the types of analogue computing-machine available for the numerical solution of a wide variety of physical and engineering problems. It is a very useful reference book but it will only be of interest to those few meteorologists who are concerned with obtaining numerical results to complicated problems, and to those it will probably only be of academic interest as nearly all of the large meteorological computing problems can be better done by electronic digital computers than by analogue computers.

The first two chapters of the book deal with the use of mechanical, electrical and electronic computing elements for carrying out simple arithmetic operations. Subsequently it is shown how these elements can be assembled in order to solve simultaneous linear algebraic equations and non-linear algebraic equations.

Chapters 5 and 6 are devoted to the mechanical and electronic differential analysers. These differential analysers have been developed primarily for the solution of ordinary differential equations, but methods are given for adapting them to solve certain types of partial differential equations.

Chapter 7 indicates ways in which the well-established correspondence between the equations of motion of dynamic (mechanical) systems, electrical circuits, and acoustical systems can be used to develop analogue machines.

One of the most interesting parts of the book is the chapter which describes the use of electrical circuits to form analogues of finite difference equations, including those corresponding to the Laplace equation, the Poisson equation and certain other special forms of linear and non-linear ordinary and partial differential equations.

F.H.B.

Wladimir Köppen; ein Gelehrtenleben. By Else Wegener-Köppen. Stuttgart (Wissensch. Verlag.), 1955. Pp. 195, 13 Figs., Portrait.

Wladimir Köppen, who died on 22 June 1940 in his 93rd year, was something of a phenomenon for the long continuance of his activity. This account of a long life's work is of great interest in bridging the gap between the day before yesterday and today. The first part, up to 1903, was written by Köppen himself from his own recollections; the subsequent pages were compiled by his daughter.

Köppen was of German descent, but he was born in Russia, where his father was employed, and his early interests were in Russian climatology and their bearing on biology, but he returned to Germany after the death of his father, and attended the University of Heidelberg. In Germany he met many of the founders of modern meteorology – men like Helmholz, Hann, Kämtz, Wild, Jelinek and Fritsch, whose names today have passed into history, but were household words among meteorologists even forty years ago. This directed his work towards climatology, and his first paper, published in 1868, dealt with the observations of temperature at Karabagh in the Crimea. His doctorate followed in 1870, with a dissertation on warmth and plant growth. After that papers followed thick and fast. In 1872-3 he returned to St. Petersburg as assistant, but in 1875 he removed to the Deutsche Seewarte where he remained until 1919.

It would take too long to follow all Köppen's activities during these fruitful years. He played an active part in the first upper-air ascents in Germany by kite and pilot balloon, but he is best known for his work on the classification of climates, first published in 1900. At the end of the first world war Köppen was 72, and he retired, not to sink into idleness but to devote his energies to writing and research. In 1926 he began, in collaboration with R. Geiger, the editorship of the great *Handbuch der Klimatologie*, actually writing several parts himself and stepping into the breach when the Russian collaborator fell out. He continued writing almost to the day of his death.

C.E.P.B.

Obituary Notices

Mr. B. W. LEE, B.A.

We have to record the sudden death at Lincoln on 21 January 1955 of Basil Westcott Lee. He wa born in London in 1908 and graduated from Keble College, Oxford. In 1931 he became tutor at Victori College, Alexandria, and two years later transferred to Gordon College, Khartoum. From 1938 to 1940 h was education officer for the Sudan provinces of Kordofan and Darfur. He then saw war service in variou capacities, ultimately receiving an appointment as senior civil-affairs officer at Acchele Guzai and Massawa with the rank of major. On release from the British Army in July 1945 he returned to educational administratio at Khartoum, and at the time of his retirement in 1954 held the post of deputy director of education for th Sudan. Since his schooldays Mr. Lee had taken an enthusiastic interest in weather study. Before goin abroad he kept meteorological records at Anwick, Lincolnshire, where his father was vicar, and when h came back to England, about a fortnight before he died, it was his intention to set up a climatological statio He had been a Fellow of this Society since 1938. A memorial service for Mr. Lee was held in the cathedra of All Saints, Khartoum, on 27 January 1955.

A. B. Tinn

Mr. J. R. PIKETT, F.R.A.S., F.P.S., M.Inst.Pet., M.R.S.G.B.

Mr. John Robert Pikett, a Fellow of the Society since 1943, died suddenly at the age of 51 on 6 Februar 1955. He was a man of wide interests : in addition to meteorology these included observational astronomy electrical research and various branches of chemistry. During the 1939-45 war, and for some years after Mr. Pikett served as technical consultant both to the chief quartermaster of the U.S. Army and to the are petroleum officer of the U.S. Navy in London. Later he became senior executive officer at the South Ruislip headquarters of the U.S. Air Force. Among his other war-time activities was the development of a metho of weather forecasting which he submitted for consideration by the Air Ministry.

E.L.H.

Corrigendum to this issue

W. H. Munk, p. 322, line 10 and p. 327, line 25—

Since this paper was sent to press, the author has asked for the addition of the following explanator note :

In Section 3 the term ' pressure tappings ' should have been used instead of ' pitot tubes.'

Jeffreys has assumed a pressure distribution proportional to $(U - C)^2$ (Eq. 9); I took $U(U - C)$. T be consistent one must re-evaluate the sheltering coefficient. In the former case it is proportional to th minimum value of $U^3 C^{-1} (U - C)^{-2}$, or 27/4 for $C = \frac{1}{3} U$. In the latter case it is proportional t $U^2 C^{-1} (U - C)^{-1}$, or 4 for $C = \frac{1}{2} U$. Jeffreys's value of 0·27 is then changed to 0·27 (16/27) = 0·16, unde the present hypothesis, and the values of $s\,b$ in Section 6 have to be reduced in the same ratio.

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SELECTED AND ARRANGED BY

R. G. K. LEMPFERT, C.B.E., M.A., F.R.Met.S.
Formerly Assistant Director of the Meteorological Office, sometime President of the Royal Meteorological Society

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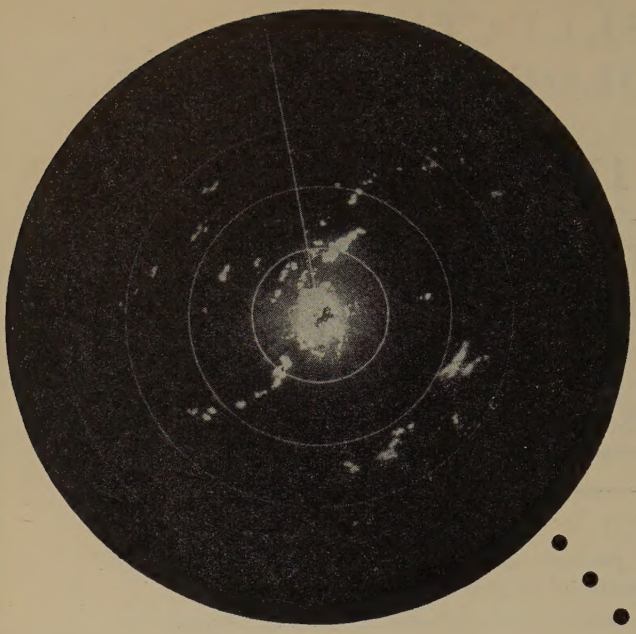

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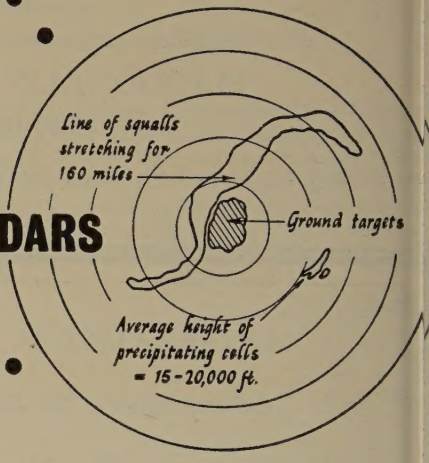

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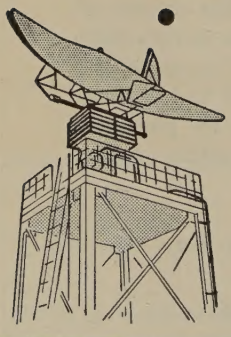

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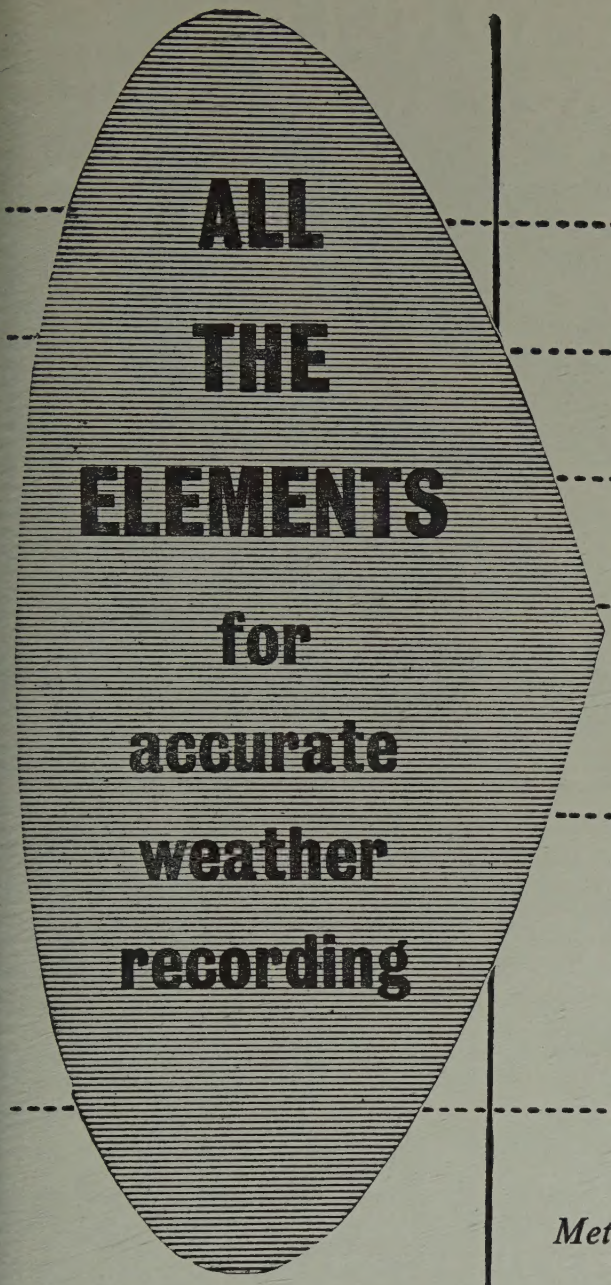

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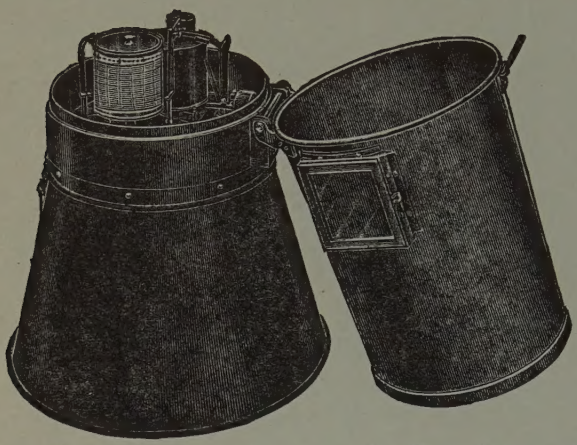

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